

# Air-Water Rockets

Air-water rockets are good fun to make and you can learn a lot from studying how they work. The following is a brief explanation of how they work by Peter Nielsen, Associate Professor in Civil Engineering, The University of Queensland. A spreadsheet designer/simulator based on the theory is also available on <http://uq.edu.au/civeng>.



## What does the thrust force $F_{thrust}$ depend on?

The thrust force  $F_{thrust}$  from any rocket exhaust is the mass flow  $\dot{m}$  times the velocity  $u_{ex}$  of the exhaust relative to the rocket

$$F_{thrust} = \dot{m} u_{ex} = \rho A u_{ex}^2 \quad (1)$$

where  $\rho$  is the density of the exhaust fluid, and  $A$  is the outlet cross section.

For air-water rockets, *Bernoulli's equation* gives  $u_{ex} = \sqrt{2P^+ / \rho_w}$  where  $P^+$  is the internal over-pressure and  $\rho_w$  the density of water. The thrust force is then, remarkably

$$F_{thrust} = 2P^+A \quad (2)$$

which is twice the force required to hold the water back before start.

## Finding the enclosed air volume $V$

The pressure inside the rocket decreases as the water escapes and the air volume  $V$  increases. The expansion process is complicated by the fact that some water vapour condenses and some heat is received from the surroundings. However, a reasonably simple model is obtained by assuming the process to be like the adiabatic expansion of an ideal gas. In that case:

$$P = P_o (V_o/V)^{1.4} \quad (3)$$

if the volume and absolute pressure at take-off are  $V_o$  and  $P_o$ .. The rate of increase of the enclosed air volume due to the exhaust flow is then

$$u_{ex}A = \sqrt{\frac{2P^+}{\rho_w}}A = \sqrt{\frac{2[P_o(V_o/V)^{1.4} - P_a]}{\rho_w}}A, \quad V < V_{tot} \quad (4)$$

$P_a$  is the atmospheric pressure and  $P^+ = P - P_a$ . The air volume at a slightly later time  $t + \delta_t$  can then be calculated as

$$V(t + \delta_t) \approx V(t) + \delta_t u_{ex}A = V(t) + \delta_t \sqrt{\frac{2[P_o(V_o/V)^{1.4} - P_a]}{\rho_w}}A, \quad \text{for } V < V_{tot} \quad (5)$$

These formulae hold until the water is used up and  $V = V_{tot}$ .

## Finding the mass $m$ of the rocket

If the solid parts of the rocket have the mass  $M_s$ , and the total volume of enclosed air and water is  $V_{tot}$ , the mass of the rocket as function of time is

$$m(t) = M_s + M_a + \rho_w(V_{tot} - V) \quad \text{for } V < V_{tot} \quad (6)$$

$M_a$  is the mass of enclosed air which can be estimated as

$$M_a = \frac{P_o V_o}{288 T_o} \quad (7)$$

where  $T_o$  is the starting temperature in degrees Kelvin.

## The temperature of the enclosed air

As the enclosed air expands, it cools down considerably. Under adiabatic conditions with no heat received from the surroundings or from condensing water vapour, it would vary as

$$T = T_o \left( \frac{P}{P_o} \right)^{0.286} = T_o \left( \frac{V}{V_o} \right)^{-0.4} \quad (8)$$

### The drag force from the surrounding air

As the rocket moves through the air it is slowed down by air drag. The magnitude of the drag force  $F_D$  depends on the density of the surrounding air  $\rho_{air}$  ( $\approx 1.2 \text{ kg/m}^3$ ), on the speed  $u$  of the rocket, its cross sectional area  $A_R$  and on a drag coefficient  $C_D$ . The drag coefficient is of the order 1.0 for non-streamlined objects but may be as low as 0.1 for very streamlined objects. The formula for the drag force is

$$F_D = \frac{1}{2} \rho_{air} C_D A_R |u| u \quad (9)$$

### Finding the acceleration $a$ of the rocket

If the rocket is moving vertically upwards against gravity, Newton's Second Law gives its acceleration  $a$  by

$$ma = F_{thrust} + F_D - mg = 2P^+ A - \frac{1}{2} \rho_{air} C_D A_R |u| u - mg \quad (10)$$

where  $g$  is the acceleration due to gravity,  $9.8 \text{ m/s}^2$  at the surface of planet Earth.

Using the expression (6) for the instantaneous mass and rearranging, this means

$$a = \frac{2P^+ A - \frac{1}{2} \rho_{air} C_D A_R |u| u}{M_s + M_a + \rho_w (V_{tot} - V)} - g, \quad \text{for } V < V_{tot} \quad (11)$$

After the water is used up ( $V \rightarrow V_{tot}$ ), air is expelled at great speed. Initially (for  $P^+ > 1.89 P_a$ ), this gives  $F_{thrust} = 0.89 P^+ A$ , and the air is escaping at the speed of sound.

### Finding the speed $u$ and height $h$ of the rocket

The formulae above can be used to calculate speed  $u$  and height  $h$  of the rocket in a spreadsheet. You start with  $u=0$  and  $h=0$  at time zero and then update  $u$  and  $h$  after each time increment  $\delta_t$  using

$$u(t + \delta_t) = u(t) + \delta_t \frac{[a(t) + a(t + \delta_t)]}{2} \quad (12)$$

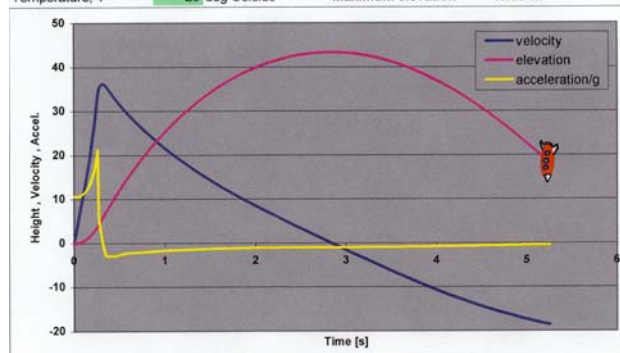
$$h(t + \delta_t) = h(t) + \delta_t \frac{[u(t) + u(t + \delta_t)]}{2}$$

In the spreadsheet you can try different parameters and optimize by trial and error. For example, how much water to put in to get the best result.

It will also help you find the best bottle dry mass ( $M_s$ ). You will find that the lightest bottles are not the best!

You can also experiment with jet propulsion of aircraft.

Simulation of air-water rockets, fired vertically.				Parameters may be set in the green cells			
For explanations, see the "Word" file "Rockets".							
Solid mass =	120 grams =	0.12 kg	Cross section, A=	0.01 m <sup>2</sup>			
Total volume =	1250 ccm =	0.00125 m <sup>3</sup>	Drag coeff, Cd=	0.3			
Exhaust diameter=	0.8 cm → A=	6.4E-05 m <sup>2</sup>	Air density =	1.2 kg/m <sup>3</sup>			
Filling ratio=	0.35		Water density =	1000 kg/m <sup>3</sup>			
Start overpressure P+=	5 atm → Po=	6.1E+5 Pa	g=	9.8 m/s <sup>2</sup>			
Atm. pressure Pa=	1013 hPa						
Temperature, T=	25 deg Celsius						
			Maximum elevation	43.36 m			



time count	time [s]	V/Vtot [-]	P/Pa [-]	mass [kg]	Thrust [N]	ration [m/s <sup>2</sup> ]	speed [m/s]	Plot Speed [m]	height [m]	Drag [N]
0	0	0.65	6.00	0.563	64.4	104.6	0.00	0	0.00	0
1	0.010	0.686	5.80	0.543	61.8	104.1	1.04	1.04	0.01	0.0
2	0.020	0.683	5.60	0.523	59.3	103.7	2.11	2.11	0.02	0.0
3	0.035	0.706	5.35	0.493	56.0	103.7	3.67	3.67	0.06	0.0



For this type of aircraft you will find that the thrust force has to be reduced by reducing  $A$ . Otherwise, the wings get ripped off the plane at take-off.