

# Development of a mechanistic model for the prediction of slug length in horizontal multiphase flow

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## Synopsis

A mechanistic model for the prediction of slug length in multiphase flow is presented based on a unique concept involving the Froude number. It is shown that the Froude number in the liquid film ahead of the slug is greater than unity. It decreases to values less than unity inside the mixing zone of the slug and then gradually increases within the body of the slug. The slug tail occurs as the Froude number tends to unity once more. Agreement with experimental data is good. The model also closely predicts the data of other researchers in large diameters pipes.

## Notation

- $a_{GF}$  = area occupied by the gas above the stratified layer of liquid
- $a_{LF}$  = area occupied by the stratified layer of liquid
- $S_L$  = perimeter of liquid contact with wall over which shear stress acts.
- $S_G$  = perimeter of gas contact with wall over which gas phase shear stress acts.
- $S_i$  = width of gas-liquid interface.
- $\tau_{wL}, \tau_{wG}$  = wall shear stress for liquid and gas respectively
- $\tau_i$  = shear stress at gas-liquid interface.
- $\rho_L, \rho_G$  = density of liquid and gas respectively
- $\theta$  = pipe inclination (small values, close to horizontal).
- $f_i$  = interfacial friction factor
- $f_G$  = gas phase friction factor
- $v_t$  = translational velocity of the slug.
- $v_{LF}$  = liquid film velocity
- $h_{EF}$  = effective film height ahead of the slug

$u_L$	=	average velocity of the liquid
$p_{LF}, p_s$	=	pressures in the liquid film and the slug
$a_{LF}, a_{LS}$	=	areas of the liquid film and the liquid in the slug
$x_o, x_1$	=	points in the liquid film and slug respectively
$\rho_L$	=	density of liquid
$\bar{a}_{LF}, \bar{a}_{LS}$	=	fraction of area occupied by the liquid in film and slug
$\bar{v}_{LF}, \bar{v}_{LS}$	=	average liquid velocity in film and slug
$\bar{a}_{GF}$	=	fraction of area occupied by the gas over the liquid film.
$p_{GF}$	=	pressure in gas pocket over liquid film
$F_D$	=	drag force exerted by the gas bubbles on the liquid.
$p'_o$	=	$p_{LF} - \rho_G \zeta'_o$ , and $\zeta'_o$ is the depth from the surface of the center of pressure in the gas over which $p'_o$ acts.
$v_{GS}, v_{GF}$	=	gas velocity in the slug and gas pocket over liquid film
$\bar{a}_{GS}, \bar{a}_{GF}$	=	fraction of area occupied by gas in slug and by gas pocket over liquid film
$h_F, h_j$	=	hydrostatic head corresponding to the pressures $p_o, p_1$
$v_{LS}$	=	the effective average liquid velocity in the slug at any point
$h_j$	=	the effective "height" of the jump.
$q_l$	=	volumetric flow rate of liquid as part of slug
$v(y)$	=	velocity at distance $y$ from the pipe wall
$v_o$	=	centerline velocity
$\delta$	=	thickness of the boundary layer
$\bar{h}$	=	nondimensional height from the bottom of the pipe
$a, b$	=	nonlinear regression coefficients for void fraction distribution
$\alpha_{avg}$	=	average void fraction over a cross sectional area
$l_m$	=	The length of the mixing zone
$m, c$	=	linear regression coefficients for the mixing zone
$u_o$	=	centerline velocity in a moving coordinate system = $v_t - v_o$
$f$	=	the Fanning friction factor
$C$	=	constant in friction factor calculation = 0.046
$n$	=	exponent in friction factor calculation = - 0.2
$C_D$	=	drag coefficient
$d_b$	=	diameter of bubble
$v_r$	=	measured rise velocity of bubbles in the slug body
$Re_b$	=	bubble Reynolds Number

## 1 INTRODUCTION

Multiphase flow has become a technology central to oil and gas production today. With the development of "full well stream transportation" systems (Fairhurst, 1988), both from remote sites, such as Alaska, and in offshore production, the emphasis is on the transport of all components of the produced fluids from the well over long distances to central processing facilities. An accurate prediction of multiphase flow characteristics is essential for the design and economical operation of these systems.

Several flow regimes occur in multiphase flows, including, stratified, slug, and annular flows. The production rates from the wells are such that these multiphase flow pipelines are expected to be in slug flow at some time in their lives. This is a highly turbulent flow regime, leading to increased pipe damage from internal corrosion and mechanical impacts. This is related to the slug length. It is therefore, important to obtain a detailed, mechanistic understanding of slug flow characteristics, and to determine the overall slug length.

Mathematical models have been developed that describe the relationship between different variables as knowledge of slug flow features have increased over the last decade. However, a detailed understanding of the motion of gas within the slug, and the distribution of phases in the different zones of the slug is not known. This is essential information that may be used to develop a complete mathematical model to predict slug length.

This paper describes a mathematical model that has been developed to predict slug length utilizing the phase distribution and velocity profiles within the slug. The experimental techniques have been described elsewhere (Gopal and Jepson, 1997, 1998a,b).

## 2 BACKGROUND

Figure 1 shows the profile of a slug. Waves form on the liquid film, that grow to bridge the pipe. This causes the liquid to be accelerated by the gas. As the slug front moves through the pipe, it overruns the slow moving liquid film ahead of it and accelerates it to the velocity of the slug. A mixing vortex is created in this process. This leads to a scouring mechanism on the pipe wall with high rates of shear. Also, as the liquid is assimilated by the slug, a considerable amount of gas is entrained (Jepson, 1987). This leads to the creation of a highly frothy, turbulent region behind the slug front called the mixing zone.

Beyond the mixing region of the slug, the level of turbulence is reduced, and buoyancy forces move the gas towards the top of the pipe. The cross sectional area available for liquid flow increases, a boundary layer develops, and the liquid velocity decreases. This is the slug body. Eventually a point is reached where the liquid velocity is no longer sufficient to sustain the bridging of the pipe, and the slug body is curtailed. This is called the slug tail. The liquid velocity decreases in the liquid film, its height rebuilds with waves forming on its surface, and the next slug is initiated.

Dukler and Hubbard (1975) published the first realistic mechanistic model for slug flow characteristics. They established fundamental equations that could predict several slug flow characteristics. The agreement with experimental data was good and a better understanding of the mechanisms was achieved. However, many parameters, such as slug frequency and the void fraction within the slug, were required to complete the calculation. Also, their definition of the mixing zone is not adequate (Gopal and Jepson, 1998b). The slug lengths in their studies varied from 12 to 25 pipe diameters.

Nicholson et al. (1978) found a non-zero gravity induced drift velocity, even for horizontal pipes, and determined that this velocity needed to be incorporated in the calculation of the slug translational velocity. They modified and extended the model of Dukler and Hubbard to apply

to the entire intermittent flow regime. The slug lengths in their case varied from 10 to 60 pipe diameters. It is to be noted that the model ignores any slip between gas and liquid within the slug. It has been found (Jepson, 1987) that for large diameter pipes, this is not true for high slug velocities.

Maron et al. (1982) derived a model for slug flow based on periodic distortion of the hydrodynamic boundary layer followed by a recovery process. As the slug front overruns the liquid film, the boundary layer is destroyed by the mixing eddy. At the end of the mixing zone this boundary layer begins to redevelop. The model considered two separate types of slug, one with aeration and the other without. In the first case, the entrained gas bubbles in the slug leave the boundary layer region due to buoyancy effects and tend to agglomerate in the upper portion of the pipe. The slug length is the distance required for complete separation of the gas from the liquid. In the second case, the slug length is given by the distance required for the boundary layer to fully develop and reach the center of the pipe. They developed their boundary layer analysis in a coordinate system moving with the slug front and introduced a one-seventh power law model to describe the velocity profile within the boundary layer. They showed that the model could be applied to predict pressure drop for a wide quasi-steady frequency range of slugs. However, stability analysis indicated that the slug pattern stabilized over a narrow frequency range corresponding to a minimum pressure drop. It is to be noted that no information about the pipe diameter or working fluids were given.

Dukler et al. (1985) applied the concepts developed by Maron et al. (1982) and formulated a generalized model for the prediction of the minimum stable slug length for horizontal and vertical slug flow. The model utilized the velocity profile developed by Maron et al. (1982) in the boundary layer, and combined this with an inviscid potential core. An assumption was made that a flat velocity profile resulted at the end of the mixing zone. The velocity at the center was allowed to decrease due to frictional effects predicted by a Blasius-type equation. The slug length was predicted by the distance required for the complete development of the boundary layer. The results of the model were applied to 5 cm I.D. vertical and horizontal, and 3.8 cm horizontal pipes, with air and water as the working fluids. It was found that the experimental results were bounded between the value predicted by the model and twice that value. It should be noted that the model ignored the contribution of gas to the slug characteristics, and hence, the slug length. Their slug length could have been the mixing length of the slug (Gopal and Jepson, 1998a).

Kouba (1986) formulated a model to account for both liquid and gas phase distribution as well as velocity distribution within the slug. By considering a mass balance between the slug and the liquid film, he was able to formulate a generalised expression for liquid film velocity ahead of the slug. From observations in a 7.5 cm I.D., 418 m long pipeline, using kerosene and air as the working fluids, he concluded that the drift velocity,  $v_d$ , was significant even for horizontal slug flow. Utilising the theory of shearing flow over a wavy boundary developed by Benjamin (1968), he developed an expression for the drift velocity as a function of pipe diameter. The incorporation of the drift velocity improved slug length predictions by twenty percent. The range of slug lengths in this study were twenty-five to one hundred pipe diameters. This was due to the length of his pipeline, which was 418 m. There was an effect of gas expansion over this length resulting in an acceleration of the slugs and an increase in slug length.

Figure 2 describes the physical model for stratified flow developed by Taitel and Dukler (1976). The gas and liquid flow in stratified layers within the pipe. They used this model along with other parameters to determine the transition to other flow regimes. This model is used as the foundation for predicting the liquid film height in slug flow in this study. From a consideration of a momentum balance in the liquid and gas phase the following equation is obtained:

$$\tau_{WG} \frac{S_G}{a_{GF}} - \tau_{WL} \frac{S_L}{a_{LF}} + \tau_i S_i \left( \frac{1}{a_{LF}} + \frac{1}{a_{GF}} \right) + (\rho_L - \rho_G) \quad (1)$$

Equation (1) can be used to predict the liquid film height in slug flow as well. Details are given elsewhere (Gopal and Jepson, 1998b).

Jepson (1989) presented a physical model for the prediction of transition to slug flow. The model assumes that the slug is formed as a result of a hydraulic jump propagating along the pipe. He defined a dimensionless Froude Number for the film ahead of the slug using a comparison between slugs and hydraulic jumps. Using equations of continuity and momentum conservation at a condition where the jump just touches the top of the wall of the pipe, he provided the necessary conditions for the existence of slugs. This theory forms the foundation of the proposed model for slug length in this paper.

Several investigations on slug flow have been conducted for large diameter pipes. Crowley et al. (1986) conducted slug flow studies in 17 cm pipes, with water and glycerine (viscosity 400 cP) for the liquid phase and Freon for the gas phase at densities one to twenty times that of air. They found that the translational velocity of the slug was predicted for all cases using the drift flux model. There was a negligible effect of the gas density on the slug velocity. However, there was a large effect of liquid viscosity. The slug velocities for glycerine were fifty percent greater than water for the same conditions. It was found that the interfacial friction factor was about ten times the gas phase friction factor in this case. The slug lengths ranged from 0.5 m to about 3.5 m.

Jepson and Taylor (1993) found that there was an increase in slug length with an increase in pipe diameter. Below a gas superficial velocity of 5 m/s, the slugs appeared to be growing. This indicates that there may be gas expansion in large diameter pipes, or a significant drift velocity causing the the slug to grow.

Scott et al. (1986) developed correlations for predicting slug length in large diameter pipes using data from 30, 40, 50, and 60 cm I.D. pipes in oil fields in Prudoe Bay, Alaska. They found that in these pipelines there was an additional factor in slug length analysis. They termed this factor the "long term growth". This was related to gas expansion within the pipeline due to pressure changes.

Fairhurst (1988) discussed several important issues related to slug flow in large diameter pipes. From an analysis of the data gathered in the 30, 40, 50, and 60 cm pipes in Alaska, he concluded that generally available design methods for slug flow failed to predict the behavior of real oil and gas pipelines, and that future work should be tested with field data.

Recently statistical characterizations have been developed to describe the variations of slug lengths in pipes (Nydal et al., 1992, Barnea and Taitel, 1993). A log-normal distribution of slug lengths was found to be applicable by Nydal et al. (1992), while Barnea and Taitel (1993) found that the mean slug length was 1.5 times the minimum stable slug length and the maximum length was about 3 times the minimum stable length.

### 3 MATHEMATICAL MODEL

A mathematical model is developed to relate the variables involved in slug flow. Figure 4 shows a schematic flow chart of the model development.

Step 1 involves the calculation of the Froude number in the liquid film ahead of the slug. Knowing the input superficial velocities of liquid and gas, the pipe diameter, and the fluid properties, the liquid film height,  $h_{LF}$ , and the average velocity of the liquid in the film,  $v_{LF}$ , are calculated. Using the superficial liquid and gas velocity, the slug translational velocity,  $v_t$ , is also calculated. These three variables,  $v_t$ ,  $v_{LF}$ , and  $h_{LF}$ , are then used to calculate a film Froude number.

Step 2 involves the calculation of the Froude Number in the slug. Once the film Froude number is known, the length of the mixing zone in the slug,  $l_m$ , and the effective height of the slug,  $h_j$ , are calculated. Next, knowing the superficial gas and liquid velocities, the slug velocity  $v_s$ , is computed. Then, using the void fraction distribution and the velocity profile equations, the effective average liquid velocity in the slug,  $v_{LS}$ , is calculated. Finally, using  $v_s$ ,  $v_{LS}$ , and  $h_j$ , the Froude Number in the slug is determined.

In step 3, the decrease of the liquid velocity in the slug is calculated using pressure drop relations. The decrease in the effective average liquid velocity in the slug is then related to the increase in Froude number in the slug. The point where the Froude Number equals unity is found, and the slug length is calculated.

#### 3.2 Liquid Film height

As mentioned previously, the liquid film height is predicted using a modified Taitel and Dukler (1976) model. The modification of the interfacial friction factor as given by Equation (1) is insufficient to predict the liquid film height in slug flow. It has been found that the interfacial friction factor needs to be increased over those predicted by Equation (7), due to the presence of large three dimensional waves on the liquid film. Details of this explanation can be found elsewhere (Gopal and Jepson, 1998b). Hence it was estimated as a constant multiplied by the gas phase friction factor as follows:

$$f_i = k_1 f_G \quad (2)$$

where,  $k_1 = 50$

#### 3.2 Liquid Film Velocity

It is very difficult to estimate the liquid film velocity, since it is not constant between slug. The liquid drains from the rear of the slug and mixes with new incoming liquid. Further, at high

gas velocities, roll waves are formed on the liquid surface that can also affect the liquid film velocity. In this study the liquid film velocity was estimated to be, on average, equal to the liquid superficial velocity. The details are given elsewhere (Gopal and Jepson, 1998b). This model sets:

$$v_{LF} \approx v_{SL} \quad (3)$$

This is used in the model for slug length prediction

### 3.3 Film Froude Number

It has been shown that slugs are hydraulic jumps (Kouba and Jepson, 1987) and their strengths may be determined by the Froude Number ahead of the slug. The Film Froude Number,  $Fr_f$ , is defined as follows:

$$Fr_f = \frac{v_t - v_{LF}}{\sqrt{g h_{EF}}} \quad (4)$$

The details of the Film Froude number derivation are given in a previous paper (Gopal and Jepson, 1998b).

#### Pressure Relationship in Slugs

In a coordinate system moving with the slug front, the momentum equation becomes the same as that for a hydraulic jump (Jepson, 1989). This is given for channel flow by Stoker (1957) as:

$$\frac{d}{dt} \int_{x_s}^{x_t} \rho_L a_L u_L dx = \int_{a_{LF}} p_{LF} dA - \int_{a_{LS}} p_s dA \quad (5)$$

In a coordinate system moving with the slug front, the relative velocities of the fluids in the slug and in the film are described by  $(v_t - v_s)$  and  $(v_t - v_{LF})$  respectively, and the slug front itself becomes stationary. Under such a condition, following Stoker (1957), Equation (5) can be integrated to give:

$$L \left[ \bar{a}_{LS} (v_t - v_{LS})^2 - \bar{a}_{LF} (v_t - v_{LF})^2 \right] = p_{LF} \bar{a}_{LF} - p \quad (6)$$

A momentum equation for the gas phase can be similarly written:

$$G \left[ \bar{a}_{GS} (v_t - v_{GS})^2 - \bar{a}_{GF} (v_t - v_{GF})^2 \right] = p'_o \bar{a}_{GF} \quad (7)$$

Adding equations (6) and (7) and neglecting the terms involving  $\rho_G$ , since  $\rho_G \ll \rho_L$  we get:

$$\rho_L (\bar{a}_{LS} (v_t - v_{LS})^2 - \bar{a}_{LF} (v_t - v_{LF})^2) = p_{LF} \quad (8)$$

Next, dividing throughput by  $\rho_L g$ , the following expression is obtained:

$$\frac{(v_t - v_{LS})^2}{g} \bar{a}_{LS} - \frac{(v_t - v_{LF})^2}{g} \bar{a}_{LF} = \frac{p_{LF} - p_{LS}}{\rho_L g} = h \quad (9)$$

This can then be rearranged, after some manipulation, to give:

$$\frac{(v_t - v_{LS})^2}{gh_j} \frac{h_j}{h_F} \bar{a}_{LS} - \frac{(v_t - v_{LF})^2}{gh_F} \bar{a}_{LF} = 1 - \frac{h_j}{h_F} \quad (10)$$

The second term in Equation (10) provides a basis for the definition of a Froude number in the slug. The term  $(v_t - v_{LF}) / \sqrt{gh_F}$ , is similar to the film Froude number defined in Equation (4)

In open channel flow, the film Froude Number is defined as:

$$Fr_f = \frac{v_t - v_{LF}}{\sqrt{g h_F}} \quad (11)$$

In pipe flow, the definition of the film Froude Number must be modified to account for the geometry. In this case,  $h_F$  is the height of the liquid film at the center of the pipe. The definition of the film Froude Number is then written as in Equation (4). Equation (11) is also based on channel geometry. To use it for a pipe,  $h_F$  is replaced by  $h_{EF}$ . The term  $(v_t - v_{LF}) / \sqrt{gh_F}$  would then be replaced by the term  $(v_t - v_{LF}) / \sqrt{gh_{EF}}$ . This would then be the Froude Number in the film, as defined in Equation (4). The term  $(v_t - v_{LS}) / \sqrt{gh_j}$ , may then be said to be the Froude Number in the slug.

### 3.5 Slug Froude Number Definition

As is indicated from Equation (10), a Froude number in the slug,  $Fr_s$ , may be defined as:

$$Fr_s = \frac{v_t - v_{LS}}{\sqrt{gh_j}} \quad (11)$$

It should be remembered, that  $v_{LS}$  is not constant in the slug. Farther into the body of the slug, momentum losses occur and, the fraction of the total pipe cross sectional area available for liquid flow, increases with distance into the slug body. Here the gas is pushed towards the top of the pipe decreasing the overall void fraction. This leads to a decrease in the slug liquid velocity,  $v_L$  and consequently, gives an increase in the slug Froude number.

### 3.6 Ratio of Heights in Slugs

A relationship to predict  $h_j$  as a function of the Film Froude number is now needed. Chow (1959) gives a correlation between  $h_F$  and  $h_j$  for channel flow:

$$\frac{h_j}{h_F} = \frac{1}{2} \left[ \sqrt{1 + 8Fr_f^2} + 1 \right] \quad (12)$$

where,  $Fr_f$  = Film Froude Number in open channel flow as given by Equation (10).

This equation also needs to be modified for the geometry of the pipe. Figure 5 shows the variation of the ratio  $h_j/h_{EF}$  and  $h_j/h_F$  for a stationary slug (Kouba and Jepson, 1987) where,  $h_{EF}$  is the effective height of the liquid film and  $h_F$  is the height of the film at the center. The data shown in Figure 5 were obtained from single point pressure measurements. Table 1 shows the variations of  $h_j/h_{EF}$  as a function of distance into the slug at various Film Froude Numbers for water-carbon dioxide stationary slug systems. It is seen that beyond a distance of 45 cm from the slug front, the ratio  $h_j/h_{EF}$  varies similar to the values predicted by Equation (12), with  $h_{EF}$  used to estimate  $h_j$ .

It is seen that Equation (12) provides a reasonable estimate for  $h_j$  for slug flow in pipes, if the effective film height,  $h_{EF}$ , is used. This is due to the curved geometry of pipes. In channels, the depth is uniform across the width. Hence, a single height is sufficient to describe the liquid film. Using the value of  $h_{EF}$  for the liquid film height in Equation (12) gives a good estimate of the effective height of the slug,  $h_j$ .

### 3.7 Average Liquid Velocity in Slug

The volumetric flow rate of liquid in the slug is given by:

$$q_l = \int_0^A v (1 - \alpha) dA \quad (13)$$

The local velocity,  $v$ , and the void fraction,  $\alpha$ , within the differential area in the slug body are given by the following equations:

$$v(y) = v_o \left( \frac{y}{\delta} \right)^{1/7} \quad (14)$$

and,

$$\alpha(\bar{h}) = \frac{a \bar{h}}{(1 - b \bar{h})} \quad (15)$$

In Equation (15), the values of  $a$  and  $b$  are taken as 0.1 and 0.085. The details of Equations (14) and (15) are given elsewhere (Gopal and Jepson, 1997).

Once  $q_l$  is known, the effective average velocity in the slug is calculated as:

$$v_{LS} = \frac{q_l}{a_{LS}} = \frac{q_l}{A(1 - \alpha_{avg})} \quad (16)$$

where,  $\alpha_{avg}$  = average void fraction over a cross sectional area.

Once  $v_{LS}$  is known, the Froude Number at any distance in the slug can be calculated using Equation (10).

### 3.8 Slug Length Model

The slug front is a propagating hydraulic jump (Jepson, 1989) and represents a transition from subcritical flow to supercritical flow when viewed in a coordinate system moving with the slug front. Figure 6 shows how the Froude Number varies throughout the film and the slug in this case. When a moving slug is observed in a stationary coordinate system, the reverse occurs and the flow is subcritical in the film ahead of the slug and supercritical in the slug.

The above analysis implies that in a coordinate system moving with the slug front, the Froude Number in the film is always greater than unity, indicating supercritical flow, and in the slug is always less than unity, indicating subcritical flow. The Froude Number decreases rapidly at the slug front and subsequently, gradually rises back up to unity within the slug body. The point where it tends to unity corresponds to the end of the slug and the formation of a new liquid film. The increase in Froude Number, according to Equation (10), may be used to find the point where the Froude Number in the slug equals unity, and hence the slug length. The slug is broadly composed of a mixing zone, and the slug body. The total slug length is then a sum of the individual lengths of each of these zones.

### 3.9 Length of Mixing Zone

The mixing length was defined as the minimum distance into the slug required for the void fraction profile to reach a quasi-steady state described by Equation (15). The mixing length was estimated as an empirical function of the film Froude number. The dependence is given by:

$$l_m = m.Fr_f + c \quad (17)$$

The values of the regression coefficients are found to be  $m=0.13$ , and  $c = -0.31$ , to calculate the length of the mixing zone in metres.

### 3.10 Slug Body Length

In the slug body, the void fraction distribution is obtained by using equation (15) and the velocity profile may be obtained by Equation (14). The effective average liquid velocity may be calculated using Equations (13) and (16). The liquid velocity decreases due to momentum losses. This is expressed by the following relation:

$$u_o \frac{du_o}{dx} = -\frac{1}{\rho_L} \frac{dp}{dx} \quad (18)$$

It is noted that the velocity profile in the slug body is similar to that given by the one-seventh power law, with  $v_0$  as the centerline velocity. However, unlike a fully developed flow in a pipe, the center line velocity decreases in a slug due to momentum losses. The profile, however, has the same shape throughout the body of the slug (Gopal and Jepson, 1997).

The pressure gradient,  $dp/dx$ , in the slug body is given by the sum of two components, the pressure drop due to friction alone,  $dp/dx_f$ , and, an excess pressure drop due to agitation by the gas bubbles in the slug  $dp/dx_e$ .

The pressure drop due to friction is given by the well known equation:

$$\frac{dp}{dx_f} = \frac{4f\rho_L v_{LS}^2}{D} \quad (19)$$

where  $f$  is calculated using a Blasius-type equation:

$$f = C \left( \frac{D v_{LS} \rho_L}{\mu_L} \right)^n \quad (20)$$

The pressure drop due to agitation by bubbles is more difficult to evaluate. This is given by the expression:

$$\frac{dp}{dx_e} = \frac{3C_D \alpha_{avg} \rho_L v_r^2}{2d_b} \quad (21)$$

The drag coefficient  $C_D$ , is a complicated function of several dimensionless groups. Wallis (1969) modified the derivation of Peebles and Garber (1953) and used the following:

$$Re_b = \frac{\rho_L d_b v_r}{\mu_L} \quad (22)$$

Also, a dimensionless group  $G_1$ , is defined as follows:

$$G_1 = \frac{g\mu_L^4}{\rho_L \sigma^3} \quad (23)$$

Then a table for the drag coefficient may be used as shown in Table 2. Hence, the total pressure gradient can be calculated. Starting at the end of the mixing zone, the centerline velocity,  $u_c$ , can be updated using Euler's method in Equation (19).  $v_{1,S}$  can then be found as before. The slug Froude number can then be checked using Equation (12) and a value of unity yields the length of the slug body.

The sum of the mixing length and the slug body length would then yield a measure of total slug length.

## 4 RESULTS AND DISCUSSION

The detailed results of the slug translational velocity, mixing length, and the detailed profiles of velocity and voids within the slug are given elsewhere (Gopal and Jepsen, 1997, 1998a,b). Here the emphasis is on Froude number results, and slug length modeling. Froude numbers in the slug were calculated for water-carbon dioxide systems.

### 4.1 Liquid Film Height

Figures 7a and 7b show the variation of the mean film height ahead of the slug as a function of slug velocity. It is seen that the mean height of the film, in both cases, lie between 0.3 and 0.5 pipe diameters. The mean height does not change significantly over the range of velocities studied for either cases. The modified Taitel and Dukler model used in this study, gives reasonable value of film height prediction. The slight overprediction of the model for the case of water is due to the neglect of liquid flow rate as part of slugs and large roll waves that occur in slug flow. These do not seem to have a major effect for the ARCOPAK90™ slug systems.

### 4.2 Froude Number

The details of the film Froude number have been given in a separate paper (Gopal and Jepsen, 1998a). Here the details of the variation of the Froude number in the slug is given.

Figures 8a to 8c show the variation of Froude number with distance in the slug, for a water-carbon dioxide slug system. Figure 8a describes the Froude number variations for a superficial liquid velocity of 0.2 m/s, and a superficial gas velocity of 1.07 m/s. The film Froude number in this case is 3.8. It is seen that the Froude Number decreases rapidly to a value of about 0.8 to 0.9 in the slug, at a distance of 20 cm from the slug front. This also corresponds to the end of the mixing zone in this case. The Froude Number then gradually rises in the slug body and tends to a value close to unity near the tail of the slug. The total length of the slug at which the Froude Number tends to unity is about 60 to 80 cm.

Figure 8b shows the Froude Number variation in the slug for a superficial liquid velocity of 0.3 m/s, and a superficial gas velocity of 1.07 m/s. The film Froude number in this case is also about 3.8. Again, it is seen that the Froude number drops to a value between 0.8 and 0.9 at 20 cm into the slug. However, there is an oscillation of the Froude Number near the end of the mixing zone at approximately 30 cm into the slug. This is due to a release of pulses of gas bubbles observed in the mixing zone. The pulse of bubbles results in increased local void

fraction, and an increase in the average velocity of the liquid. In the slug body, these effects are dissipated and the Froude Number rises again to a value close to unity, near the tail of the slug. This is at a distance of 40-50 cm into the slug.

Figure 8c shows the Froude number variation in the slug for a superficial liquid velocity of 0.4 m/s, and a superficial gas velocity of 1.43 m/s. The film Froude number is about 4.6. In this case, the increase in the Froude number value is rapid. As is seen, the Froude number drops rapidly below unity to a value of around 0.7 at a distance of 15 cm into the slug. At this Froude number, there are high levels of turbulence within the mixing zone. The Froude number can therefore be expected to oscillate rapidly in this zone. From this point, the Froude number increases rapidly and reaches a value near unity at the tail of the slug. This occurs at approximately 55 cm.

Figure 9 shows the variation of the Froude Number from the film to the slug and beyond the tail of the slug for slug velocities of 1.27 m/s, 1.37 m/s, and 1.83 m/s. It is seen that the Froude Number in the film ahead of the slug in each case is of the order of 4 to 5. This decreases rapidly to a value of about 0.7-0.9 within the slug. The Froude number then gradually increases within the body of the slug and reaches a value close to unity at the tail of the slug. Beyond the slug tail, it is seen that the Froude number begins to rise rapidly again. For slug velocities of 1.27 m/s and 1.37 m/s, the Froude number in the tail rises to 2.5, while for a slug velocity of 1.83 m/s, the Froude number beyond the tail rises back to about 4.5.

Figures 8a, b, and c, and Figure 9 strongly indicate the existence of a transition in the flow characteristics based on Froude number. It is seen that when viewed in a coordinate system moving with the front of the slug, the Froude number in the film is greater than unity. It drops to below unity in the slug and rises again to above unity beyond the slug tail. There is a transition from supercritical flow in the film, to subcritical flow in the slug, and a transition back to supercritical flow at the end of the slug. This point of transition marks the end of the slug.

#### 4.3 Total Slug Length

Figures 10a and 10b show the variations of total slug length as a function of slug velocity for water-carbon dioxide slug and ARCOPAC90™-carbon dioxide slug system. Tables 3 and 4 list the variations for the two systems.

It is seen that the slug length is distributed about the mean value within  $\pm 2$  standard deviations. In general, the total slug length increases with an increase in slug velocity, from 0.7 m at a slug velocity of 1.27 m/s to about 1.3 m at a slug velocity of 3.2 m/s. Table 3 shows the variations of total slug length as a function of slug velocity for ARCOPAK90™-carbon dioxide slug system. Again, it is seen that the slug length is within  $\pm 2$  standard deviations. In general, the total slug length increases with an increase in slug velocity, from 0.6 m at a slug velocity of 1.27 m/s to about 1.16 m at a slug velocity of 5.88 m/s.

Figure 11 shows the variation of slug lengths for different systems. Data from this study is shown along with those of Nicholson et.al. (1978), Kouba (1986), Crowley et.al. (1986), and Jepson (1988). The slug lengths have been represented as a number of pipe diameters, to compare slug lengths from systems of different pipe diameters. Nicholson et.al. performed experiments in 2.58 cm pipes.

It is seen that, in general, the slug length varies from about five to twenty pipe diameters. The data from Kouba (1986) are twice as large as the rest of the data. This is ascribed to gas expansion over the length (418 m) of the pipe. The data in this study for both water and ARCOPAK90 are very similar to those of others.

Figure 12a and 12b shows a comparison of the model with average total slug length for water-carbon dioxide and ARCOPAK90<sup>TM</sup>-carbon dioxide slug systems. The model agrees well with the experimental data for the mean slug length, in the case of water-carbon dioxide slug systems. The mean slug length varies from about 0.7 m at a slug velocity of 1.3 m/s, to 1.3 m at a slug velocity of 3.2 m/s. This translates from about 10 pipe diameters to about 17 pipe diameters. It should be noted that there is a distribution of the slug length of approximately two standard deviations from the mean, and the model predictions are well within the range of this distribution.

Figure 12b shows the results for slug lengths for ARCOPAK90<sup>TM</sup>-carbon dioxide slug system. The mean experimental slug length varies from about 0.6 m at a slug velocity of 1.6 m/s to about 1.2 m at a slug velocity of 6 m/s. The model agrees well with the mean slug length results up to a slug velocity of 3 m/s. Beyond that velocity, it begins to over predict the slug length. This may be due to variations in the length of the mixing zone for this mixture. Again it is to be noted that there is a distribution of lengths around the mean slug length and the model predictions are within this range.

Figure 13 shows the model predictions for the data of Jepson and Taylor (1988) and Crowley et al. (1988). The lengths vary from about 3 pipe diameters to 17 pipe diameters. It is seen that there is good agreement between the model and the experimental data.

## 5 CONCLUSIONS

Slug flow characteristics have been studied and modelled for two-phase gas-liquid systems. The effect of liquid properties were investigated by the use of two different liquids, water and ARCOPAK90<sup>TM</sup>.

Experimental conditions maintained in this study were in the slug flow regime. The liquid superficial velocity ranged from 0.2 m/s to 0.7 m/s for water, and 0.15 m/s to 0.88 m/s for ARCOPAK90<sup>TM</sup>. The superficial gas velocity ranged from 1 m/s to 5 m/s.

There was a wide variation of slug lengths for both water-carbon dioxide slugs and ARCOPAK90<sup>TM</sup>-carbon dioxide slugs. The slug lengths were distributed within  $\pm 2$  standard deviations. The data agrees with those of Jepson and Taylor (1988) and Crowley et al. (1988) as well as Nicholson et al. (1978) and Dukler and Hubbard (1975).

A slug Froude number was defined which was used to determine slug lengths. From knowledge of the Film Froude Number, the effective height of the slug was defined. An effective average velocity of the liquid in the slug was also defined and used in the slug Froude number.

It was found that the film Froude Number was always greater than unity and the slug Froude

number was always less than unity. The slug Froude number tended to unity near the tail of the slug, and this criteria was used to determine the slug length. Beyond the tail, the film Froude Number was seen to increase beyond unity once more.

A mathematical model was developed to predict slug length. Knowing the superficial gas and liquid velocities, the fluid properties, and the pipe diameter, the liquid film height ahead of the slug was predicted. The Film Froude Number was then calculated. The mixing length in the slug was then determined as a function of the Film Froude Number. A mechanistic model was developed that incorporated the void fraction distribution and the velocity profiles in the slug body to predict the variation of the slug Froude Number. The slug Froude Number variation was then computed, and the point where it equalled unity was determined. This provided the criterion to determine the slug body length. The total slug length was then estimated as the sum of the mixing length and the slug body length.

The model gives good prediction of the mean slug length for the data in this study. It also is able to predict the data of Jepson and Taylor (1988), and Crowley et al. (1988).

**Table 1: Table for  $h_j/h_{EF}$  with distance for a range of film Froude numbers**

Froude Number	$h_j / h_{EF}$ with distance into slug					
	0 cm	15 cm	30 cm	45 cm	60 cm	Eq. (19) $h_F = h_{EF}$
6.6	6.8	9.4	10.2	10.2	10.3	8.9
9	3.6	9.3	11.42	11.9	11.95	12.4
12		5.7	11.05	14.6	14.0	16.4
14			11.1		15.8	19.3

**Table 2: Summary of drag coefficient equations**

Region	Drag Coefficient, $C_D$	Range of Applicability
Region 1	$24 Re_b^{-1}$	$Re_b \leq 2$
Region 2	$18.7 Re_b^{-0.68}$	$2 \leq Re_b \leq 4.02 G_1^{-0.214}$
Region 3	$0.0275 G_1 Re_b^4$	$4.02 G_1^{-0.214} \leq Re_b \leq 3.10 G_1^{-0.25}$
Region 4	$0.82 G_1^{0.25} Re_b$	$3.10 G_1^{-0.25} \leq Re_b$

**Table 3: Variation of total slug length with slug velocity for water-carbon dioxide slug**

Slug velocity, m/s	Total slug length, m			
	Minimum	Maximum	Mean	St. Deviation
1.27	.44	1.04	.70	.19
1.37	.34	.82	.61	.15
1.63	.51	1.03	.68	.16
1.73	.35	.81	.54	.14
1.83	.43	.85	.61	.14
2.7	.73	1.45	1.0	.23
3.2	1.01	1.70	1.31	.22

**Table 4: Variation of total slug length with slug velocity for ARCOPAC90™-carbon dioxide slug systems**

Slug velocity, m/s	Total slug length, m			
	Minimum	Maximum	Mean	St. Deviation
1.6	.40	.83	.61	.17
2.3	.46	.76	.67	.14
2.47	.50	1.07	.73	.17
3.3	.50	1.10	.78	.16
4	.60	1.60	.94	.25
5.88	.98	1.76	1.16	.34

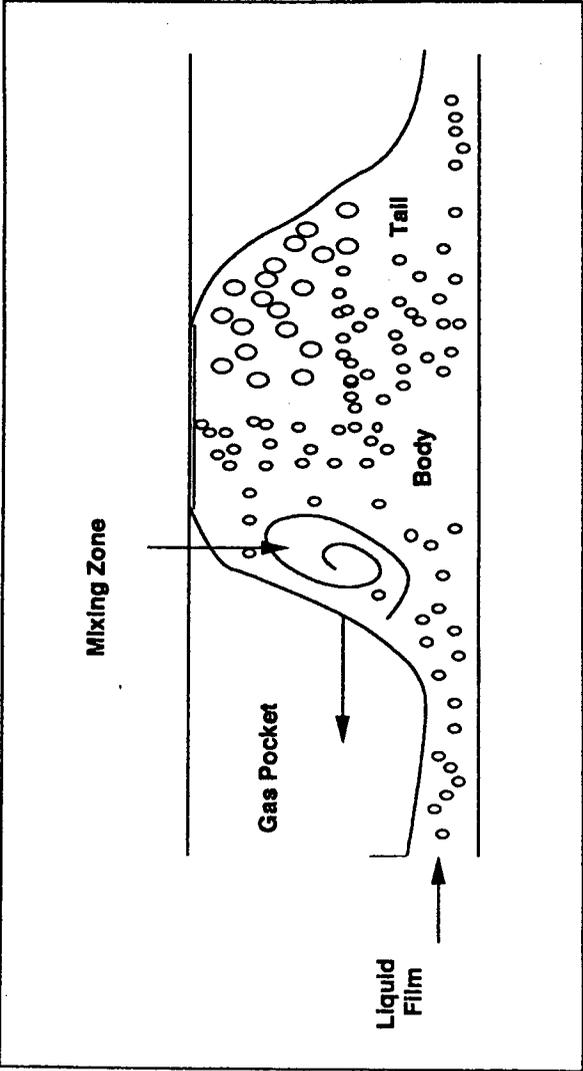


Figure 1: Profile of a slug unit

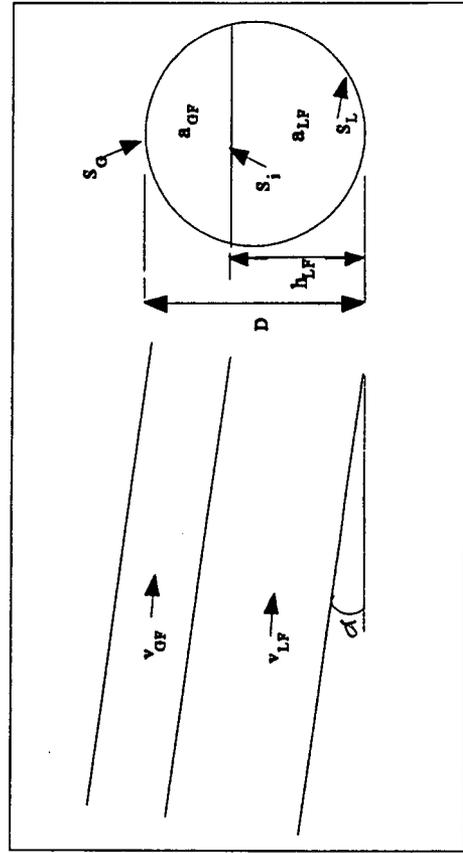


Figure 2: Physical model for stratified flow

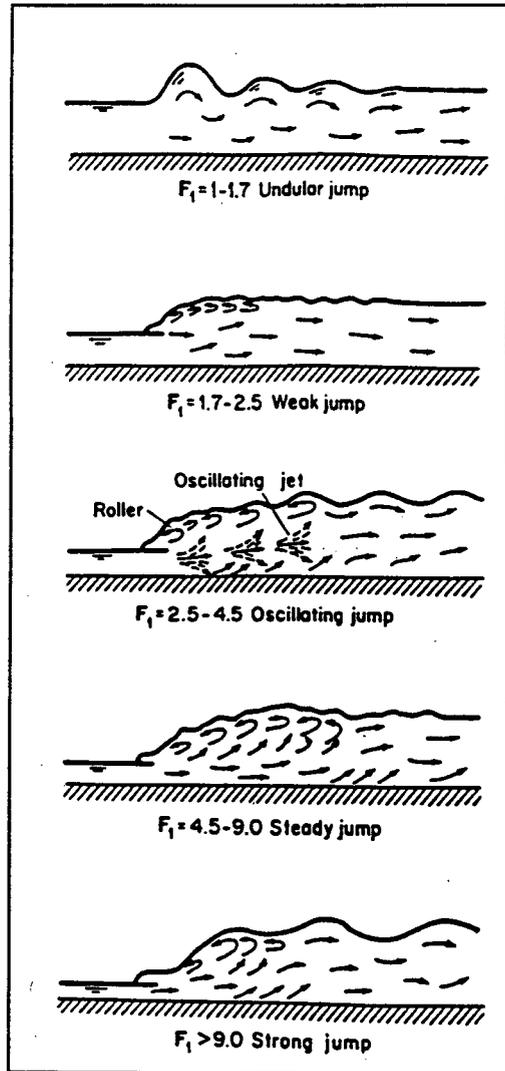


Figure 3: Various types of hydraulic jumps

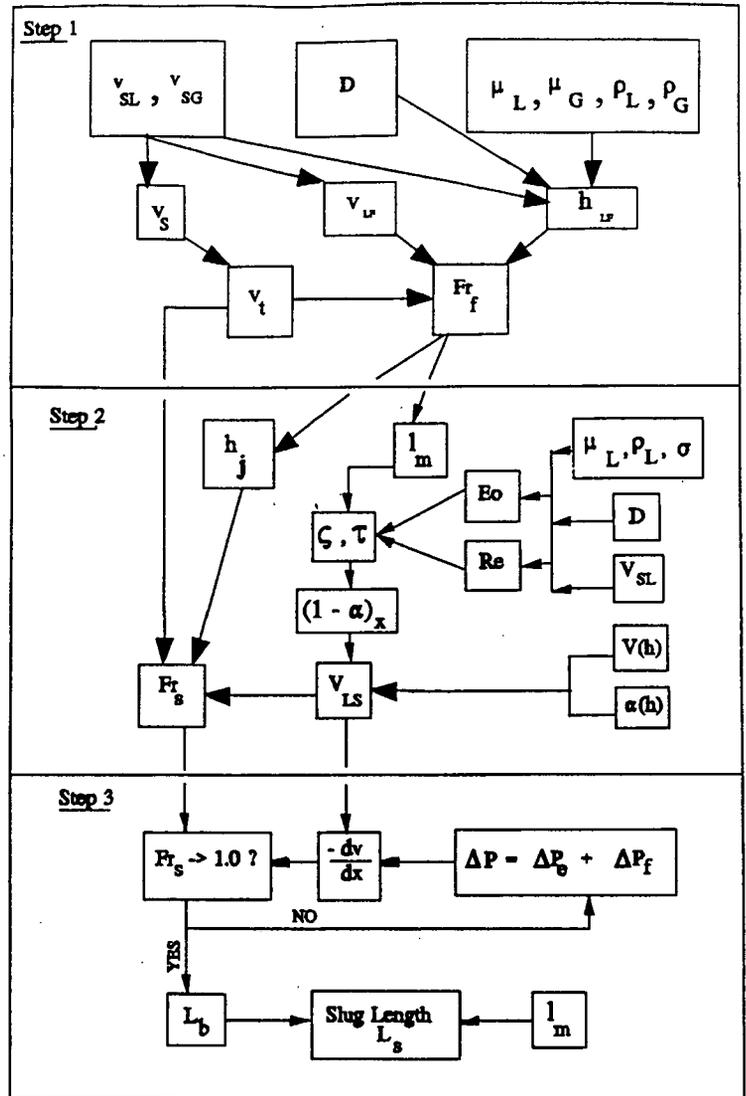


Figure 4: Schematic flow chart of slug length model

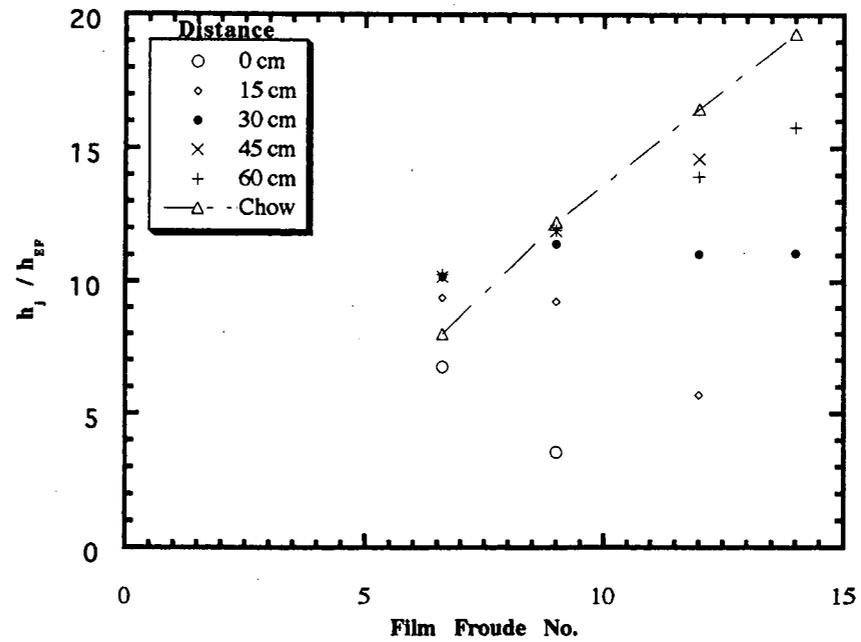


Figure 5: Variation of equivalent height ratio with distance into slug as a function of Film Froude No. for stationary slug

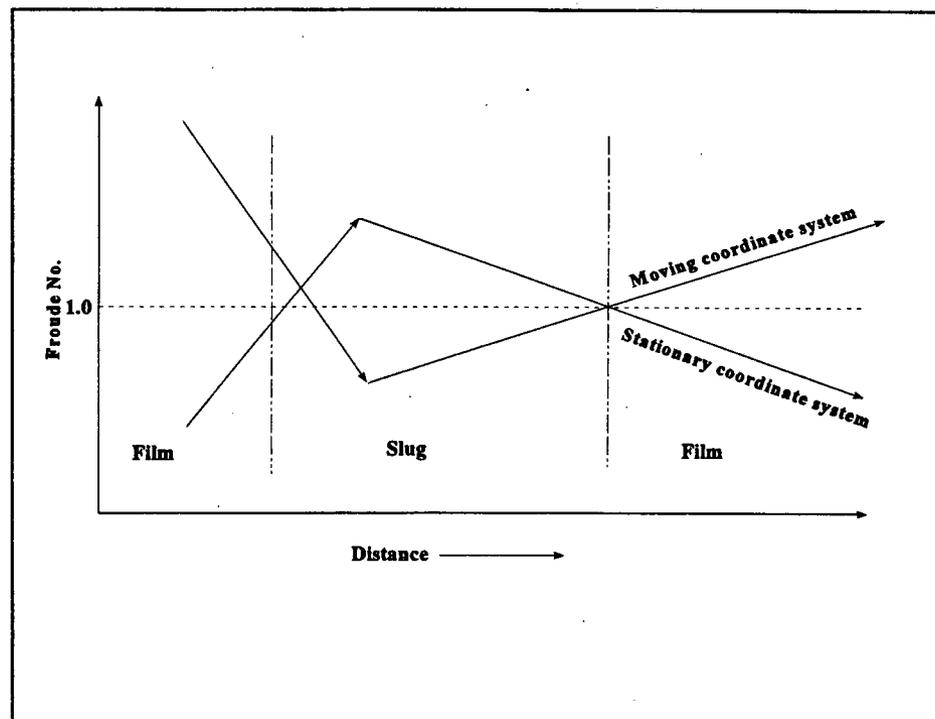


Figure 6: Schematic diagram of Froude number variation with distance into the slug

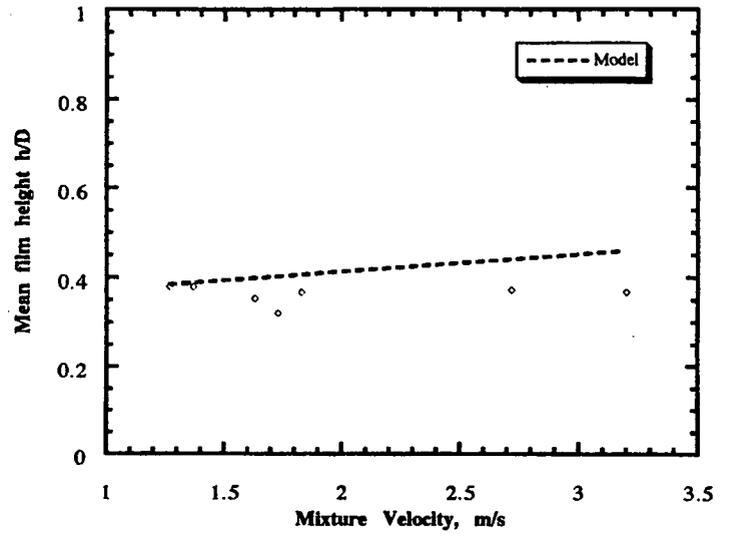


Figure 7a: Variation of mean liquid film thickness for water-carbon dioxide systems. Comparison with model.

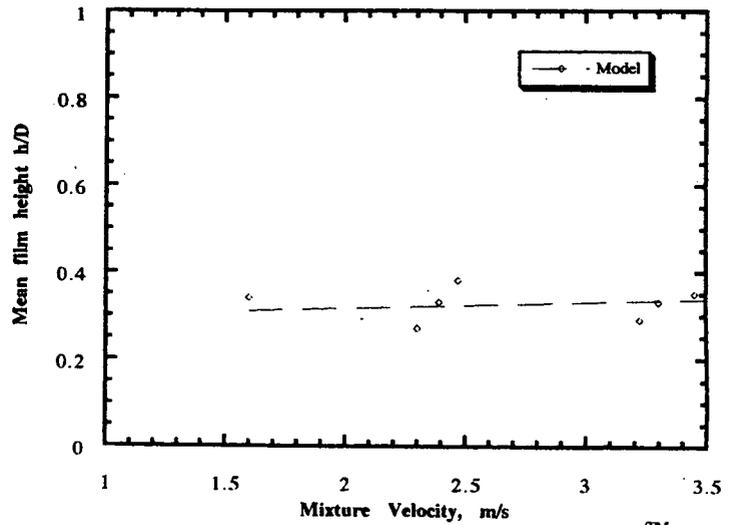


Figure 7b: Variation of mean liquid film thickness for ARCOPAK90™-carbon dioxide system. Comparison with model.

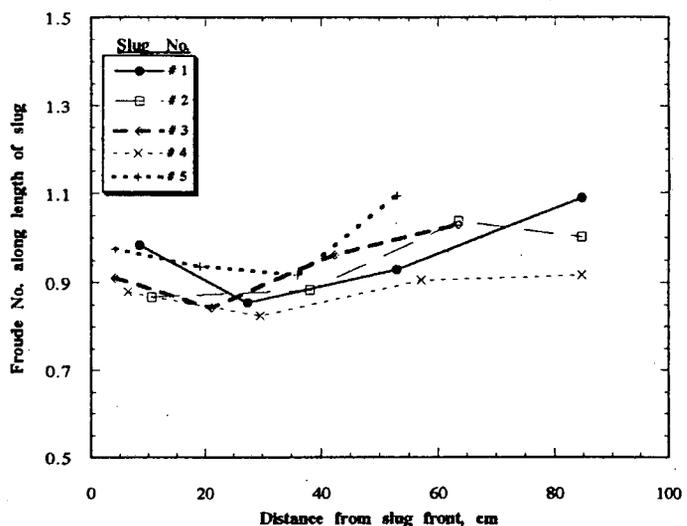


Figure 8a: Variation of Froude number with distance into the slug for water-carbon dioxide system.  $v_{s1} = 0.2\text{m/s}$ ,  $v_{s2} = 1.07\text{m/s}$ .

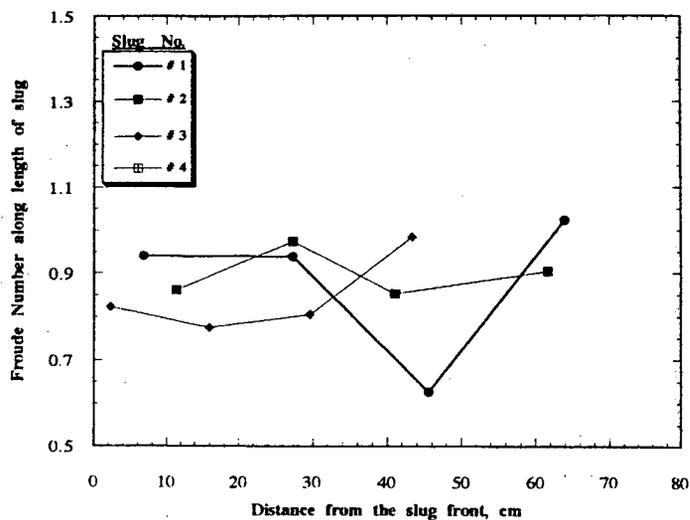


Figure 8b: Variation of Froude number with distance into the slug for water-carbon dioxide system.  $v_{s1} = 0.3\text{m/s}$ ,  $v_{s2} = 1.07\text{m/s}$ .

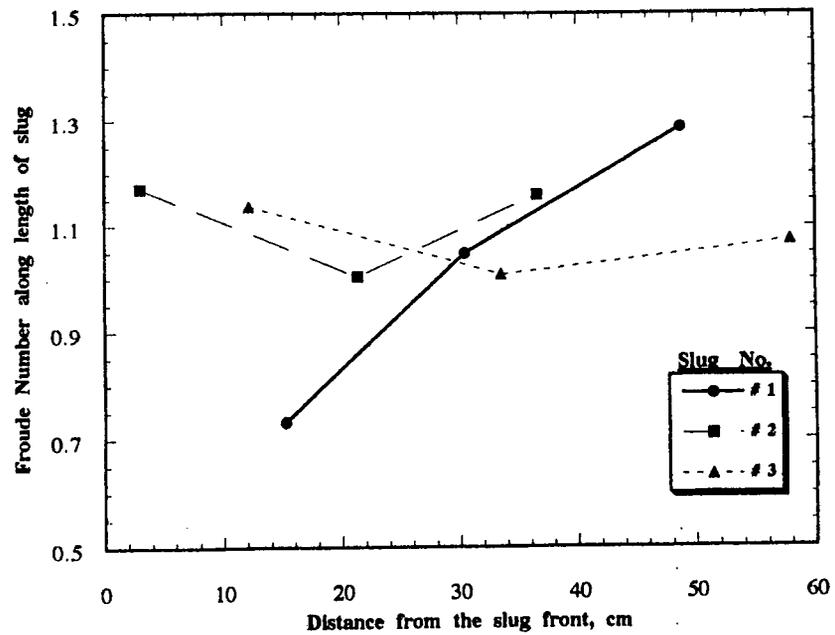


Figure 8c: Variation of Froude number with distance into the slug for water-carbon dioxide system.  $v_{sl} = 0.4\text{m/s}$ ,  $v_{sg} = 1.43\text{m/s}$ .

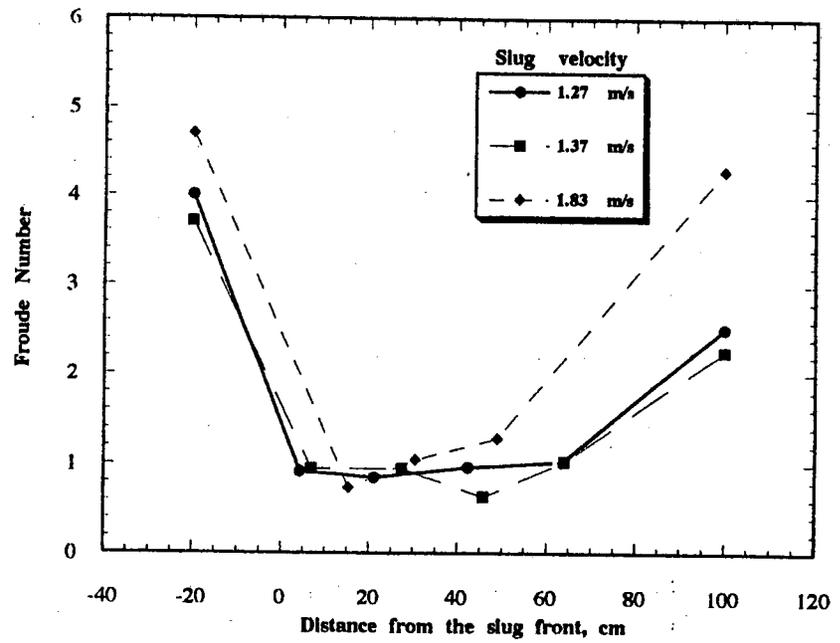


Figure 9: Variation of Froude number across a slug for different slug velocities.

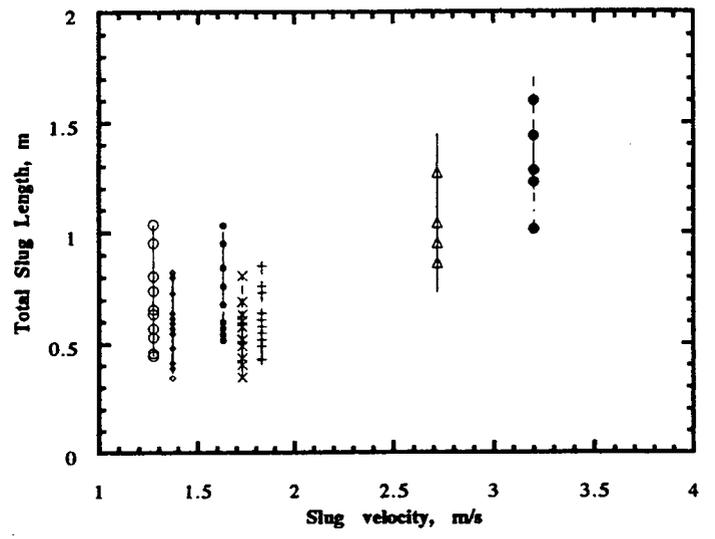


Figure 10a: Variation of total slug length with slug velocity for water-carbon dioxide system.

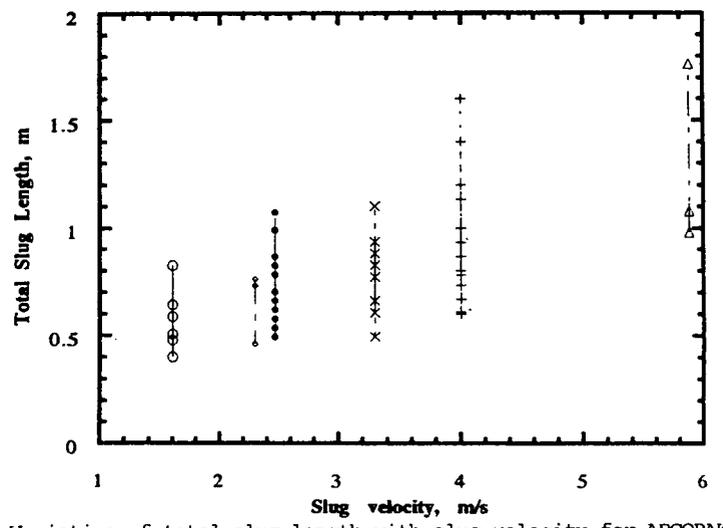


Figure 10b: Variation of total slug length with slug velocity for ARCOPAK90™-carbon dioxide system.

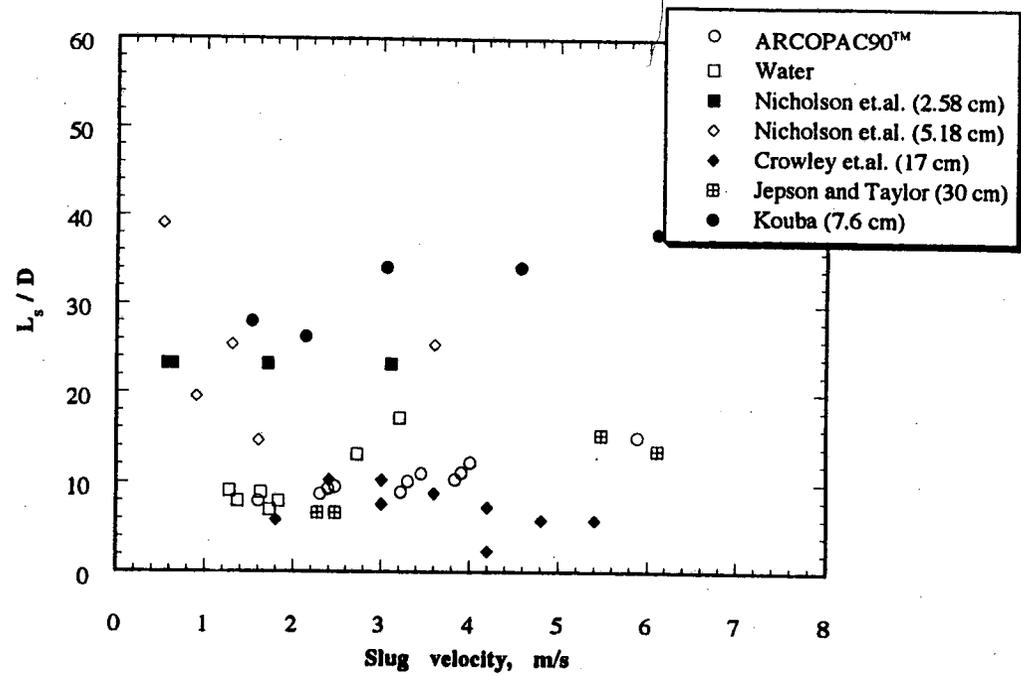


Figure 11: Comparison of total slug length from different systems.

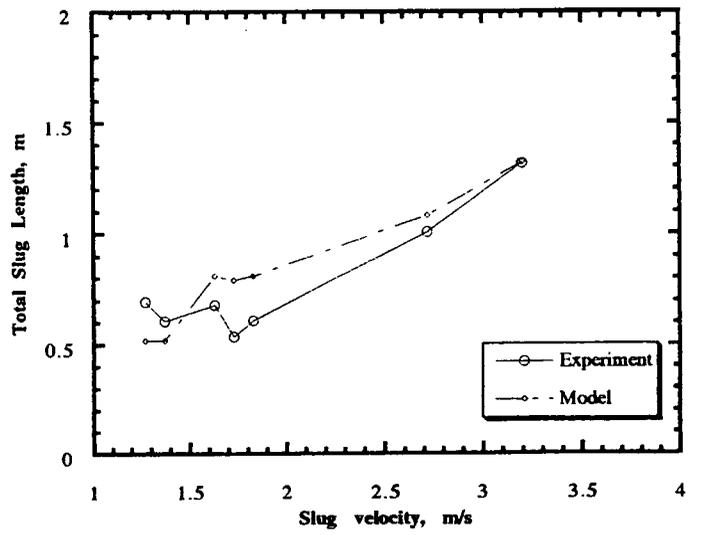


Figure 12a: Comparison of slug length model with average slug length for water-carbon dioxide system.

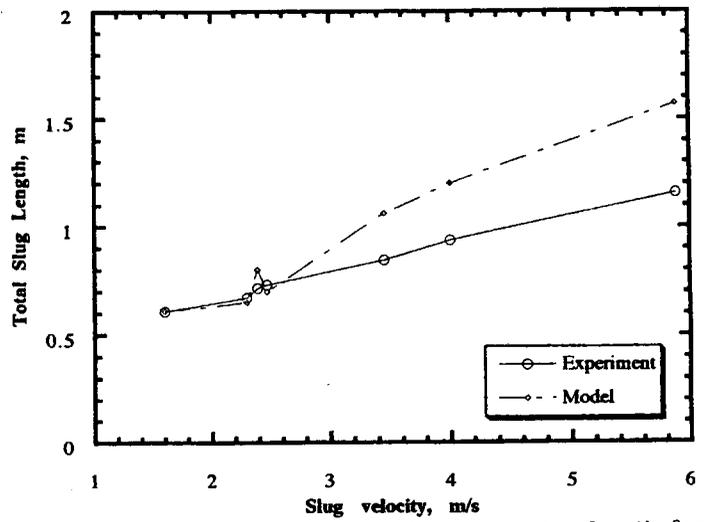


Figure 12b: Comparison of slug length model with average slug length for ARCOF carbon dioxide system.

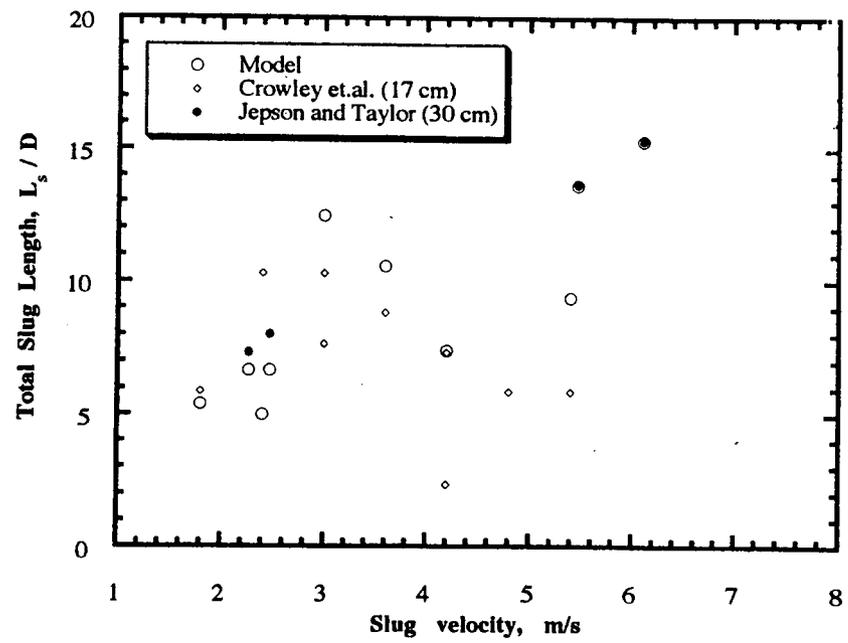


Figure 13: Comparison of model predictions with slug length from other systems.

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