Error Analysis and Metacognitive Awareness as Tools in Multi-digit Subtraction

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Abstract

In the present study, the author determined the utility of error analysis to guide instruction in multi-digit subtraction in a 4th-grade classroom. The participants were introduced to metacognitive awareness within the scope of this study. A total of 29 students participated in this study by completing a pre-assessment to identify error categories. Analysis of error types present in this assessment determined the focus of a short-term intensive instruction series. This series was designed to clarify mechanics of operations leading to most common error types and metacognitive awareness strategies. Analysis indicated incorrect regrouping as the predominant error category present on the pre-assessment. A 3-day instruction series was developed to clarify mechanics of regrouping and to present metacognitive strategies useful in multi-digit subtraction. Following post-assessment, analysis of results indicated a reduction in incorrect regrouping errors as well as a decrease in the total number of errors committed. Measurement of metacognitive application proved difficult and provided limited data. Analysis indicated that error analysis can be utilized to determine common errors and guide instruction to target these errors.
Chapter 1

Introduction

Students make mistakes. This is a part of the growing and learning process. We all make mistakes as we are learning new things. It helps us to understand how and why things work, or do not work, in a certain manner. Students are in the business of learning; it is their entire world. They are learning social skills; they are learning verbal skills; they are learning academic skills, and in all of this learning they are making mistakes. It would be unrealistic to think that any person could do that much learning without making mistakes. After all, they are only human, and all humans make mistakes. As teachers, we do not like to see students make mistakes as it makes us question our methods and strategies. However, we may be overlooking a valuable resource of information in the very mistakes we are striving to eradicate. These mistakes can help shed light on the issues the students are finding challenging. Research has demonstrated that analysis of student errors can be a powerful tool in the teaching process. The error is now seen as a natural stage in the knowledge construction and is thus inevitable (Fiori and Zucchari, 2005).

A common challenge faced by educators is the task of determining the actual knowledge possessed by their students. Through the varied processes of formative and summative assessment, slowly a picture reveals itself describing the body of knowledge possessed by an individual student. Complicating this process is the understanding that not all students learn in the same manner, nor do they all possess the same “test-taking” skills. A superb student with poor test-taking skills can easily be dismissed if only the ratio of correct to incorrect answers is considered. Error analysis gives the educator a powerful tool to aid in this difficult task. Research has shown that students tend to repeat errors in a consistent manner (Coker, 1991) and by analysis of this error pattern the teacher can better understand the thinking process that the
Another aspect of error analysis is the benefit that students can gain from thinking about and understanding reasons for, the errors they are repeating. This is an introduction for the student to the concept of metacognition. Metacognition is a broad concept that is vaguely defined in the research literature (Roebers, Cimeli, Röthlisberger, and Neuenschwander, 2012). Schneider (2011) defined metacognition as “higher-order self-reflective cognitive processes that may be used for regulating information processing,” or, in other words thinking about our thinking. At its core, metacognition, for the students within the context of this study consists of, considering why they are applying a given algorithm to a problem and asking “does this algorithm work within the parameters of the given problem?”. A student who can be elevated to this level of metacognition is given the opportunity to become aware of his own mistakes and how they differ from the correct application (Bell, Brekke and Swan, 1987). Students will also have a clearer self-perception of their work, and will be less likely to place blind acceptance in a learned algorithm simply because they believe they understand how the operation works.

Students operate under a misguided notion that if they “know” how to do an operation, such as subtraction, then their work with that the problem must be correct; like inputting digits into a calculator, selecting a function, and not asking if the output makes sense. If the student is
faced with a problem that is beyond their understanding of the operation, research shows that the student is likely to modify a known procedure and improperly apply it to the task at hand (Blando, Kelly, Schneider and Sleeman; 1989). The cognitive level on which most 4th-grade students operate does not allow them to consider the possibility that if they know how to do something, they could make a mistake doing it. The students view mathematics as a black or white situation; they either know how to perform an operation or they do not. The concept of knowing how to perform a function but performing this function incorrectly or at the wrong time is not a possibility in their young minds. Mathematics as a subject appears decidedly abstract to a 4th-grader if they are utilizing algorithms of which they do not fully understand the mechanics.

**Research Question**

The purpose of this study was to investigate the following questions:

1. Can error analysis be utilized to determine the most common errors 4th grade students commit during multi-digit subtraction?

2. Does short-term intensive instruction reduce student errors in this category thereby improving assessment scores?

3. Do metacognitive prompts help students to verify answers in multi-digit subtraction?

This study was conducted in my mentor’s 4th-grade mathematics class. It involved a pre-assessment to determine the students’ subtraction error category. Data collected from this assessment was utilized to design a lesson to scaffold the students’ misconceptions about subtraction and strategies to check. At the conclusion of the lesson, the students were given a post-assessment to measure the change in ability and understanding of the concept.
Summary

Error analysis can be a valuable tool for teachers and students. The mathematical error, once considered simply a wrong answer can now be utilized as a tool to direct instruction and highlight areas of incomplete comprehension. A teacher utilizing error analysis is given a valuable window into the inner workings of the students’ minds. In order for students to utilize this tool they must be introduced to the concept of metacognition, essentially to think about their own thinking. Students operating on this higher level of thinking are capable of self-diagnosis and error correction. The goal for this study is to apply error analysis to a mathematical operation, in this case multi-digit subtraction, to uncover the limitations and/or strengths of this tool. This error analysis will then be utilized to shape the instruction component of the study. The participants will also be given metacognitive prompts during the instructional period. This study will culminate in a post-assessment that will be compared to a similarly instructed pre-assessment. The results of this comparison will be utilized to weigh the benefits of the instructional component as well as the use of error analysis as a directional tool.
Chapter 2

Literature Review

The concept of error analysis is not a new idea. Error analysis has been a topic of research for many decades. Individual studies have focused on the value of error analysis for; uncovering error patterns (Blando, Kelly, Schneider, and Sleeman, 1989), directing instruction (Drucker and McBride, 1987), and categorization of common error types (Fiori and Zuccheri, 2005). Each of these studies focused on a specific function of error analysis.

Along with the error analysis being conducted by the researcher in this study, it was also the intention to introduce a concept of self-analysis to the participants. To utilize self-analysis in an effective manner, participants were required to “think about their own thinking”. This self-perception of their task mastery (Roebers, Cimeli, Röthlisberger, and Neuenschwander, 2012) is a key component of the concept of metacognition. Studies have attempted to measure the effect of metacognition upon error-analysis in students (Jacobse and Harskamp, 2012; Kramarski and Zoldan, 2008). These studies have focused on the value of metacognition in raising students to a higher-level thinking about their own work. There is a significant level of importance of metacognition within the scope of error analysis; therefore, this aspect was included in this study. This study aimed to interconnect aspects of many of these previous studies to provide a picture of the value of error analysis within the 4th-grade mathematics classroom.

Error Analysis

An error is not merely an incorrect answer. It represents a faulty link in the chain of education somewhere between the teacher’s understanding of the concept and the student. It also
represents an opportunity to understand the creation of that faulty link, and an opportunity to prevent further building upon this misunderstanding. In the subject of arithmetic, this presents a unique opportunity to trace the source of the error to the specific misunderstanding that caused the error. By carefully analyzing the operations and calculations, it is possible to pinpoint the exact misunderstanding. The academic community realized this opportunity and that made the topic of calculation errors in arithmetic one of the most prevalent topics of research for the period of 1900 to 1970 (Fiori and Zuccheri; 2005). The significant change that has occurred in recent decades is that the negative aspect of the error is no longer the focus. The error is now seen as a part of the natural growth of the students’ understanding and is inevitable (Bouvier, 1987). Moreover, research has shown that the student’s errors are valuable resources for suggesting strategies to improve not only the teaching process, but also the learning.

Self-Analysis

By viewing the error, not as a mistake but an opportunity, allows the teacher and students to utilize the errors in a positive manner. For the students, the errors can serve as a springboard for reflection and exploration, introducing students to the habits of metacognition that can be utilized thereafter. Error analysis allows students to explore their own process, allowing the students the opportunity to analyze not only the problem but also the solution. In the process, the students can discover the structure and dependencies and consider the alternatives. When this process is utilized in arithmetic, this analysis can stimulate cognitive conflict. The cognitive conflict enables students to become aware of the choices made and the process through which the choices were made. By making the learners aware of the incorrect thinking within the situation, enables learners to see how the correct approach differs from the current one (Kramarski and Zoldan, 2008). Making the students aware of the thinking being utilized allows
students to make a considerable step towards awareness of the process that can be known as executive function or metacognition. Executive function is described as a “variety of self-regulatory processes including goal-directed intentional behavior, cognitive processes that allow flexibility, error detection and conflict resolution” (Roebers, Cimeli, Röthlisberger and Neuenschwander; 2012). While executive function is an understood concept, the definition is still vague and can vary from source to source. What is agreed upon by researchers of the subject is that executive function has been found to be a reliable predictor of school readiness (see Blair and Razza 2007) and school achievement (see Duncan et al. 2007). These studies and others also verify that the effects of executive function are strongly tied to mathematics success.

**Metacognition**

The concept of metacognition has also been vaguely defined by the research literature as “referring to higher-order self-reflective cognitive processes that may be used for regulating information processing” (Roebers, Cimeli, Röthlisberger and Neuenschwander; 2012). This regulation of information processing is the component of the definition that is of primary importance in this study. Metacognition has also been recognized as a reliable predictor of accomplishing complex learning tasks (Van der Stel and Veenman, 2010) The process of monitoring the information processing, or the recognition of the cognitive process, is of vital importance in the ability of the student to recognize the use of improper algorithm in the mathematical computation. Large numbers of studies have been completed to show that, through the use of metacognitive training, students have improved their ability to solve mathematical problems (Jacobse and Harskamp, 2012). Respecting the strong correlation of metacognition to self-analysis, metacognitive aspects were utilized in this study; however due to the limited scope
of this study, metacognitive aspects were limited to actively monitoring the individual’s own cognitive processes.

This self-analysis is a valuable tool to the student that can wield it properly. Research has shown that average students rated their performance higher on assessments than they actually performed (Bradshaw, 2001). The growth of metacognition as a focus in the classroom has come about as more research has shown the positive attributes of metacognition as a skill, just as vital as problem solving and critical thinking. One research study found that students that were more frequently exposed to metacognitive self-questioning and provided reasons for ideas demonstrated greater gains in mathematical reasoning. This finding supported the researchers’ concept that metacognitive instructional approach evokes explicit awareness of the concepts, rules, and mathematical explanations relevant to the solution process (Kramarski and Zoldan, 2008). This study promoted the importance of reflection as a cognitive mechanism that promoted critical thinking about the errors, allowing students to be better equipped to focus on reducing their errors. While the purpose of the study was to compare two different instructional approaches, the findings supported the initial concept that the students exposed to both metacognitive approaches improved their strategic use of a general problem solving, and “showed the importance of analyzing and monitoring errors as a means of reducing conceptual errors” (Kramarski and Zoldan, 2008).

In a study undertaken by researchers at Stanford University in 1989, a mathematics test was designed to highlight common mathematical errors. This test was delivered to a limited number of seventh-grade students and covered a broad range of operations. Upon delivery of the test, the students’ final answers and computations were analyzed, and errors categorized. The researchers focused their study on the types of errors the students were making, whether they
were stable errors or theoretical errors (Brown & Burton, 1978). The theoretical errors “occur when a student is faced with a difficult or unfamiliar feature of a task that leads the student to an impasse. This impasse is resolved by modifying a known procedure and incorrectly applying it to the task (Brown & VanLehn, 1980).

Summary

This study undertook the goal of determining the level of correction that can be obtained through a combination of student error analysis, small-group training focusing on specific error formats, and the overt inclusion of metacognitive reminders. The results of this study will be relevant to educators that find themselves confronted with the same errors time and time again, with no guidance as to how to proceed to eliminate the errors. This study utilized methods suggested in prior studies, reworking the methods to fit the parameters of this study.
Chapter 3

Methods

This study was conducted in a 4th-grade mathematics class to determine if the students comprehended the concept of self-checking mathematical tasks. A variety of tools were utilized to determine this. An assessment was administered to differentiate the specific types of subtraction errors the students are making, based on the operational mechanics of the task. A 3-day course of instruction was then implemented to address the most prevalent error type, along with strategies for self-checking the results of the tasks. A post-assessment was then administered, and the student’s scores from the pre- and post-assessment were compared and analyzed to determine if any improvement had been observed and to what extent.

Setting

The participants in this project were 31 4th-grade students. These students are currently enrolled in a local elementary school, in a city school district. The population served by this school is a mix of in-town, suburban and rural students. The building’s poverty status, as designated by the State of Ohio is Medium-Low Poverty. This elementary school maintains an average daily student enrollment of 292 students, 89.1% of whom are white, non-Hispanic; 4.3% are Asian or Pacific Islander and 4.3% of whom are Multi-Racial. 33.9% of the student population is economically disadvantaged, and 20.3% are students with disabilities. There are no students listed as Limited English Proficient.

One hundred percent of the teachers at this school have at least a Bachelor’s Degree, with 64.3% having at least a Master’s degree. Published statistics also show that 93.9% of the core academic subjects are taught by properly certified teachers. This school is currently designated as
Excellent in the School Report Card for the 2010-2011 school year. The school had 85.7% of their students scoring at or above the Proficient Level in 4th Grade Mathematics in this same year.

**Participants**

The participants in this project were 31 4th-grade mathematics students. These students are made up of two classes, constituting the entire 4th-grade of this elementary school. There are 17 students in one class, consisting of 10 boys and 7 girls. The other class contains 16 students, consisting of 9 boys and 7 girls. Both classes are inclusive for math and contain 5 students with IEP’s. Of these 5 students, 3 specify math deficiencies on the IEP.

These students are predominantly from a middle-class background, and represent the full spectrum of middle-class socioeconomic living situations. The two 4th-grade classes have mathematics classes at different times of the day. One class has math in the morning, and the other has math in the afternoon. Both math classes have the same math teacher and both use the same textbook. They maintain the same pace with lessons and course work.

Each class is split into two mixed-ability groups within their class, Omega and Delta. These groups are not static as students are regularly assessed to determine the group that would provide the best support and pace of curriculum. The time allotted to the math period varies from 75 to 90 minutes, depending on the class schedule for that particular day. The allotted time is split in halves. These groups split the classes approximately in half, allowing for a better teacher to student ratio. While Delta group is receiving the main mathematics lesson, Omega group is split again, with half of Omega working at a math game station while the other half of Omega works on skills practice. When the timer sounds the quarter of the lesson mark, these two smaller
groups switch stations, and Delta continues the main lesson. At the conclusion of the half period timer, Delta and Omega switch places; Omega goes to the main mathematics lesson and Delta splits for the two skills stations.

This station design allows the primary teacher to work more closely with the students during the main lesson and lessens the number of students engaged in the lesson at any one time. It was during the skills practice portion of the lesson that the instruction module of this study was conducted.

**Instruments**

**Pre-Assessment.** The participants were given a pre-assessment to determine the specific subtraction errors they were making most commonly. This assessment was in addition to their continuing mathematics assessments. The pre-assessment was designed to highlight problem areas that have attracted the attention of the co-teachers throughout the first semester of the year. The problems on the assessment focused on specific recurring issues that have been noticed and continue to cause errors on assessments.

The pre-assessment contained a balanced mixture of multi-digit subtraction tasks. The multi-digit subtraction tasks focused on regrouping errors, specifically when regrouping is required in multiple stages, as well as general computational errors. Both of these issues have been recognized as recurring problems that could likely be eliminated with instruction focusing on self-check strategies for participants. The participants were given space to work the problems as well as additional space for any self-check they may have wanted to compute. The participants were instructed to check their work before they began the assessment. The instruction was stated on the assessment form, as well. The pre-assessment can be seen in Appendix A.
Analysis. The pre-assessment was analyzed to differentiate the errors made by the participants. This analysis determined the primary errors hindering the participants’ mathematical development. Errors were analyzed to determine the specific nature of the error, whether simple computation error or a perceived misunderstanding of the mechanics of the required operation. The participants’ errors were separated into four categories; subtraction error, incorrect regrouping, regrouping when not necessary and flipping digits to place the larger digit in the subtrahend. Computation of the maximum number of error type will determine the focus of the lesson within this study.

Lesson. Lessons were developed to provide the participants strategies to assist with the errors occurring most often on their pre-assessment. Analysis of the pre-assessment indicated that incorrect regrouping was the most common error, with over half of the participants recording errors in this category. Three lessons were developed, designed with the aim to strengthen the participants understanding of regrouping; within these lessons were included metacognitive tools the participants could utilize to self-analyze their work. Intensive instruction began with a 15 minute lesson focusing on the self-check strategies that could be employed during multi-digit subtraction, including estimation and the usefulness of addition as a tool to check answers. The second lesson in this series consisted of a 30 minute lesson focusing on place value and logical processing. The final instruction in this series was a 15 minute lesson with a focus of correlation between place value and regrouping. All lessons focused on teaching the participants both proper operational mechanics as well as strategies to self-check their work to verify their answers. The two 15 minute lessons were add-on to the primary mathematics lessons and took place while the classes were split, enabling the researcher to work with groups of four participants at a time. The 30 minute lesson replaced the main lesson; therefore the researcher
was working with a group of eight participants. All lessons consisted of a guided practice component followed by a student centered discussion component.

**Post-Assessment.** At the completion of the 3 day lesson period, the participants were given a post-assessment to determine the improvement, if any, of the participants in the area the lessons targeted. The post-assessment was of a similar construct of the pre-assessment, with a mixture of multi-digit subtraction tasks, each task designed to highlight a specific error type. As with the pre-assessment, the participants were given ample time and space to perform calculations, and additional space to do self-check computations. They were instructed to check their work before they began the assessment, and this reminder was stated on the assessment as well. The post-assessment can be found in Appendix B.

**Data Collection**

The results of the post-assessment were collected in a similar manner as the pre-assessment, and the results were calculated in the same manner, to allow for direct comparison. The post-assessment was analyzed for the same four categories of errors as the pre-assessment: incorrect regrouping, subtraction error, regrouping when not necessary, and digits flipped to place the larger digit in the subtrahend. This will allow for direct comparison of specific error patterns. The pre-assessment and post-assessment scores will be compared, based on this differentiation.
Data Analysis

The participant’s performance on the pre-assessment and post-assessment will be analyzed in the following manner. Incorrect answers were highlighted then the individual operations performed by the participant were examined to determine the cause of the incorrect answer. Each incorrect digit was identified by the cause of the error. The errors were identified by one of four tags; (IR) - incorrect regrouping, (SE) - subtraction error, (RNN) - regrouping not necessary, and (F) – digits flipped. Example of this can be seen in the example of participant’s pre-assessment found in Appendix D.

After errors had been categorized, error totals were calculated for both the pre-assessment and post-assessment. Categorized errors were also totaled for individual participants to all comparison for individual participant’s change from pre-assessment to post-assessment. All of the participants in the study will have their data analyzed in the same manner. The participant’s scores from the other operation will also be analyzed to look for overall improvement in general mathematical skills and self-check improvement.

Summary

This study aimed to determine the value of utilizing error-analysis as a guiding method for instruction in a 4th-grade mathematics class focusing on multi-digit subtraction. A variety of tools were utilized to determine this including a pre-assessment and post-assessment designed to highlight common error types recognized from observation of 4th-grade students. The error types recognized within the assessments were categorized and compiled to ascertain the predominant error type. This error type was then highlighted in lessons aiming to scaffold the participants’ knowledge and introduce self-analysis strategies. After the post-assessment was administered
and the categorized error types were analyzed and compiled, individual participant’s error totals as well as total errors were compared to reveal if any improvement had been observed and to what extent.
Chapter 4

Results of Study

The pre-assessment questions were designed to highlight a specific multi-digit subtraction error common to 4th-grade students. There are 4 categories of mistakes highlighted by the pre-assessment; subtraction errors, incorrect regrouping, regrouping not necessary, and digit flipping. Majority of mistakes committed during the pre-assessment determined that incorrect regrouping was the most common problem participants encountered, followed by subtraction errors. The intensified instruction, therefore, focused on clarification of the mechanics of regrouping, as well as self-check mechanisms.

The post-assessment questions highlighted the same categories and results indicated that incorrect regrouping remained the most common error, followed by subtraction errors, as indicated in the pre-assessment results. It was noticed that the number of incorrect regrouping, subtraction errors, and the total number of errors decreased following the instruction period.

The error analysis utilized during this study determined where scaffolding of the students’ prior knowledge was indicated. Results of this study revealed an apparent improvement in participants’ recurrence of error resulting from the intensive instruction component of this study. Analysis of participants’ post-assessment revealed an apparent lack of utilization of written self-check verification.

Results of this study will be grouped according to the research question they address.
Participants

After collecting data from all the students participating in this study, including the post-assessment, each participant’s contributions were analyzed. Any student who did not complete either the pre-assessment or post-assessment was eliminated from the study. Final count of participants in this study was 29, with a gender breakdown of 16 boys and 13 girls.

Can error analysis be utilized to determine the most common errors 4th-grade students commit during multi-digit subtraction?

Analysis of the pre-assessment revealed specific areas of difficulty the participants were experiencing. High occurrence of errors in problems requiring multiple regrouping revealed a weakness in a significant number of participants mathematical knowledge and directed the intensive instruction to take place prior to the post-assessment.

After collecting all the data from all students participating in the pre-assessment each participant’s errors, if any, were analyzed. This analysis focused on the errors committed by the participant and categorization of these errors using pre-defined categories.

Each of the questions on the pre-assessment was designed to highlight a specific error common in multi-digit subtraction. Breakdown of the error highlighted by each problem, the number of participant errors committed on that problem, as well as the number of highlighted errors is shown in Table 4.1.
Table 4.1

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Common Error Highlighted</th>
<th>Number of Student Errors</th>
<th>Number of Student Errors in Highlighted Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>Repeated regrouping – Incorrect Regrouping (IR)</td>
<td>19</td>
<td>15</td>
</tr>
<tr>
<td>#2</td>
<td>Flipping digits – Digits Flipped (F)</td>
<td>13</td>
<td>2</td>
</tr>
<tr>
<td>#3</td>
<td>Repeated regrouping – Incorrect Regrouping (IR)</td>
<td>17</td>
<td>14</td>
</tr>
<tr>
<td>#4</td>
<td>Regrouping required by regrouping – Incorrect Regrouping (IR)</td>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>#5</td>
<td>No regrouping required – Subtraction Error (SE)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>#6</td>
<td>Repeated regrouping not required – Regrouping Not Necessary (RNN)</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>#7</td>
<td>Single regrouping – Incorrect Regrouping (IR)</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>#8</td>
<td>Regrouping required by regrouping – Incorrect Regrouping (IR)</td>
<td>14</td>
<td>11</td>
</tr>
<tr>
<td>#9</td>
<td>No regrouping required – Subtraction Error (SE)</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>#10</td>
<td>Flipping digits – Digits Flipped (F)</td>
<td>13</td>
<td>0</td>
</tr>
</tbody>
</table>
Analysis of the types of errors recorded for the pre-assessment is shown in Table 4.2 and Table 4.3.

**Table 4.2**

<table>
<thead>
<tr>
<th>Total Errors</th>
<th>SE Subtraction Error</th>
<th>IR Incorrect Regrouping</th>
<th>RNN Regrouping Not Necessary</th>
<th>Digits Flipped</th>
</tr>
</thead>
<tbody>
<tr>
<td>111</td>
<td>35</td>
<td>69</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

**Table 4.3**

**Categorization of Participant Errors for Pre-Assessment**

- Subtraction Error (SE): 31%
- Incorrect Regrouping (IR): 62%
- Regrouping Not Necessary (RNN): 3%
- Digits Flipped (F): 4%
Categorization indicated that the majority of errors (62%) were due to incorrect regrouping (IR). Regrouping is closely tied to a student’s understanding of number sense, specifically place value. Evaluation determined to focus primary instruction in this subject area for this study. The second largest cause of errors (31%) was subtraction errors (SE). A single lesson focusing on self-check methods was decided to be included to benefit the participants.

Total number of errors possible on the pre-assessment is 40. The average number of errors recorded per participant was 3.83. The maximum number of errors committed by a participant was 20. Eight participants out of the 29 participants committed no errors on the pre-assessment. Three participants out of the 29 participants committed 10 or more errors.

Occurrence of errors averaged for all participants in the pre-assessment revealed that, on average, each participant committed 3.83 errors; 2.38 resulting from incorrect regrouping (IR), 1.21 resulting from subtraction error (SE), 0.10 resulting from regrouping when not necessary (RNN) and 0.14 errors resulting from flipping digits (F).

When the pre-assessment was analyzed on a per participant basis, it was revealed that 14 participants committed at least one incorrect regrouping (IR) error and that five of these committed more than five errors. Analysis also revealed that 14 participants committed at least one subtraction error (SE), with two of these committing more than five errors. No participant committed more than two digit flipping (F) or regrouping when not necessary (RNN) errors on the pre-assessment.

Illustration 4.2 and 4.3 indicate the clear prevalence of the incorrect regrouping (IR) error on the pre-assessment. Utilizing this data, it was determined that instruction to scaffold the participants’ prior knowledge on regrouping was to be administered during this study.
Does short term intensive instruction reduce student errors in this category thereby improving assessment scores?

Analysis of the post-assessment revealed a continuation of high occurrence of errors in problems requiring multiple regrouping. The total number of errors was shown to decrease from 111 to 75, as did the number of errors occurring in the incorrect regrouping category, decreasing from 69 to 53.

After collecting all the data from all students participating in the post-assessment each participant’s errors, if any, were analyzed. This analysis focused on the errors committed by the participant and categorization of these errors using pre-defined categories.

Each of the questions on the post-assessment was designed to highlight a specific error common in multi-digit subtraction. Breakdown of the error highlighted by each problem, the number of student errors committed on that problem, as well as the number of highlighted errors is shown in Table 4.4.
Table 4.4

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Common Error Highlighted</th>
<th>Number of Student Errors</th>
<th>Number of Student Errors in Highlighted Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>Repeated regrouping not required – Regrouping Not Necessary (RNN)</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>#2</td>
<td>Repeated regrouping – Incorrect Regrouping (IR)</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>#3</td>
<td>No regrouping required – Subtraction Error (SE)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>#4</td>
<td>Regrouping required by regrouping – Incorrect Regrouping (IR)</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>#5</td>
<td>Repeated regrouping – Incorrect Regrouping (IR)</td>
<td>7</td>
<td>4</td>
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<td>#6</td>
<td>No regrouping required – Subtraction Error (SE)</td>
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<tr>
<td>#7</td>
<td>Flipping digits – Digits Flipped (F)</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>#8</td>
<td>Regrouping required by regrouping – Incorrect Regrouping (IR)</td>
<td>14</td>
<td>9</td>
</tr>
<tr>
<td>#9</td>
<td>Repeated regrouping – Incorrect Regrouping (IR)</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>#10</td>
<td>Repeated regrouping – Incorrect Regrouping (IR)</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>
Analysis of the types of errors recorded during post-assessment is shown in Table 4.5 and Table 4.6

### Table 4.5

<table>
<thead>
<tr>
<th>Total Errors</th>
<th>SE Subtraction Error</th>
<th>IR Incorrect Regrouping</th>
<th>RNN Regrouping Not Necessary</th>
<th>F Digits Flipped</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>15</td>
<td>53</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

### Table 4.6

**Categorization of Participant Errors for Post-Assessment**

- Subtraction Error (SE): 20%
- Incorrect Regrouping (IR): 71%
- Regrouping Not Necessary (RNN): 5%
- Digits Flipped (F): 4%
Categorization indicated that the majority of errors (71%) were due to incorrect regrouping (IR). The second largest cause of errors (20%) was subtraction errors (SE). The total number of errors committed by the participants dropped from 111 on the pre-test to 75 on the post-assessment. The number of incorrect regrouping (IR) and subtraction error (SE) errors also decreased from the pre-assessment to the post-assessment, as indicated in Table 4.7.

**Figure 4.7**

<table>
<thead>
<tr>
<th>Error Type</th>
<th>Pre-Assessment</th>
<th>Post-Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subtraction Error (SE)</td>
<td>35</td>
<td>15</td>
</tr>
<tr>
<td>Incorrect Regrouping (IR)</td>
<td>69</td>
<td>53</td>
</tr>
<tr>
<td>Regrouping Not Necessary (RNN)</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Digits Flipped (F)</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Total number of errors possible on the post-assessment is 40. The average number of errors recorded per participant was 2.59, a reduction from 3.83 on the pre-assessment.

The maximum number of errors committed by a participant was 20. Ten participants out of the 29 participants committed no errors on the post-assessment. Three participants out of the 29 participants committed 10 or more errors, two of which also committed 10 or more errors on
the pre-assessment. Figure 4.8 shows the change in number of total errors committed by participants between the pre-assessment and post-assessment. This figure indicates a negative trend in number of total errors committed by participants after the short-term intensive instruction.

**Figure 4.8**

![Change in number of total errors from pre-assessment to post-assessment](image)

Occurrence of errors averaged for all participants in the post-assessment revealed that, on average, each participant committed 2.59 errors, down from 3.83 per participant on the pre-assessment. Breakdown of the 2.59 errors revealed the following: 1.83 resulting from incorrect regrouping (IR), 0.52 resulting from subtraction error (SE), 0.14 resulting from regrouping when not necessary (RNN) and 0.10 errors resulting from flipping digits (F).

The number of participants that made at least one incorrect regrouping (IR) error on either assessment declined from 14 on the pre-assessment to 7 on the post-assessment. Of the 14 participants that made at least one IR error on the pre-assessment, 2 increased the number of IR errors on the post-assessment. Figure 4.9 shows the change in number of IR errors from the pre-
assessment to the post-assessment recorded by each participant. The participants committing more IR errors on the post-assessment than on the pre-assessment are shown on the far left.

Figure 4.9

![Change in number of IR errors from pre-assessment to post-assessment](image)

Analysis also revealed that the total number of participants that committed subtraction errors (SE) on either assessment declined from 14 on the pre-assessment to 9 on the post-assessment. Of the 14 participants that made at least one SE on the pre-assessment, all recorded equal or less SE on the post-assessment. It should be noted that the five participants recording more subtraction errors on the post-assessment than on the pre-assessment recorded no subtraction errors on the pre-assessment.

Taking all this data into account reveals a possible correlation between the short-term intensive instruction the participants received in this study and the decrease in overall errors.
committed as well as a decrease in the specific categories of incorrect regrouping (IR) and subtraction errors (SE) recorded on the post-assessment.

**Do metacognitive prompts help students to verify answers in multi-digit subtraction?**

Analysis of all participants’ pre-assessment revealed no apparent attempts to verify answers. Similar analysis of post-assessments revealed decidedly limited apparent attempts to verify answers. Three of the participants in this study utilized the self-check strategies that were included in the instruction portion of this study, to verify at least one of their answers on the post-assessment.

The limited adoption of self-check strategies by participants in this study may correlate to the limited exposure they received to the strategies during the study, as well as their lack of exposure to these strategies prior to the study.

**Summary**

Analysis of results received from this study revealed possible correlation between the error analysis utilized to determine the most common errors committed on the pre-assessment and the reduction of similar errors on the post-assessment following the short-term intensive instruction designed to strengthen the participants understanding of related concepts. The lack of evidence for adoption of self-check strategies by the majority of participants may reveal a limited exposure to the strategies both prior to and during the study.
Chapter 5

Discussion

This chapter of the paper will discuss the findings of the study in relation to the research questions posed at the beginning of the study. Each of the three original research questions will be addressed beneath a heading stating the original question. The chapter will then discuss implications of the information revealed during this study.

Can error analysis be utilized to determine the most common errors 4th grade students commit during multi-digit subtraction?

Many prior research studies have suggested that error analysis can be utilized to determine recurring errors (Kramarski and Zoldan, 2008; Fiori and Zuccheri, 2005; Blando et. al, 1989). The process of error analysis began prior to the participants taking the pre-assessment. From observation in the 4th grade mathematics class, recurring errors were noted. The four most commonly noted recurring errors were;

1. students subtracting the smaller number from the larger number regardless of their position in the problem

2. students incorrectly regrouping when dealing with a 0 in the minuend

3. students regrouping when there was no need for regrouping

4. students making simple subtraction errors due to speed or incomplete subtraction facts

Recognizing these recurring errors in student work involved careful observation while assisting students with their multi-digit subtraction. This background aided in the creation of the pre-
assessment that would highlight these error patterns. Prior to this study, no formal analysis of the errors was undertaken to calculate the frequency of which they occurred in the students’ work.

This study intended to use the error analysis to guide the instruction that the participants would receive. The design of the subtraction problems was such that the categorization of errors would be simplified by the separation of recognized error types into different problems. It was the intention that each problem on the pre- and post-assessment would task the participant to deal with a single common error category. Figure 5.1 illustrates this intention with two problems from the pre-assessment.

**Figure 5.1**

<table>
<thead>
<tr>
<th>1) 9000</th>
<th>2) 1273</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1538</td>
<td>-546</td>
</tr>
</tbody>
</table>

In problem #1, the participant is faced with a problem that requires repeated regrouping to enable the standard algorithm for subtraction to take place. Utilizing the standard algorithm, the participant would begin on the right and recognize that it is not possible to subtract 8 from 0 in the ones column. Faced with this task the participant, following the practiced algorithm, would look to the tens column, the next column to the left, to regroup. Recognizing no value in the tens column, the participant is now forced to move to the next column to the left, in this case the hundreds column to regroup. In this problem, the participant will find no value in the hundreds column and is forced to move another column to the left, in this case to the thousands column, where the value is 9. Following standard regrouping protocol, the participant then regroups 1000 from the 9000, and that 1000 becomes 10 100’s, a necessary step to allow this value to be utilized in the hundred’s column. After removing the 10 100’s, the participant is left with 8000
in the thousands column. Protocol mandates that the next step for this task is to regroup 10 10’s from the 10 100’s, once again, to make it utilisable in the ten’s column. This algorithm now continues once more to regroup 10 1’s into the one’s column. At this point in the problem, the participant now has a larger digit in each place of the minuend; subtraction can now occur by standard means. Effectively the participant now has 8000, 900, 90 and 10 in each respective place value. Following standard subtraction algorithm protocol at this point the participant will begin subtracting the subtrahend from the minuend beginning on the right in the one’s column, and working towards the left to the thousands column, in this problem. 10-8 will result in a remainder of 2, 90-30 will result in a remainder of 60, 900-500 will result in a remainder of 400 and 8000-1000 will result in a remainder of 7000; the final remainder is revealed as 7,462.

This problem was designed to highlight the errors encountered when students are required to repeatedly regroup. This problem highlighted this error group by removing the minuend values and placing zeros. This forced the participants to apply a known algorithm to allow subtraction. The instructions received by participants to “show your work” allowed the researcher to see what algorithm the participant was using and any errors in the application that were present. This simplification of the analysis was performed on all assessments, to allow distinction between the preset error categories.

Problem #2 represented a different error category, specifically the flipping of digits. It was observed in prior research studies (Coker, 1983) that a common solution for students facing a problem where the subtrahend was larger than the minuend, was to subtract the smaller digit from the larger digit, regardless of location. This problem highlighted that possibility by supplying digits that are easily recognized as simple subtraction, such as 6-3=3.
For a participant following standard multi-digit subtraction protocol, the participant would begin on the right of the problem and recognize the impossibility (at this level of mathematics) of subtraction 6 from 3. A regrouping would then take place from the ten’s column and the 3 would be added to the newly regrouped 10 1’s to make a minuend of 13. Subtraction would then take place in the ones column to produce a remainder of 7, then the tens column would produce a remainder of 20. Upon reaching the hundred’s column, the participant is again faced with an impossible task, subtracting 500 from 200. Regrouping would be necessary to regroup 10 100’s from the thousands column and adding this to the 2 100’s already present, this would provide 1200 to subtract the 500 from, leaving a remainder of 700.

*How Error Analysis was applied to this study*

Upon completion of the pre-assessment by all participants, the pre-assessments were collected and error analysis was completed. A key was produced to compare individual answers. Each pre-assessment question was evaluated for correctness. Any problem with an incorrect answer was highlighted and then the error analysis was completed. Beginning on the right side of the remainder, each digit was considered, and connections were sought. If the participant had shown work on the problem, this was taken into account during analysis. An example of analysis is shown in Figure 5.2.
To analyze this problem, the researcher began with the incorrect answer, the remainder. The correct digit in the one’s column along with the regrouping shown verified that the participant correctly completed that function. Moving to the left, the incorrect digit in the remainder indicated an error. Upon inspection, it is clear from the participant’s work that a regrouping error occurred. The participant failed to reduce the value when repeated regrouping occurred, leading to an incorrect minuend digit. This error was labeled “IR” to indicate an incorrect regrouping error. Analysis continued to the next column, revealing an incorrect regrouping “IR” error. Once again the participant failed to reduce the minuend when regrouping occurred. The final column was found to be correct, and analysis shows that the participant correctly reduced the minuend when regrouping was required.

All pre-assessments and post-assessments were analyzed in this manner. In the event of two errors occurring in the same column, the credit for the error was given to the error that preempted the next process. In the event that a subtraction error and an incorrect regrouping error occurred in the same column, the process that occurred first was credited. In this example the regrouping would precede the subtraction, therefore the incorrect regrouping would be credited.
No effort was made to distinguish between types of incorrect regrouping errors based on how the participant dealt with the regrouping, due to the limited scope of this study.

During this study, error analysis was utilized to determine the most common errors committed by 4th-grade students on both assessments. Utilizing prior research studies into common mathematical errors (Kramarski and Zoldan, 2008; Hartman, 2001), it was possible to design assessments that highlighted the most common errors. This study reinforces prior research findings supporting the use of error analysis as a tool to indicate recurrence of common errors. Further research in a broader scope could be utilized to apply this finding in a standard curriculum setting. The methods utilized in this study could be applied to measure the utility of error analysis in this setting. Further refinement of analysis methods would be necessary for practical use of this application is a classroom setting.

**Does short-term intensive instruction lessen student errors in this category thereby improving assessment scores?**

Utilization of the results from the first part of this study appeared to indicate a lack of knowledge, within the participants of this study, in the regrouping category of errors. This category of errors indicated a need for scaffolding of knowledge in the area of regrouping and associated content. This included focus on the mechanics of regrouping, place value and logical processing. Instruction was developed to strengthen the participants understanding in all these areas. This intense instruction was restricted to: 1) only three days and 2) fit within the curriculum and 3) fit within the participants daily mathematics schedule.

**Day 1 - Self-check strategies and logical processing**
Day 1 of the instruction took place within the context of the mathematics class, in the work station. The researcher worked with four participants at a time, for a period of 15 minutes. Participants were given a handout with five mathematic problems, three subtraction and two addition. Handout can be seen in Appendix C. Participants were instructed to put their pencils down before beginning the lesson. The instructions were written at the top of the handout and instructed the participants to make an estimate for each problem and to write the estimate next to the problem, prior to solving the problem. Once all participants had estimates for all problems, they were instructed to solve problems and to show their work. At the conclusion of this exercise, all participants were asked to share their estimates. Estimates that fell outside of the group’s range were questioned and revised if needed. Participants then shared their answers and were instructed to state whether or not their answer conformed with the estimate they had made prior to the mathematical work. In the event that their estimate did not conform to the estimate produced, further investigation was made. In many cases, the participant was not confident in his/her estimation skills and chose to rely on subtraction skills. As the group worked through each answer, and differences were brought to attention, and the participants discussed why they had arrived at the different answers by working through each step of the process they utilized to arrive at their answer. In all cases, the participants were able to recognize the mistake they had made and correct it during this period of metacognition.

At this point in the lesson, the participants were asked if they noticed any similarities between the numbers in the problems. Upon this prompt, at least one participant in each group noticed that the #2 subtraction problem and the #4 addition
problem contained all of the same numbers. The researcher used this as an entry into a discussion about using addition to check the answer of a subtraction problem.

Verification of the difference can be achieved by adding the difference to the subtrahend. If this results in an answer equal to the minuend, then the difference is correct. At this point, most participants shared that this was new information to them and that they had never noticed this connection before. This concluded the first day’s lesson. All participants received the same lesson with only slight variations based upon the mistakes made by the participants in that particular group or the discussion that followed.

Day 2 - Number sense, place value and logical processing

Day 2 of the instruction took place within the context of the mathematics class as the main lesson. The researcher worked with seven or eight participants at a time, for a period of 30 minutes. This lesson took the form of a game called “4 Strikes, You’re Out!” This game was an adaptation of a game designed by Marilyn Burns from her book *Teaching Arithmetic: Lessons for Addition and Subtraction*. This game was selected for the focus on number sense and place value that are highlighted skills. Throughout the lesson, the researcher observed participants for number sense and strategies.

The researcher wrote the following on the board;

\[
\begin{array}{ccccccccc}
\_ & \_ & - & \_ & \_ & = & \_ & \_ \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9
\end{array}
\]

Participants were informed that they had to guess the subtraction problem that the researcher had hidden in a folded piece of paper. The participants were instructed that
they were to guess a single digit. If that digit was in the equation, it would be written in all the appropriate spaces it occurred. If the digit was not in the equation, then it would count as a strike. Once a digit was guessed by the participants, it was crossed off the list below the equation. Participants were allowed four incorrect guesses before the game ended. The researcher altered this lesson to include only subtraction operations.

While engaged in the lesson, participants were prompted to explain their reasoning for guesses. This observation enabled the researcher to informally gather information about the participants’ number sense and place value prior knowledge. It was observed that many participants (>75%) were unable to explain why a digit would occur in the equation, even though they knew it would. For example, during a round of play, the digits in the one’s place presented the following order: 5 - __ = 2. Majority of participants were confident that the missing digit was 3, but few (<10%) were able to articulate why this was the correct response. The researcher observed this scenario repeatedly throughout the lesson. Discussion amongst the participants occurred throughout the lesson, with limited prompts by the researcher. By the conclusion of the lesson, it was observed that a higher percentage of participants (>50%) were able to articulate the reasoning behind their choices. This increase could signify an improvement in the participants’ number sense. Further research in this area would be needed to verify these results. In this study, the goal was to improve the participants’ understanding of logical processing and number sense within the context of multi-digit subtraction.
Day 3 - Regrouping mechanics and place value

Day 3 of the instruction took place within the context of the mathematics class, in the work station. The researcher worked with four participants at a time, for a period of 15 minutes. This lesson focused on the mechanics of the regrouping operation. Base 10 blocks were utilized to activate the visual and tactile senses during this lesson. The purpose of this lesson was to strengthen the participants’ understanding of the individual steps within the process of regrouping as it was observed through prior lessons that the process was unfamiliar to the majority of participants in this study. The researcher utilized questioning prompts and discussion with the participants throughout this lesson. Base-10 blocks were utilized to visualize the need for regrouping when moving an amount from one place value to the next.

Discussion during the lesson, focusing on talk-aloud metacognition (Jacobse and Harskamp, 2012) confirmed earlier suspicions that the mechanics of regrouping were not well understood by the participants in this study. During this lesson, the researcher explained each step of a repeated regrouping problem, with the participants responding to prompts when possible. Remarks heard during this discussion when asked why they regroup were summed up by one participant who remarked “I don’t know why, but this is what they taught me in 3rd grade.” Other participants in this group, as well as other groups, confirmed similar remarks.

In total, all participants received 60 minutes of instruction over the course of three days within their regular mathematics curriculum. This instruction focused on logical processing, metacognition, self-check strategies, number sense and regrouping mechanics.
The day following lesson 3, the participants were given a post-assessment, designed identically to the pre-assessment, but utilizing different problems. Participants were given the same instructions as with the pre-assessment and were given the same time frame in which to take the post assessment.

Raw scores of the post-assessment indicated an improvement from scores recorded on the pre-assessment. Overall number of errors committed decreased from 111 to 75, a decrease of 36 total errors. Of those 36 errors, 16 were incorrect regrouping errors and 20 were subtraction errors. This number does not indicate the true reduction recognized in the analysis as all participants who had made incorrect regrouping errors on the pre-assessment, minus two, committed fewer incorrect regrouping errors on the post-assessment. 15 of the 29 participants showed no improvement due to the fact that they committed no incorrect regrouping errors on either the pre- or post-assessment.

There were two participants that committed more errors on the post-assessment than the pre-assessment. Further analysis of these two participants revealed that all the increase in errors were incorrect regrouping errors. Both participants committed subtraction errors on the pre-assessment but committed no subtraction errors on the post-assessment. Participant #17 committed 10 incorrect regrouping errors on the pre-assessment and 17 on the post-assessment. Participant #23 committed 1 incorrect regrouping error on the pre-assessment and 10 on the post-assessment. This researcher attributes the increase in incorrect regrouping errors committed by these two participants to a further confusion of the concept of regrouping created by the intensive short-term instruction. It was noted that both of these participants indicated no prior knowledge concerning the mechanics of regrouping, they were applying an algorithm learned in a previous grade.
The overall trend revealed by the data indicates a possible correlation between the reduction in total errors as well as incorrect regrouping errors following the intensive short-term instruction period. The researcher is attributing this reduction to the clarification of the regrouping mechanics as well as the focus in instruction highlighting number sense. Further research would be necessary to separate the specific cause for the reduction.

**Do introductory metacognition concepts prompt students to verify answers in multi-digit subtraction?**

Previous studies have indicated the correlation between improved assessment scores and metacognitive strategies (Mevarech and Kramarski, 2003; Bradshaw, 2001). Metacognition was included in this study due to this correlation as well as the interconnection of the process of metacognition with self-analysis. The difficulty inherent in this portion of the study involved a reliable method of metacognition measurement. Studies revealed previous researchers’ difficulty with this measurement as well (Kramarski, Mevarech, and Arami, 2002). One of the most reliable methods to measure metacognition is the talk-aloud method. The researcher decided to eliminate this measurement, on the basis that it has been shown in other studies, to alter participants’ processing, due to the required slowing down of the participants’ responses while working through the thought process ((Kramarski, Mevarech, and Arami, 2002).
Summary

This study was designed to investigate the possibility of utilizing error analysis as a tool for guiding instruction. The utility of this method enables teachers to direct instruction, targeting the problem area highlighted by the analysis. In this study, an assessment was designed with error analysis as a focus, but this method shows promise for use in standard curriculum assessments as well. The results of this study showed a possible correlation between the short-term instruction the participants received in multi-digit subtraction and a reduction in errors on a post-assessment. The area of interest for instructors desiring to utilize this system is to identify the error categories that are prevalent in the area of interest. Observation and research should be utilized in the design of future studies in this subject. Future study should be directed to eliminate the time constraints present in this study, both in the overall study length and the instruction.
References


Gerver, R., & Sgroi, R. (1989). *Focus on computational errors for teachers of basic skills mathematics programs. monograph 139*


Appendix A

Pre-Assessment
9000  - 1538

3006  - 437

3869  - 1365

5756  - 4846

9989  - 3675

403
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Appendix C

Lesson #1 handout
1. Make an estimate about your answer.
2. Write your estimate.
3. Solve the problem. Show your work.
4. Does your answer make sense?

<p>| | | | | |</p>
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Appendix D

Participant pre-assessment with error identification
Chapter 1 Introduction Students make mistakes. This is a part of the growing and learning

Chapter 2

Literature Review
Chapter 3

Methods

67 issues found in this text
Score: 75 of 100
Report generated on Wed, Aug 14 2013 06:31 PM

Contextual Spelling Check 6 issues
× Spelling (6)

× Use of articles (1)
× Comparing two or more things (1)
× Confusing modifiers (2)
× Subject and verb agreement (2)
× Sentence structure (24)
× Wordiness (6)
× Passive voice use (22)

Grammar 58 issues

Punctuation 1 issue
× Punctuation within a sentence (1)

Style and Word Choice 2 issues
× Vocabulary use (2)

Chapter 4

Results of Study
Chapter 5 Discussion This chapter of the paper will discuss the findings of the study in relation to