“If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.” – John Louis von Neumann

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Course Web Site: www ohio edu/cas/math/undergrad/courses-resources/calculus cfm
O.U. MATLAB Web Site: www ohio edu/cas/math/undergrad/courses-resources/matlab/index cfm
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1 Review the Student Handbook for MATH 2301

Welcome to MATH 3300 Calculus III at Ohio University. We hope that you excel. If you did not take Calculus I or Calculus II at Ohio University, you need to get the Student Handbook for MATH 2301 and review it carefully. If you did take Calculus I or II here, then you need to review the Handbook for MATH 2301. It contains among other things:

- Policy on Students with Disabilities and Policy on Academic Integrity.
- Some general advice about studying Calculus.
- Material you should know before studying Calculus.
- Study Aids & instructions about MATLAB Assignments.

2 Material You Should Know Before MATH 3300

Math is cumulative. To help you succeed in Calculus III these pages summarize the material from Calculus I & II. Your Math 3300 instructor will take for granted that you know and can use the following material. You should review it before starting the course and before each test.

If you are new at Ohio U. or did not take MATH 2301 and 2302 here you should obtain copies of the Student Handbook for MATH 2301 & Student Handbook for MATH 2302 from the course web site and read them carefully. If you already have copies of the handbooks for MATH 2301 and 2302 then you should review them now.

Pre-Calculus Material
As a general rule you should understand and be able to use all the material in reference pages 1 - 4 in the inside front and back covers of the textbook. You should memorize most of the formulas on those pages. Page 4 of the Student Handbook for MATH 2301 contains more detailed information.

Limits and Derivatives.
See the Student Handbook for 2302.

Definitions related to Integration:
\( F(x) \) is an Antiderivative of \( f(x) \) means: \( F'(x) = f(x) \)

Riemann Sum - \( R_n \), \( L_n \) and \( M_n \) are examples.

Definite Integral - The limit as \( n \to \infty \) of any Riemann sum.

Average of a Function: \( f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) \, dx \)

Integration Theorems:
If \( f(x) \) is continuous on \([a, b]\) then \( \int_a^b f(x) \, dx \) exists, however, it might not be expressible in terms of elementary (usual) functions.

Fundamental Theorem of Calculus: If \( f \) is continuous, and \( F \) is an antiderivative of \( f \), then

Part 1: \( \frac{d}{dx} \int_a^x f(s) \, ds = f(x) \). Part 2: \( \int_a^b f(x) \, dx = F(b) - F(a) \).
Riemann sums and numerical integration:

Let \( (x_0, x_1, x_2, \ldots, x_n) \) be evenly spaced, \( a = x_0, b = x_n, \Delta x = x_i - x_{i-1} = (b-a)/n \)

Let \( (y_0, y_1, y_2, \ldots, y_n) \) be values of \( f(x) \), i.e. \( y_i = f(x_i) \)

left sum - \( L_n = \Delta x \sum_{i=0}^{n-1} y_i = \Delta x (y_0 + y_1 + \ldots + y_{n-1}) \)

right sum - \( R_n = \Delta x \sum_{i=1}^{n} y_i = \Delta x (y_1 + y_2 + \ldots + y_n) \)

trapezoid rule - \( T_n = \Delta x/2 (y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n) \)

Simpson’s rule - \( S_n = \Delta x/3 (y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n) \)

midpoint sum/rule - \( M_n = \Delta x \left( f(\bar{x}_1) + f(\bar{x}_2) + \ldots + f(\bar{x}_n) \right) \)

where \( \bar{x}_i = (x_{i-1} + x_i)/2 \), i.e. the mid-points of the intervals: \([x_{i-1}, x_i]\).

L'Hopital’s rule:

\[
\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \text{ if the first limit is } \frac{0}{0} \text{ or } \frac{\infty}{\infty} \text{ and the 2nd limit exists.}
\]

Change \( \infty \cdot 0 \) and \( \infty - \infty \) to \( \frac{0}{0} \) or \( \frac{\infty}{\infty} \). Use \( \ln \) for \( 0^0, \infty^0, 1^\infty \)

Integrals to memorize:

\[
\begin{align*}
\int u^n \, du &= \frac{u^{n+1}}{n+1} + C, \quad n \neq -1. \\
\int \frac{1}{u} \, du &= \ln |u| + C \\
\int e^u \, du &= e^u + C \\
\int \cos u \, du &= \sin u + C \\
\int \sin u \, du &= -\cos u + C \\
\int \frac{1}{\sqrt{1-u^2}} \, du &= \sin^{-1} u + C \\
\int \frac{1}{1+u^2} \, du &= \tan^{-1} u + C \\
\int \sec^2 u \, du &= \tan u + C \\
\int \sec u \tan u \, du &= \sec u + C \\
\int \sinh u \, du &= \cosh u + C \\
\int \cosh u \, du &= \sinh u + C
\end{align*}
\]

\[
\int f(x) + g(x) \, dx = \int f(x) \, dx + \int g(x) \, dx
\]

Three major integration techniques:

Substitution: Recognize \( \int f(g(x)) \cdot g'(x) \, dx \), set \( u = g(x) \), and get: \( \int f(u) \, du \).

By parts: \( \int u \, dv = uv - \int v \, du \), first identify \( dv \) that can be integrated.

Partial fractions: First reduce by dividing

Real roots: \[
\begin{align*}
\cdots \quad \frac{A_1}{ax + b} + \frac{A_2}{(ax + b)^2} + \cdots + \frac{A_n}{(ax + b)^n}
\end{align*}
\]

Complex roots: \[
\begin{align*}
\cdots \quad \frac{Ax + B}{ax^2 + bx + c}
\end{align*}
\]

Arc Length:

Parametric: \( x = f(t), y = g(t), \alpha \leq t \leq \beta \): \( L = \int_{\alpha}^{\beta} \sqrt{f'(t)^2 + g'(t)^2} \, dt \)

Polar: if \( r = r(\theta), L = \int_{\alpha}^{\beta} \sqrt{r^2 + (dr/d\theta)^2} \, d\theta \)

Physics:

Work: \( W = \mathbf{F} \cdot \mathbf{D} \) (constant force & linear motion)

Torque: \( \tau = \mathbf{r} \times \mathbf{F} \)
3 Syllabus for 3300


Review of Conic Sections*
10.1 – 3-D Coordinate Systems
10.2 – Vectors
10.3 – The Dot Product
10.4 – The Cross Product
10.5 – Equations of Lines and Planes
10.6 – Cylinders and Quadric Surfaces
10.7 – Vectors Functions and Space Curves
10.8 – Arc Length and Curvature**
10.9 – Motion in Space
11.1 – Functions of Several Variables
11.2 – Limits and Continuity
11.3 – Partial Derivatives
11.4 – Tangent Planes and Approximation
11.5 – The Chain Rule
11.6 – Directional Derivatives and Gradient
11.7 – Maximum and Minimum Values
11.8 – Lagrange Multipliers

12.1 – Double Integrals Over Rectangles
12.2 – Double Integrals On General Regions
12.3 – Double Integrals in Polar Coord.
12.4 – Applications of Double Integrals
12.5 – Triple Integrals
12.6 – Triple Ints. in Cylindrical Coords.
12.7 – Triple Ints. in Spherical Coords.
12.8 – Changes of Variables in Integrals

13.1 – Vector Fields
13.2 – Line Integrals
13.3 – Fund. Theorem for Line Integrals
13.4 – Green’s theorem
13.5 – Curl and Divergence
13.6 – Parametric Surfaces
13.7 – Surface Integrals
13.8 – Stokes’ Theorem
13.9 – The Divergence Theorem

* – Note that Conic Sections are no longer covered in MATH 1300
** – Skip Normal and Binormal Vectors.

This is a TAGS (Ohio Transfer Assurance Guides) course and the above topics follow closely the material prescribed by TAGS.
# Homework Problems for MATH 3300

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* - optional, but your instructor might assign some problems.