“If a healthy minded person takes an interest in science, he gets busy with
his mathematics and haunts the laboratory.” - W.S. Franklin

Department of Mathematics
Ohio University
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1 Review the Student Handbook for MATH 2301

Welcome to MATH 2302 Calculus II at Ohio University. We hope that Calculus II is an exciting, challenging and rewarding experience.

If you did not take Calculus I at Ohio University, you need to get the Student Handbook for MATH 2301 and review it carefully. If you did take Calculus I here, then you also need to review the Handbook for MATH 2301. It contains among other things:

- Policy on Student Accessibility.
- Policy on Academic Integrity.
- Some general advice about studying Calculus.
- Material you should know before studying Calculus.
- Web Assigns.
- Instructions about MATLAB Assignments.

2 Material to know before starting MATH 2302

Because math is cumulative, mastery and review of previous material is essential for your success. Your MATH 2302 instructor will take for granted that you have mastery of the material below. You should always review it before tests. As a general rule you should understand and be able to use all the material in reference pages 1 - 4 in the inside back cover of the textbook. You should memorize most of the formulas on those pages.

From Calculus I:

Limits:
Meaning of Limits
Limits Laws
One-sided Limits
Infinite Limits
Basic Trig. Limit: \( \lim_{x \to 0} \frac{\sin x}{x} = 1 \)

Continuity:
Continuity means: \( \lim_{x \to a} f(x) = f(a) \). Polynomials, Rational, Algebraic, Exponential, Logarithmic, Trigonometric Functions all are continuous where defined.

Definition of derivative:
Wherever it exists: \( f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \).

Differentiation rules:
Linear Combinations: \( \frac{d}{dx}(af(x) + bg(x)) = af'(x) + bg'(x) \).
Compositions (Chain rule): \( h(x) = f(g(x)), \Rightarrow h'(x) = f'(g(x))g'(x) \)
Products: \( \frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x) \).
Quotients: \( \frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \).
**Derivatives you must memorize:** Reference Page 5 formulas: 1-9, 11, 13-15, 17, 19, 21, 25-27

**Implicit Differentiation:** Means: Differentiate the whole equation: \[ \frac{d}{dx}(\ldots = \ldots). \]
Assume \( y \) is a function of \( x \), so when you differentiate \( y \) you get \( dy/dx \).

**Graphs:**

**Vertical Line Test**
Know the graphs of functions in the reference pages.

**Graphing by hand:**
- Find zeros \( f(x) = 0 \).
- Find vertical asymptotes at places where \( f(x) \) is undefined.
- Check for horizontal and slant asymptotes.
  - Divide quotients \( p(x)/q(x) \) if \( \text{degree}(p) \geq \text{degree}(q) \).
- Find \( f'(x) \) and check for critical points.
  - Either \( f'(x) = 0 \) or \( f'(x) \) does not exist.
- Find \( f''(x) \) and check for possible inflection points.
  - Either \( f''(x) = 0 \) or \( f''(x) \) does not exist.
- List critical and inflection points.
- Graph.
- Clearly label all features.

**Linear approximations:** For \( x \approx x_0 \):
\[
 f(x) \approx L(x) = f(x_0) + f'(x_0)(x - x_0).
\]

**Max-Min:**
- Global vs. Local extrema
- Find critical points: \( f'(x) = 0 \) or \( f'(x) \) does not exist.
- On a closed interval \([a, b]\), must also check \( f(a) \) and \( f(b) \).
- First Derivative Test: Check sign of \( f'(x) \) on each side of the critical point.
- Second Derivative Test: Check sign of \( f''(x) \) at the critical point.

**Intermediate Value Theorem** If \( f(x) \) is continuous on \([a, b]\) and \( d \) is any real number between \( f(a) \) and \( f(b) \), then there exists a point \( c \), \( a < c < b \), such that \( f(c) = d \).

**Max/Min Theorem** If \( f \) is continuous on the interval \([a, b]\), then \( f \) has a maximum value and a minimum value on \([a, b]\).

**Mean Value Theorem** If \( f(x) \) is continuous on \([a, b]\) and differentiable on \((a, b)\), then there exists a point \( c \), \( a < c < b \), such that
\[
 f'(c) = \frac{f(b) - f(a)}{b - a}.
\]

**Definitions related to Integration:**
- \( F(x) \) is an Antiderivative of \( f(x) \) means: \( F'(x) = f(x) \)
- Riemann Sum - \( R_n, L_n \) and \( M_n \) are examples.
- Definite Integral - The limit as \( n \to \infty \) of any Riemann sum.

Average of a Function:
\[
 f_{\text{avg}} = \frac{1}{b - a} \int_a^b f(x) \, dx
\]
Integration Theorems:
If \( f(x) \) is continuous on \([a, b]\) then \( \int_{a}^{b} f(x) \, dx \) exists, however, it might not be expressible in terms of elementary (usual) functions.

Fundamental Theorem of Calculus: If \( f \) is continuous, and \( F \) is an antiderivative of \( f \), then

Part 1: \( \frac{d}{dx} \int_{a}^{x} f(s) \, ds = f(x) \). Part 2: \( \int_{a}^{b} f(x) \, dx = F(b) - F(a) \).

Riemann sums:
Let \((x_0, x_1, x_2, \ldots, x_n)\) be evenly spaced, \( a = x_0, b = x_n, \Delta x = x_i - x_{i-1} = (b - a)/n \)
Let \((y_0, y_1, y_2, \ldots, y_n)\) be values of \( f(x) \), i.e. \( y_i = f(x_i) \)
left sum - \( L_n = \Delta x \sum_{i=0}^{n-1} y_i = \Delta x (y_0 + y_1 + \ldots + y_{n-1}) \)
right sum - \( R_n = \Delta x \sum_{i=1}^{n} y_i = \Delta x (y_1 + y_2 + \ldots + y_n) \)
midpoint sum - \( M_n = \Delta x \sum_{i=1}^{n} f(\bar{x}_i) \)
where \( \bar{x}_i = (x_{i-1} + x_i)/2 \), i.e. the mid-points of the intervals: \([x_{i-1}, x_i]\).

Draw the diagrams for each of these sums.

Antiderivatives (indefinite Integrals) to memorize:
\[
\begin{align*}
\int u^n \, du &= \frac{u^{n+1}}{n+1} + C, \quad n \neq -1. \\
\int \frac{1}{u} \, du &= \ln |u| + C \\
\int e^u \, du &= e^u + C \\
\int \cos u \, du &= \sin u + C \\
\int \sin u \, du &= -\cos u + C \\
\int \frac{1}{\sqrt{1-u^2}} \, du &= \sin^{-1} u + C \\
\int \frac{1}{1+u^2} \, du &= \tan^{-1} u + C \\
\int \sec^2 u \, du &= \tan u + C \\
\int \sec u \tan u \, du &= \sec u + C \\
\int \sinh u \, du &= \cosh u + C \\
\int \cosh u \, du &= \sinh u + C \\
\int f(x) + g(x) \, dx &= \int f(x) \, dx + \int g(x) \, dx
\end{align*}
\]

Substitution: Recognize \( \int f(g(x)) \, g'(x) \, dx \), and set \( u = g(x) \).
For definite integrals, don’t forget to change the limits of integration.
3 Syllabus for 2302


6.1 – Integration by Parts
6.2 – Trig Integrals & Substitutions
6.3 – Partial Fractions
6.5 – Approximate Integration
3.7 – l'Hospital’s Rule*
6.6 – Improper integrals
7.1 – Areas between curves
7.2 – Volumes
7.6 – Application - (work only)
7.7 – Differential Equations**
8.1 – Sequences

8.2 – Series
8.3 – The Integral and Comparison Tests
8.4 – Other Convergence Tests
8.5 – Power Series
8.6 – Representations as Power Series
8.7 – Taylor and Maclaurin Series
8.8 – Applications of Taylor Polynomials
9.1 – Curves Defined by Parametric Eqs.
9.2 – Calculus with Parametric Curves
9.3 – Polar Coordinates
9.4 – Areas & Lengths in Polar Coordinates

* An introduction was given in 2301. Cover all forms, emphasize differences and powers.
** Optional, but your instructor may choose to cover this.

This is a TAGS (Ohio Transfer Assurance Guides) course and the above topics follow closely the material prescribed by TAGS.
## 4 Homework Problems for MATH 2302

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* Optional, but your instructor may assign some problems.