Factoring Expressions and Solving Equations[1]

Sample Solution

1. The command `clear` clears all variables.

   The command `syms x` declares `x` to be a symbolic variable.

   The command `expr1 = (x-1)*(x-2)*(x-3)*(x-4)*(x-5)` gives the label `expr1` to the expression $(x-1)(x-2)(x-3)(x-4)(x-5)$.

   The command `expand` is used to expand or multiply out an expression. Expanding `expr1` yields

   $$x^5 - 15x^4 + 85x^3 - 225x^2 + 274x - 120$$

   The command `factor` is used to factor an expression. Here we factor the expression that results from expanding `expr1`. Thereby, we recover `expr1`, which is what one would expect.

   Solving $expr2 = 0$ gives $x = 1, 2, 3, 4, 5$.

   The relationship between solving and factoring is as follows:
   Let $p(x)$ be any polynomial. $x = x_0$ is a solution of $p(x) = 0$ if and only if $x - x_0$ is a factor of $p(x)$.

2. MATLAB is not able to factor $x^4 + 3x^3 + 3x^2 + x + 3$.

   MATLAB *is* able to solve $x^4 + 3x^3 + 3x^2 + x + 3 = 0$ symbolically; however, the solutions it gives are extremely long and complicated.

   The command `double(ans)` numerically evaluates `ans`, in this case the symbolic solutions to $x^4 + 3x^3 + 3x^2 + x + 3 = 0$. (Note. `double(ans)` does not mean `2 * ans`; `double` is short for double precision.) The numerical solutions we obtain are

   $$x = 0.2289 \pm 0.8595i, -1.7289 \pm 0.8959i$$

   One reason an exact, symbolic solution may not be as useful as an approximation is that when we measure things we usually use decimals or very simple fractions.

3. Solving $expr3 - 3$ gives $x = 0, -1, -1, -1$. The reason the answer is so nice is that $expr3 - 3$ is $x^4 + 3x^3 + 3x^2 + x = x(x^3 + 3x^2 + 3x + 1) = x(x+1)^3$.

4. MATLAB is unable to factor $expr4$ or to solve $expr4 = 0$, symbolically. MATLAB gives the numerical solutions (to four decimal places)

   $$x = 0.9615, 2.2093, 2.7342, 4.1510, 4.9541$$

   None of the algorithms MATLAB uses to obtain symbolic solutions to polynomial equations work for this equation, so MATLAB provides an approximate numerical solution.

   It is known from higher mathematics that, for polynomial equations of degree five or higher, a symbolic solution is not always possible.