Linear versus Nonlinear [1]

1. Try the following commands (at the prompt and then press Enter):
   (a) `syms t y`
   (b) `dsolve('D2y+y=0', 'y(0)=2', 'Dy(0)=2')`
   (c) `ezplot(ans, [0, 50])`
   (d) Change the initial conditions to $y(0) = .2, y'(0) = .2$
   (Type command b as: `dsolve('D2y+y=0', 'y(0)=.2', 'Dy(0)=.2')`).
   How does this affect the solution?
   (e) Explain exactly what happened.

2. Repeat the above procedure to solve the the following differential equation. Use the initial conditions: $y(0) = 1, y'(0) = 1$.

$$y''(t) - y(t) + y^3(t) = 0$$

   Why is MATLAB unable to solve this equation symbolically?

3. Note that the equation in #2 may be written as a system by the substitution $y_1 = y, y_2 = y'$. This produces the system:

$$\frac{y_1}{dt} = y_2, \qquad \frac{y_2}{dt} = y_1 - y_1^3 \qquad (1)$$

   Now try the following:

   (a) `F=inline('[y(2);y(1)-y(1)^3]', 't', 'y')` . . Makes `F` the r.h.s. of (1).
   (b) `T = 0:.01:50;` . . . . Don't skip the semicolon!
   (c) `[T, Y] = ode45(F, T, [2,2]);`
   (d) `plot(T, Y(:,1))`

   Try changing the initial conditions to $y(0) = .2, y'(0) = .2$. How does this effect the solution? How does this differ from the linear case?

4. Use the commands you learned in #3 to numerically solve and plot:

$$y''(t) - y(t) + y^3(t) = \sin(t), \qquad y(0) = 1, \quad y'(0) = 1$$

   on the interval $t = [0, 100]$. How does the graph of this solution differ from all the graphs of solutions you have seen for linear equations?

5. Prepare a brief ($<$ 1 page) written report answering all the questions. Use complete sentences and standard mathematical notation. Do **not** get a printout.

This assignment demonstrates that the solutions of linear equations are very "tame" compared with solutions of nonlinear equations.

 Please address comments to young@math.ohiou.edu.