Partial Derivatives[1]

1. Enter the following commands:
```
syms x y
f = x*y*(x^2-y^2)/(x^2+y^2)
fx = diff(f, x)
fx = simplify(fx)
subs(fx,{x, y},{0, y})
```
..... This is $f_x(0, y)$.

2. Define $f(0, 0) = 0$ and compute, by hand,
$$f_x(0, 0) = \lim_{h\to 0} \frac{f(h, 0) - f(0, 0)}{h}.$$
Why is it necessary to use the definition to compute $f_x(0, 0)$?

3. Try:
```
fy = simplify(diff(f, y))
subs(fy, {x, y}, {x, 0})
```
..... This is $f_y(x, 0)$.
Then, compute $f_y(0, 0)$ by hand.

4. Compute, by hand,
$$f_{xy}(0, 0) = (f_x)_y(0, 0) = \lim_{k\to 0} \frac{f_x(0, k) - f_x(0, 0)}{k}$$
and
$$f_{yx}(0, 0) = \lim_{h\to 0} \frac{f_y(h, 0) - f_y(0, 0)}{h}$$
What do you notice about $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$?

5. Try:
```
fxy = diff(fx,y)
fxy = simplify(fxy)
ezmesh(fxy)
```
What do you notice about the graph of f_{xy}?

6. Either obtain a printout of the graph, or, carefully sketch it by hand, making sure to clearly label axes.

7. Using complete sentences and standard mathematical notation, write a brief report, showing your hand calculations and answering all the questions.

The user is reminded of the definition of derivative and encounters a situation where it must be used. The user also encounters a situation where second derivatives are not continuous and $f_{xy} \neq f_{yx}$.