Lagrange Multipliers[1]

1. To find the points on the ellipse $4x^2 + 9y^2 = 36$ that are nearest to and farthest from the point (1,1), using the method of Lagrange multipliers, one needs to solve the system of equations
   $2(x-1) - 8\lambda x = 0$
   $2(y-1) - 18\lambda y = 0$
   $4x^2 + 9y^2 - 36 = 0$

   Carefully **derive** this system by hand. Do **NOT** try to solve the system by hand. Instead, solve the system using the commands:

   - `syms L x y` .... (Note that we use "L" instead of "λ".)
   - `[L,x,y]=solve(2*(x-1)-8*L*x,2*(y-1)-18*L*y,4*x^2+9*y^2-36)`
   - `double([L,x,y])` .... (Elements in square brackets must be in alphabetical order.)

   Explain what happened. What is the nearest point? What is the farthest point? Give solutions to four decimal places.

2. Adapt the procedure in #1 to find the points on the ellipsoid

   $$64x^2 + 144y^2 + 36z^2 = 576$$

   that are nearest to and farthest from the point (1, 1, 1). Write down the system you are solving and answer the questions above for this example.

3. What are your observations about symbolic versus numerical computations from #1 and #2?

4. Using complete sentences and standard mathematical notation, write a brief report (1 page only), showing your hand calculations and answering all the questions.

The system of equations resulting from relatively straightforward Lagrange multiplier problems can be very difficult, if not impossible, to solve in closed form. In this exercise MATLAB is used to solve such systems. Students are asked to compare symbolic versus numerical solutions.