Monte Carlo Integration[1]

1. Type `help rand` and read the first paragraph of resulting help page. Next enter the following:
   ```
   x = rand(10,1)
   y = x.^3
   avg = sum(y)/10
   ```
   Figure out what happens in these command before you proceed.

2. Enter the following sequence commands:

   (a) `n = 10`

   (b) `x = rand(n,1); avg=sum(x.^3)/n`

   (c) Use the ↑ key to recall this line and then press `Enter` again.

   (d) Obtain 10 estimates this way and record the values you get along with the absolute error of each estimate. You can have MATLAB calculate the absolute error for you conveniently by including at the end of line of #1(b): `error = abs(.25 - avg)`.

   (e) Explain why this is an approximation of $\int_0^1 x^3\,dx$. (It has to do with the average of a function.)

3. Enter the command `n = 100` and use the ↑ key to recall the line in #1(b) again. Press the enter key to execute this line. Obtain and record 10 estimates this way along with the absolute errors.

4. Repeat this process using `n = 1000` and `n = 10000`.

5. Make a chart showing the relationship between the sample size `n` and the arithmetic mean of the absolute errors of the estimates with sample size `n`. The data should reflect the relationship $|E_n| \approx Kn^{-r}$. Use the data and logarithms to determine the constants K and r.

6. For the Trapezoid rule `r = 2` and for Simpson's rule `r = 4`. How does the random method introduced here compare with the Trapezoid and Simpson's methods of numerical integration? Which is the most accurate, which the least?

7. Prepare a brief written report answering all the questions. Use complete sentences and standard mathematical notation. Do **not** get a printout.

This demonstrates the connection between averages and integrals. Because this technique is efficient in higher dimensions, variants (known as Monte Carlo methods) are actually used in practice to evaluate integrals.

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