

Improper Integrals¹

1. Enter the following sequence of commands:

```
syms x
int(1/sqrt(x^6 + 1), 0, inf) ..... Calculates symbolically
double(ans) ..... Converts to a numeric format
quadl('1./sqrt(x.^6 + 1)', 0, inf) ..... Calculates numerically
```

2. Use the commands above to evaluate the following integrals (you will encounter error messages in some of them):

(a) $\int_0^{\infty} \frac{1}{x^{2/3}} dx$ (Use $1/x^{(2/3)}$.)

(b) $\int_1^{\infty} \frac{1}{x+1} dx$

(c) $\int_1^{\infty} \frac{\ln x}{x^2} dx$ (Use `log` for natural logarithm.)

(d) $\int_0^{\infty} \sin^2(x) dx$ (Use $(\sin(x))^2$.)

3. Try to use MATLAB to evaluate the following functions using commands in #1:

(a) $\int_{-1}^1 \frac{1}{x^2} dx$

(b) $\int_0^1 \frac{1}{\sqrt{x}} dx$

4. What are some problems with calculating improper integrals numerically?

5. Try the following:

```
int(1/x^5, 1, inf)
int(sin(x^3)/x^5, 1, inf)
double(ans)
```

Comparing the integrands of these two integrals, should the second one converge? Does the answer for the second integral make sense?

6. Prepare a brief (< 1 page) written report describing what happened and answering all the questions. Use complete sentences and standard mathematical notation. Writing quality will play a part in the grade.

This exercise explores improper integrals both symbolically and numerically. Evaluating improper integrals symbolically is precarious because it is hard for the computer to handle the symbol ∞ correctly. Evaluating numerically is also difficult because one cannot actually compute all the way to ∞ , one must stop at some finite place.

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