Newton's Method[1]

1. (a) Try the following commands (at the prompt and press Enter):

```
syms x
format long ................................. Sets displayed digits to 15.
f = x^3 - 3*x^2 + 1
f1 = simplify(diff(f))
g = simplify(x - f/f1)
p = .1
p = subs(g, p)
```

   (b) Repeat the command `p = subs(g, p)` until `p` stops changing. (Use the up-arrow key to recall the command instead of typing it.)

   (c) Assuming the final value is correct, how many steps did it take to get 7 decimal places of accuracy? How many steps for 14 decimal places?

2. (a) Type `p = .5` and repeat `p = subs(g,p)` until `p` stops changing. To what do the approximations converge this time?

   (b) Repeat, but start with `p = 3.0`.

   (c) Why can Newton's method give three different answers for three different starting points? (Hint: Use `ezplot(f)` to look at `f(x)`.)

3. Set `p = .11065934333376` and repeat `p = subs(g, p)` until it converges. How many iterations does it take this time?

4. Repeat the process in (1), starting with `p = .1` for the function

$$f(x) = \frac{(x - 3/4)^{1/3}}{x^{1/3}} \qquad \texttt{(f = ((x-3/4)\^{}(1/3))/(x\^{}(1/3)))}$$

   Write down the first 20 iterations. Do they seem to be converging to anything? Plot them on the interval `[0,1]`.

   Does $f(x) = 0$ have a solution on $[0, 1]$? Try that point as the initial guess and see what happens.

   Next, try starting with `p = 0.0`. What is the value of f at 0.0?

5. Can one always rely on Newton's method? What are some things to be careful about?

6. Prepare a brief ($<$ 1 page) written report answering all the questions. Use complete sentences and standard mathematical notation. Do **not** get a printout.

The user observes that Newton's method converges very fast for the certain functions and certain starting points. The convergence can be slow for other starting points and the final answer can depend on the starting point. Further, some functions lead to Newton's method iterations which are actually chaotic (random-like).

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