Final - MATH 2301 Calculus I - Spring 2016

Name: **Instructor's name :**

All problems are worth 10 points for a total of 200 points.
Partial credit will be given for correct work. Do not simplify your answers unless requested.

1. Differentiate the functions:

   (a) $g(t) = \cos(t)\ln(t)$, (b) $h(t) = \dfrac{e^{2t}}{\tan t}$.

2. Find the derivatives of the functions

   (a) $f(x) = x\sec(kx)$, k is a constant, (b) $y = \dfrac{x^2+3}{\sqrt{x}}$.

3. Find the derivatives of the functions:

   (a) $f(x) = \sin^{-1}(2x+1)$, (b) $h(x) = \displaystyle\int_1^x \frac{1}{1+t^3}\,dt$.

4. Use Newton's method with the specified initial approximation x_1 to find x_2, the second approximation to the root of the given equation.

   $x^3 + x + 3 = 0$, $x_1 = -1$.

5. Each side of a cube is increasing at a rate of 6 cm/s. At what rate is the volume of the cube changing when the volume is 8 cm^3?

6. Find the derivative of the function: $f(x) = 2x^2$ using the definition of derivative (i.e. a limit) and check your answer using differentiation rules.

7. Find the limit: $\displaystyle\lim_{x\to+\infty} \frac{x^3}{e^{x^2}}$.

8. Sketch the graph of a function f that:
   Is discontinuous at 0 but continuous from the right at 0,
   $f'(x) < 0$ for $|x| < 2$ and $f'(x) > 0$ for $|x| > 2$.
   Has vertical asymptote at $x = 3$.

9. Use implicit differentiation to find the tangent line to the curve at the given point:

   $x^2 + xy + y^2 = 3$, $(1, 1)$.

10. If $f'''(t) = \cos t$, find $f(t)$.

11. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half second intervals is given in the table. Find good upper and lower estimates for the distance that she travelled during these three seconds. Do not simplify.

t (s)	0	.5	1	1.5	2	2.5	3
v (ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2

12. Use the Midpoint rule with $n = 4$ to approximate the integral. (Do NOT simplify.) Include a drawing of your subdivision of the interval and the midpoints used in the approximation.

    $$\int_0^8 \sin\sqrt{x}\,dx$$

13. Find any critical points and the absolute maximum and absolute minimum values of the function $f(x) = 12 + 3x - x^2$ on the interval $[0, 5]$.

14. Evaluate the indefinite integral using a u substitution. (a and b are constants.)

    $$\int \frac{a + 3bx^2}{(ax+bx^3)^3}\,dx.$$

15. Find the linearization $L(x)$ of the function $f(x) = \sqrt{1-x}$ at $a = 0$ and use it to find an approximation to $\sqrt{.99}$.

16. Show that the equation has exactly one real root: $2x + \cos x = 0$.
(a) Use the Intermediate Value Theorem to show that there is at least one root.
(b) Use the Mean Value Theorem to show why there cannot be more than one root.

17. If $F(x) = \dfrac{5x}{1+x^2}$ find $F'(2)$ and use it to find an equation of the tangent line to the graph of the curve $y = F(x)$ at $x = 2$.

18. A farmer wants a rectangular pen next to a large barn, using the wall of the barn as one side of the pen. If the farmer wants the area enclosed to be 1,800 m^2, what are the dimensions of the fence that minimize the length of fencing used? Make a sketch of the fence and barn, clearly showing the variables you are using.

19. Evaluate the integral $\displaystyle\int_1^2 \left(\frac{x}{2} + \frac{2}{x}\right)\, dx$.

20. (a) Find any horizontal or vertical asymptotes. (Indicate with dashed lines on the graph.)
(b) Find the intervals on which $f(x)$ is increasing or decreasing.
(c) Find any critical points of f.
(d) Find the intervals of concavity and any inflection points.
(e) Use the information from (a)-(d) to sketch the graph.

$$y = \frac{x}{1-x}, \quad \left(y' = \frac{1}{(1-x)^2}, \quad y'' = \frac{2}{(1-x)^3}\right).$$

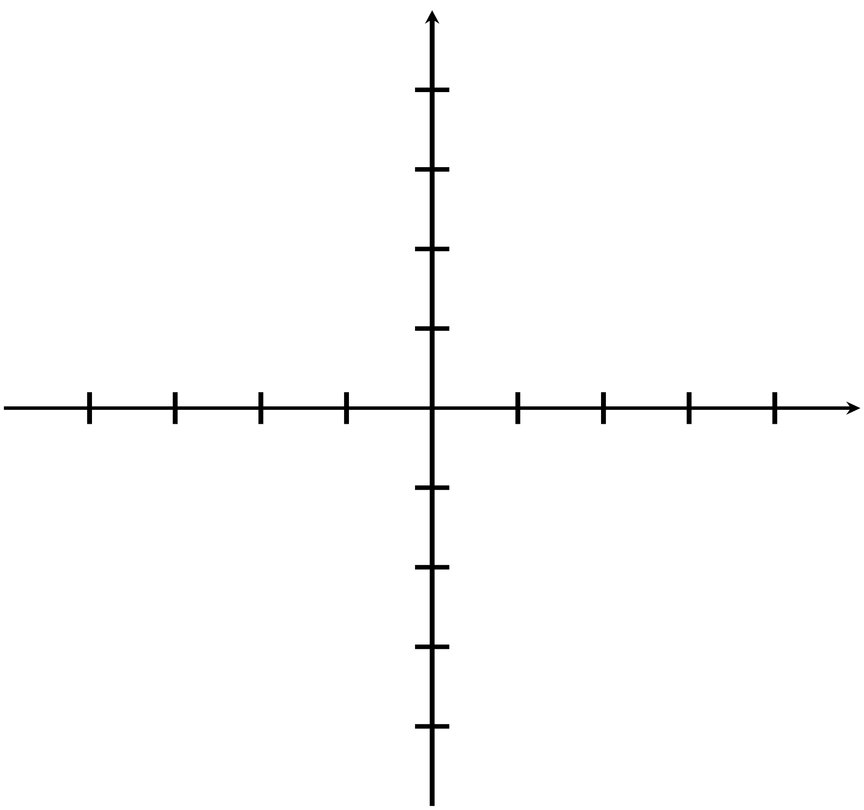

Figure 1: Example of axes printed for use in graphing problems.