

## Math 1300–Precalculus Final Exam

**Your Name:** \_\_\_\_\_

**Your Instructors Name:** \_\_\_\_\_

**Instructions:** Please justify all your answers. No partial credit will be awarded for answers without calculations or explanations. You may **Not** use any electronic technology. Please sign your name below signifying that you have read, understood and will abide by these directions and will not cheat.

**Your Signature:** \_\_\_\_\_

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Total
Possible Points	10	10	12	10	10	15	10	15	12	14	12	10	10	10	160
Your Scores															

**Problem 1:** (10 points) Choose one of the following pairs (please circle your choice).

(A)  $e^{7x-8} \geq 2$       and       $\log_6(x+4) + \log_6(3-x) = 1$

----- OR -----

(B)  $\frac{1-\ln(x)}{x^6} \leq 0$       and       $\frac{4}{1+3e^{-2x}} = 3$

Solve the inequality and express your answer in interval notation.

Solve the equation and express your answer in set (curly braces) notation.

**Problem 2:** (10 points) Given the sequence  $11, 7, 3, -1, -5, \dots$

1. Find an expression for the  $n^{\text{th}}$  term of the sequence.
2. Use your formula to explicitly find the  $50^{\text{th}}$  term.
3. Find the sum of the first 50 terms of the sequence.

**Problem 3:** (12 points) Given

$$p(x) = x^3 + 6x^2 - 9x - 14$$

1. Completely factor  $p(x)$ , use the rational roots theorem to help you.
2. Sketch a graph of  $p(x)$  and label the points where the graph intersects the  $x$ -axis and the  $y$ -axis.

**Problem 4:** (10 points) A superhero, standing on the ground, launches 50 feet of wire from a grappling gun, held at an angle of elevation of  $\pi/3$  radians. The grapple hits and catches the top edge of the building.

1. How tall is the building?
2. How far from the base of the building is the superhero standing?

**Problem 5:** (10 points) Find the exact value for each expression.

$$\sin(\arcsin(1/2) + \arccos(1/2))$$

$$\arccos\left(\tan\left(\frac{5\pi}{6}\right)\right)$$

**Problem 6:** (15 points)

1. Write the equation of the line with slope 2 that passes through the point  $(1, -7)$

2. Solve the equation  $|2x + 1| = 3$

3. Solve the inequality  $|x^2 - 9| \geq 6$

**Problem 7:** (10 points)

1. Given  $f(x) = x^2 - x$ , construct but **do not simplify**, the difference quotient for  $f(x)$

2. Simplify the following difference quotient instead

$$\frac{5(x+h) + (x+h)^2 - (5x+x^2)}{h}$$



**Problem 8:** (15 points)  $f(x) = 4x^2 - 24x + 3$

1. Complete the square to find the standard “vertex” form for  $f(x)$
2. Sketch a graph of  $f(x)$ . Mark the vertex and indicate the axis of symmetry.

**Problem 9:** (12 points) Solve for  $x$ ,  $y$ , and  $z$

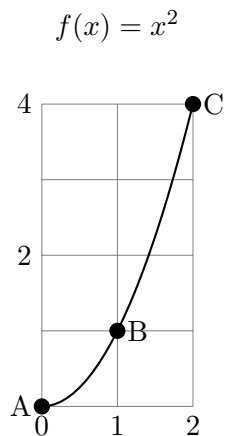
$$x + y + z = 55$$

$$y - x = 7$$

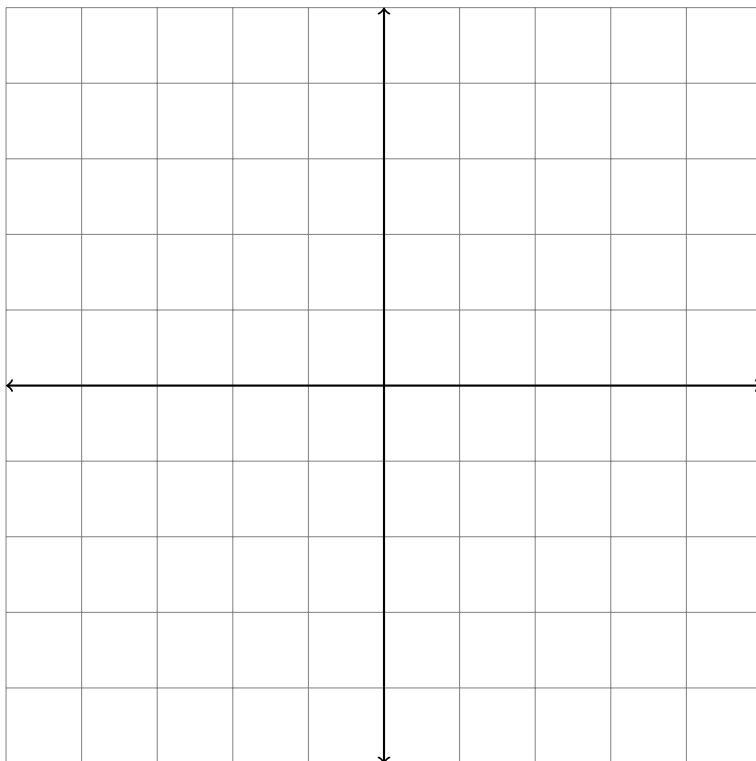
$$x + y - z = 19$$

**Problem 10:** (14 points)

1. The graph of the parent function  $f(x) = x^2$  is shown. The three points, A = (0, 0), B = (1, 1) and C = (2, 4) are marked.
2. Explain in words, the sequence of operations that transform  $f(x)$  into  $g(x) = 2(x + 1)^2 - 4$
3. Graph  $g(x)$  and label the coordinates of the three transformed points.



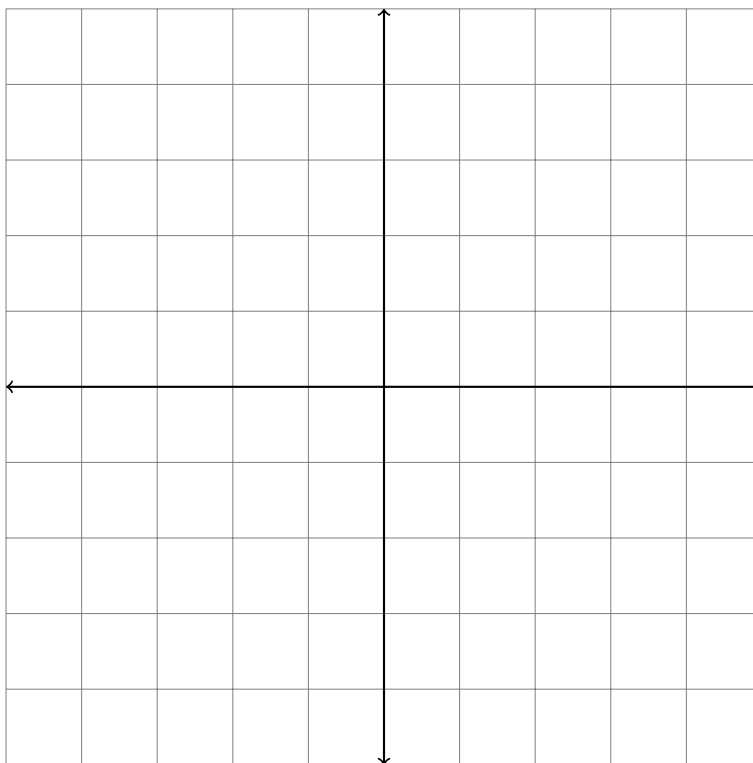
$g(x) = 2(x + 1)^2 - 4$



**Problem 11:** (12 points) For  $f(x) = \frac{x^2 - x - 6}{x^2 + x - 2}$

1. Express the domain of  $f$  in interval notation.
2. Find the vertical asymptote and the hole.
3. Find the horizontal asymptote.
4. Sketch the graph of  $f$ .
5. Solve the inequality  $\frac{x^2 - x - 6}{x^2 + x - 2} < 0$ , and express your answer in interval notation.

$$f(x) = \frac{x^2 - x - 6}{x^2 + x - 2}$$



**Problem 12:** (10 points) Sketch the graph of each of the following conic sections. **Clearly** label all centers, vertices, foci, and asymptotes where appropriate.

1.

$$\frac{(x+2)^2}{16} - \frac{(y-1)^2}{25} = 1$$

2.

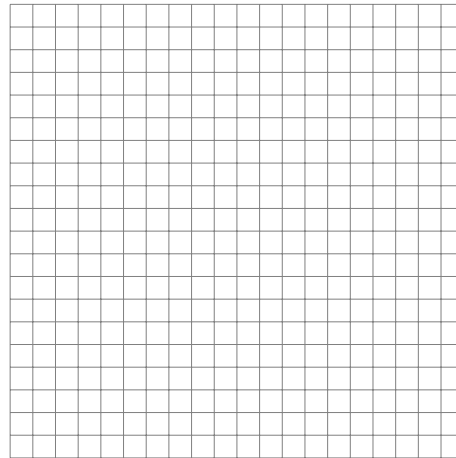
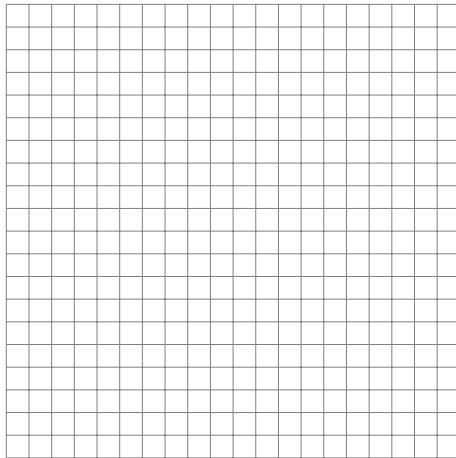
$$\frac{(y-2)^2}{4} = (x+1)$$

3.

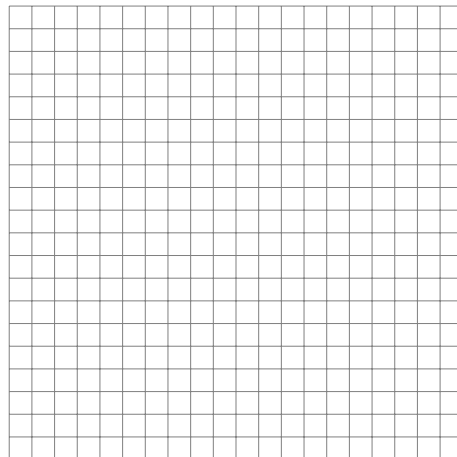
$$\frac{(y+1)^2}{9} + \frac{(x+3)^2}{16} = 1$$

**1**

**2**



**3**



**Problem 13:** (10 points) Verify the following trigonometric identities. Show all your intermediate steps.

$$\sin(x) + \cot(x) \cos(x) = \csc(x)$$

$$\sin(2\theta) = \frac{2 \tan(\theta)}{1 + \tan^2(\theta)}$$

**Problem 14:** (10 points) Given that  $\csc(\theta) = 11$  with  $\theta$  in the second quadrant. Find the exact values of all six trigonometric functions evaluated at  $\theta$ .

$$\sin(\theta) = \underline{\hspace{4cm}} \qquad \sec(\theta) = \underline{\hspace{4cm}}$$

$$\cos(\theta) = \underline{\hspace{4cm}} \qquad \csc(\theta) = 11$$

$$\tan(\theta) = \underline{\hspace{4cm}} \qquad \cot(\theta) = \underline{\hspace{4cm}}$$