# Euler's Method [1]

In this exercise, we will do some experiments to visualize the workings of Euler's method. You may submit it on Tuesday for three bonus points. You should prepare a worksheet with the answers to all questions posed below for submission.

For this exercise, you need to download the file Euler.m from my web page and save it in the "work" directory of your MATLAB folder without changing its name (and without changing the extension .m)!

First let us explore Euler's method for the autonomous differential equation $\frac{dy}{dt} = y$. Suppose you want to approximate $y(2)$ for the solution $y(t)$ of this equation that takes the initial value $y(0) = 1$. Enter:

```
>> Euler
```

At the next prompt enter:

```
>> y
```

As we discussed in class, the solution $y(t)$ that we want to approximate is given by the formula $y(t) = e^t$. Therefore, at the next prompt enter:

```
>> exp(t)
```

At the next prompts enter `0` for `a`, `1` for `y(a)`, and `2` for `b` . Finally, the program will prompt you to enter $\Delta t$. First try $\Delta t = 0.5$. The figure shows you the graph of the function and the graph of the approximations. Look closely and convince yourself that the approximations are a piecewise linear function. MATLAB'S Command Window will show you the error $y(2) - \hat{y}(2)$ and prompt you for another $\Delta t$.

1. What errors does MATLAB give you for $\Delta t = 0.5$, $\Delta t = 0.1$, and $\Delta t = 0.01$?

   Now let us consider the initial value problem $\frac{dy}{dt} = 2ty$; $y(0) = 1$. It can be easily verified that the solution of this initial value problem is $y(t) = e^{t^2}$. Let us use the program "Euler" to find approximations of $y(2)$ for this solution. Follow similar steps as before. Recall that you should enter the right-hand side of your differential equation as:

   ```
   >> 2*t*y
   ```

2. What errors does MATLAB give you for $\Delta t = 0.5$, $\Delta t = 0.1$, and $\Delta t = 0.01$?

3. Why are the errors so much bigger than for the previous example?