Taylor Approximations II [1]

In this exercise, we will do some additional visualizations of how Taylor polynomials approximate a given function. You will be able to submit this exercise for up to four bonus points on Thursday, January 30.

For this exercise, you need to download the file Tapprox.m from my web page and save it in the "work" directory of your MATLAB folder without changing its name (and without changing the extension .m)!

Now let us explore how the subsequent Taylor polynomials approach the function $\sin x$ on the interval $[-1, 10]$. For this, open MATLAB and enter:

```
>> Tapprox
```

MATLAB will now ask you to enter the formula of your function. Enter:

```
>> sin(x)
```

Next MATLAB will ask you to enter a value for a. Choose $a = 0$ here. Simply enter the number `0`. After that, you will be asked to enter the left and right endpoints of your interval $[-1, 10]$. Finally, MATLAB will ask you to put limits on the values that are displayed on the y-axis. For nice results, I recommend putting in a lower limit of -2 and an upper limit of 2.

Now you are ready to hit ENTER and observe how the Taylor polynomials approximate the graph of $\sin x$. The figure shows you the graph of $\sin x$ and of the current Taylor polynomial, and the command window simultaneously displays the degree of the current Taylor polynomial.

On a separate work sheet (to be submitted) answer the following questions:

1. What is the smallest n such that the Taylor polynomial of degree n at $a = 0$ approximates the function $\sin x$ for every x in the interval $[-1, 10]$ with an error of no more than 0.1?

2. Why does the picture in MATLAB's figure only change every other time you hit ENTER and not every time?

   Now repeat the exercise with the same function and the same interval, but let $a = 5.5$.

3. With the new value of a, what is the smallest n such that the Taylor polynomial of degree n at $a = 5.5$ approximates the function $\sin x$ for every x in the interval $[-1, 10]$ with an error of no more than 0.1? What is the reason that the answer is different from the answer to Question 1?

Now repeat the exercise with the function $f(x) = \ln x$. Remember that you need to enter:

```
>> log(x)
```

Let the interval be $[0, 2.5]$, choose $a = 1$, and make the bounds on the y-axis again -2 and 2.

4. Describe what you observe. Do the Taylor approximations appear to get better and better for all x in the interval?