MATH 1200 EXAM 4 SOLUTIONS

(1) $x+4$ is a factor of $f(x)$

Using synthetic division

-4	2	7	-14	-40
		-8	4	40
	2	-1	-10	$0 \longrightarrow$ Remainder
	x^2	x	c	

$$\Rightarrow \frac{2x^3+7x^2-14x-40}{x+4} = 2x^2-x-10$$

We can still factor $2x^2-x-10$

$$2x^2-5x+4x-10$$
$$x(2x-5)+2(2x-5)$$
$$(x+2)(2x-5)$$

$$\Rightarrow 2x^3+7x^2-14x-40 = (x+4)(x+2)(2x-5) \,//$$

(2) $$2x^3+7x^2-14x-40 = 0$$

$$\Rightarrow (x+4)(x+2)(2x-5) = 0$$

Using the zero-product property

$x+4=0$; $x+2=0$; $2x-5=0$

$\Rightarrow$ $x=-4$ $\qquad x=-2$ $\qquad x=5/2$

(3) Using synthetic Division

5	1	1	−6	−5	−15
		5	30	120	575
	1	6	24	115	560 → Remainder

$\Rightarrow f(5) = 560$

(4) Using Synthetic Division

−3	1	1	−6	−5	−15
		−3	6	0	15
	1	−2	0	−5	0 → Remainder

$\Rightarrow f(-3) = 0$

(5) For Vertical Asymptotes

$$x^2 - 9 = 0$$
$$(x-3)(x+3) = 0$$
$$\Rightarrow x = 3 \quad \text{and} \quad x = -3$$

(6) The horizontal Asymptote is $y = 0$

(Because the degree of the numerator is less than that of the denominator).

(7) As $x \to -\infty$, $f(x) \to 2$

As $x \to \infty$, $f(x) \to 2$

(8) As $x \to 4^-$, $f(x) \to -\infty$

As $x \to 4^+$, $f(x) \to \infty$

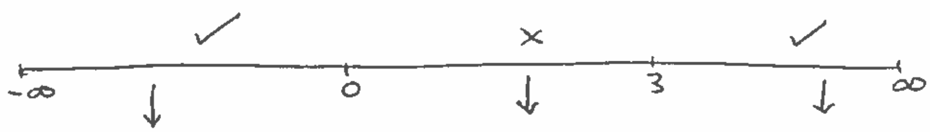
(9) Solve $a^2 \geq 3a$

$$\underbrace{a^2 - 3a}_{f(x)} \geq 0$$

Solve $f(x) = 0$

$$a^2 - 3a = 0$$
$$a(a-3) = 0$$
$$a = 0 \quad ; \quad a - 3 = 0$$

Use these as boundary points $\longrightarrow$ $a = 0$ $\qquad$ $a = 3$



$f(-1) = 4$ $\qquad$ $f(1) = -2$ $\qquad$ $f(4) = 4$

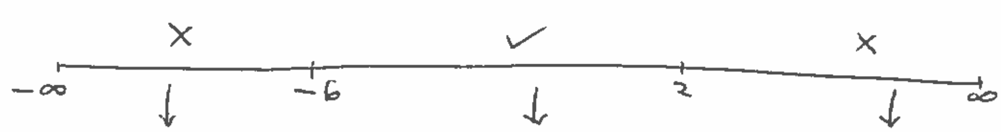
Solution Set is $(-\infty, 0] \cup [3, \infty)$

(10) Solve $\dfrac{2-x}{x+6} \geq 0$

$\underbrace{\dfrac{2-x}{x+6}}_{f(x)}$; Solve $f(x) = 0$

$$\frac{2-x}{x+6} = 0$$
$$2 - x = 0$$

Use these as boundary points $\longrightarrow$ $x = 2$ $\qquad$ but $x \neq -6$



$f(-7) = -9$ $\qquad$ $f(0) = \frac{1}{3}$ $\qquad$ $f(3) = -\frac{1}{9}$

Solution Set is $(-6, 2]$

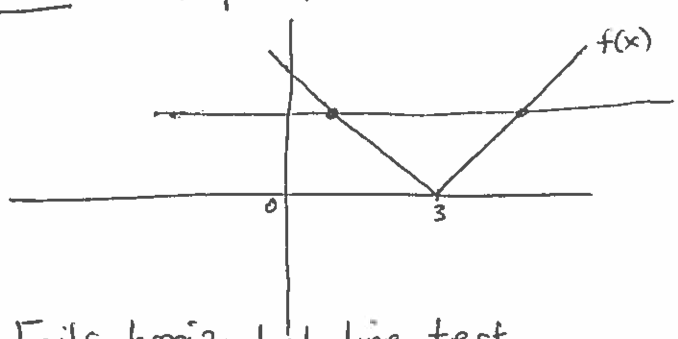
(11) $$Z = \frac{kxy}{\sqrt{W}}$$

(12)
$$A = kW$$
$$180 = k(40)$$
$$k = \frac{180}{40} = \frac{18}{4} = \frac{9}{2}$$ //

(13)
$$A = \frac{9}{2}W$$
$$135 = \frac{9}{2}W$$
$$W = \frac{2 \times 135}{9}$$
$$W = 30 \text{ lb}$$ //

(14) Using the definition of one-to-one ;
$$f(a) = |a| - 3$$
$$f(b) = |b| - 3$$
$$f(a) = f(b)$$
$$|a| - 3 = |b| - 3$$
$$|a| = |b|$$
$$a = \pm b$$
This contradicts the definition (because $a = b$)

Alternative. Graph $f(x)$ and use horizontal line test



Fails horizontal line test

(15) $(g \circ f)(x) = g(f(x)) = \dfrac{7x - 3 + 3}{7}$

$$= \frac{7x}{7}$$

$$= x$$

(16) $(f \circ g)(x) = f(g(x)) = 7\left(\dfrac{x+3}{7}\right) - 3$

$$= x + 3 - 3$$

$$= x$$

(17) f and g are inverses because $(g \circ f)(x) = (f \circ g)(x) = x$

(18)

$$f(x) = \frac{x-2}{x+2}$$

$$y = \frac{x-2}{x+2}$$

$$x = \frac{y-2}{y+2}$$

$$xy + 2x = y - 2$$

$$2x + 2 = y - xy$$

$$2x + 2 = y(1-x)$$

$$y = \frac{2x+2}{1-x}$$

$$f^{-1}(x) = \frac{2x+2}{1-x}$$

Alternative: $f^{-1}(x) = \dfrac{-2x-2}{x-1}$

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$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$P = \$4500$

$r = 4.5\% = 0.045$

$t = 5$ years

$n = 12$

$$\Rightarrow A = 4500\left(1 + \frac{0.045}{12}\right)^{12 \times 5}$$

$$= 4500\left(1 + \frac{0.045}{12}\right)^{60}$$

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Domain $= (-\infty, \infty)$

Range $= (0, \infty)$