Math 1200 (Spring'15) Exam 4A Teacher's/ your name:-----------

Show all your work in order to get full credit. Each question is worth 6 points, but # 2 is worth 4 points.

Use f(x) = $x^4 - 2x^3 - 4x^2 + 8x$ for # 1 and 2

1. What is the maximum number of turns that may occur on the graph of f(x)?

$n = 4$ $\quad n-1 = 3$

2. Describe the end behavior of f(x)

leading term $= x^4$ $\quad \uparrow \quad \nearrow$

leading coefficient $= 1$

3. Find the zeros of the function defined by f(x) = $-x^3$ + 8x^2 - 16x.

$-x^3 + 8x^2 - 16x = 0$ $\quad x = 0$ with multi. $= 1$

$x^3 - 8x^2 + 16x = 0$ $\quad x = 4$ with multi. $= 2$

$x(x^2 - 8x + 16) = 0$

$x(x-4)^2 = 0$

Use f(x) = x^4+ 11x^3+41x^2 +61x + 30 for #s 4 and 5.

4. Using synthetic division check if (x + 5) is a factor of f(x).

-5	1	11	41	61	30
	$\downarrow$	-5	-30	-55	-30
	1	6	11	6	$\boxed{0}$

↑ Remainder

Yes.

5. Use factor theorem to determine if (x + 5) is a factor of f(x).

-5	1	11	41	61	30
	$\downarrow$	-5	-30	-55	-30
	1	6	11	6	$\boxed{0}$

↑ Remainder $= 0$

Yes

6. **Build a third degree polynomial with roots of 0, $\sqrt{2}$, $-\sqrt{2}$ that is rising on the left and falling on the right**

$f(x) = -x(x-\sqrt{2})(x+\sqrt{2})$

7. *For* $f(x) = \frac{x-4}{3x^2+5x-2}$, **write equations of the vertical and horizontal asymptotes and identify the holes in this function (where the function is undefined), if any.**

V.A. $3x^2+5x-2=0$

$(3x-1)(x+2)=0$

V.A. $\boxed{x=\frac{1}{3},\ x=-2}$

No holes

since no common factor for the numerator & denominator other than 1.

H.A. degree (numerator) $=1$

degree (denominator) $=2$

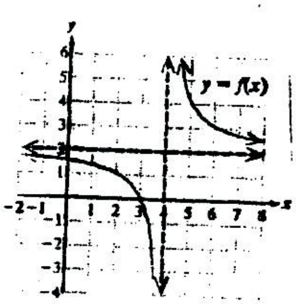
$1<2$

$\boxed{H.A:\ y=0}$

Use the graph of y = f(x) to do #s 8 – 10.

8. As x → - ∞, f(x) → 2.

9. As x → ∞, f(x) → 2.



10. The domain of f(x) is $(-\infty, 4) \cup (4, \infty)$.

11. Solve 2x(x-1) > 21 – x

$2x^2 - 2x > 21 - x$

$2x^2 - 2x - 21 + x > 0$

$2x^2 - x - 21 > 0$

$(2x - 7)(x + 3) > 0$

$f(x) = (2x - 7)(x + 3) = 0$

$x = 7/2 \quad x = -3$

$\boxed{(-\infty, -3) \cup (7/2, \infty)}$

+   -   +

$-4 \quad -3 \quad 0 \quad 7/2 \quad 4$

$f(-4) = 15 \quad f(0) = -21 \quad f(4) = 7$

12. Solve $\frac{5-x}{x-1} \geq -2$

$\frac{5-x}{x-1} + 2 \geq 0$

$\frac{5-x}{x-1} + \frac{2(x-1)}{(x-1)} \geq 0$

$\frac{5-x+2x-2}{x-1} \geq 0$

$\frac{x+3}{x-1} \geq 0$

$f(x) = \frac{x+3}{x-1} = 0 \qquad x = -3$

+   -   +

$-4 \quad -3 \quad 0 \quad 1 \quad 2$

$f(-4) = 1/5 \quad f(0) = -3 \quad f(2) = 5$

$\boxed{(-\infty, -3] \cup (1, \infty)}$

13. The amount of a pain reliever that a physician prescribes for a child varies directly as the weight of the child. A physician prescribes 180 mg of the medicine for a 40-lb child. How much medicine would be prescribed for a 50-lb child?

$Y = kw$

$180 = k \cdot 40$

$k = \frac{180}{40} = \frac{18}{4} = \frac{9}{2} = 4.5$

$Y = 4.5w$

$Y = 4.5(50)$

$= \boxed{225 \text{ mg}}$

14. The variable y varies jointly as w and v. When w is 40 and v is 0.2 and y is 16, find the constant of variation k.

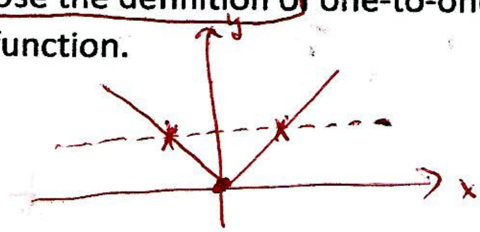
$$y = kwv$$

$$16 = k(40)(0.2) \qquad k = \frac{16}{8} = 2$$

$$16 = k(8)$$

$$\boxed{k = 2}$$

15. Use the definition of one-to-one function to determine if f(x) = $|x|$ is a one-to-one function.



Fails the horizontal line test.

$\boxed{\text{No, it's not one-to-one.}}$

Use the definition

$f(a) = f(b)$

$|a| = |b|$

$a = \pm b$

$\boxed{\text{NO, not one-to-one}}$

16. If f and g are inverse functions and f(x) = $\sqrt{x-3}$, find g(x).

$f(x) = \sqrt{x-3}$ domain $x \geq 3$

Step 1: $y = \sqrt{x-3}$

Step 2: $x = \sqrt{y-3}$

Step 3: $x^2 = y - 3$

$y = x^2 + 3$

Step 4: $g(x) = x^2 + 3, \quad x \geq 0$

17. Check if (fog)(x) = (gof)(x)

$(f \circ g)(x) = f(g(x)) = \sqrt{x^2+3-3} = \sqrt{x^2} = x$

$(g \circ f)(x) = g(f(x)) = (\sqrt{x-3})^2 + 3 = x - 3 + 3 = x$

thus: $(f \circ g)(x) = (g \circ f)(x)$