Math 1200 Exam 4A (Fall 2015) Name: Δ·T

Show all your work in order to get full credit. Each question is worth 5 points.

If $f(x) = 2x^3 + 15x^2 + 31x + 12$, use the remainder theorem to check (#s 1 – 2)

⑤ 1. $f(-3)=$ $\underline{\ 0\ }$?

$$\begin{array}{r|rrrr} -3 & 2 & 15 & 31 & 12 \\ & \downarrow & -6 & -27 & -12 \\ \hline & 2 & 9 & 4 & \boxed{0} \end{array}$$

⑥ 2. Determine if – 3 a zero of $f(x)$?

Since $f(-3)=0$ it implies that $x+3$ is a factor of $2x^3+15x^2+31x+12$ and thus -3 is a zero of $f(x)$.

⑦ 3. Write a polynomial $f(x)$ of degree 3 that has zeros $\frac{5}{2}$, $\sqrt{7}$, and $-\sqrt{7}$.

$$(x-\tfrac{5}{2})(x-\sqrt{7})(x+\sqrt{7})$$

$$= (2x-5)(x^2-7) \qquad (x-\tfrac{5}{2})(x^2-7)$$

$$= \underline{2x^3-14x-5x^2+35} \quad \text{or} \quad \underline{x^3-7x-\tfrac{5}{2}x^2+\tfrac{35}{2}}$$

For #s 4 – 6, $f(x) = \frac{2x^2-5x-3}{x-2}$

① 4. Write $f(x)$ in in the form $f(x)$ = quotient + $\frac{\text{remainder}}{\text{divisor}}$.

$$\begin{array}{r|rrr} 2 & 2 & -5 & -3 \\ & \downarrow & 4 & -2 \\ \hline & 2 & -1 & \boxed{-5} \end{array} \Rightarrow \overbrace{2x-1}^{\text{quotient}} + \frac{\overset{\text{remainder}}{-5}}{\underset{\text{divisor}}{x-2}}$$

or long division

$$\begin{array}{r} 2x-1 \\ x-2 \overline{\big)\, 2x^2-5x-3} \\ 2x^2-4x \\ \hline -x-3 \\ -x+2 \\ \hline -5 \end{array} = 2x-1+\frac{-5}{x-2}$$

② 5. Find vertical asymptote(s).

$x-2=0 \Rightarrow$ @ $x=2$ we have the VA.

③ 6. Find horizontal asymptotes.

Since the leading term of numerator has a higher power than that of denominator, we have no HA. (i.e $2x^2 > x \Rightarrow$ No H

④ 7. If $f(x) = \frac{8x^2+9x-5}{2x^2+1}$, determine the point where the graph of $f(x)$ crosses its horizontal asymptotes.

we first get HA @ $y = \frac{8x^2}{2x^2} = 4$

then $4 = \frac{8x^2+9x-5}{2x^2+1}$

$\Rightarrow 8x^2+4 = 8x^2+9x-5$

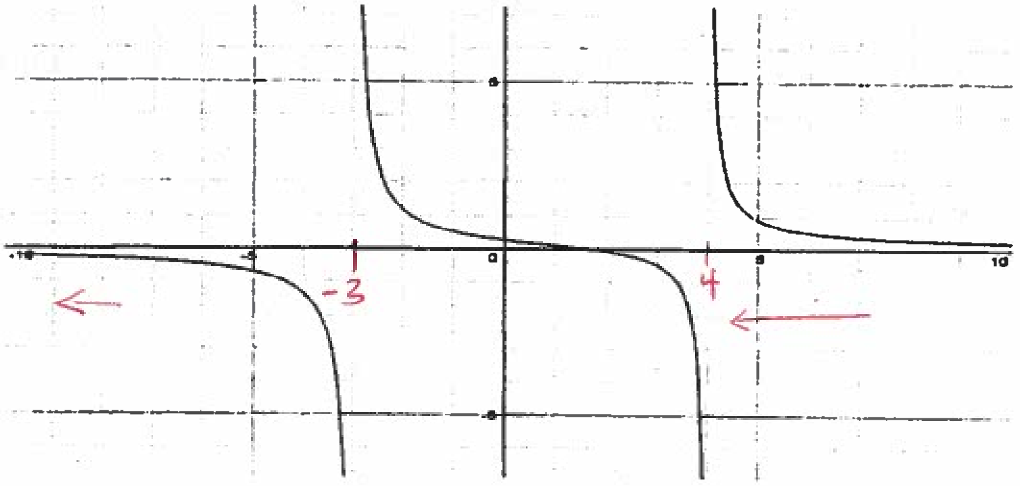
$\Rightarrow 9 = 9x$

so $x = 1$, $f(x)$ crosses the HA @ $(1,4)$.

i.e the point; $(1,4)$.

$\frac{17-5}{3} = 12/3 = 4$

Refer to the graph and complete the following statements.



⑧ 8. As $x \to -\infty, f(x) \to$ ......0........... ⑨ 9. As $x \to 4^+, f(x) \to$ .....∞.......

⑩ 10. The domain is ....$(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$ ⑪ 11. The range is ......$(-\infty, \infty)$......

Solve (#s 12 and 13)

⑫ 12. $2x(x-3) \geq 12 - 4x$

$2x^2 - 6x - 12 + 4x \geq 0$

$2x^2 - 2x - 12 \geq 0$

$\Rightarrow (2x+4)(x-3) \geq 0$

So boundary points are -2 and 3

(-3) -2 ⓪ 3 ④

\+ − +

$(-\infty, -2] \cup [3, \infty)$

⑬ 13. $\frac{5x-5}{x-2} < 4 \Rightarrow \frac{5x-5-4x+8}{x-2} < 0$

$= \frac{x+3}{x-2} < 0$

boundary points are -3 and 2.

(-4) -3 ⓪ 2 ③

\+ − +

$(-3, 2)$

Write a variation model using k as the constant of variation: (#s 14 – 15)

⑭ 14. If H is inversely proportional to Q and H is 40 when Q is 7, find the constant of variation.

$H = \frac{K}{Q} \Rightarrow$ if H is 40 and Q 7.

we have $K = HQ = 40 \cdot 7 = \underline{\underline{280}}$.

$\Rightarrow H = \frac{280}{Q}$. where $\underline{\underline{K = 280}}$.

⑮ 15. Find the value of Q when H is 28.

$H = \frac{280}{Q} \Rightarrow Q = \frac{280}{H} = \frac{280}{28} = \underline{\underline{10}}$

$Q = \underline{\underline{10}}$.

For #s 16 – 18, $f(x) = 100 + 12x$ and $g(x) = \frac{x-100}{12}$:

⑯ 16. Find $(g \circ f)(x)$

$$\frac{100+12x-100}{12}$$

$$= \frac{12x}{12} = \underline{\underline{x}}$$

⑰ 17. Find $(f \circ g)(x)$

$$100 + 12\left(\frac{x-100}{12}\right).$$

$$= 100 + x - 100$$

$$= \underline{\underline{x}}$$

⑱ 18. Determine if f and g are inverses of each other by using the property $(g \circ f)(x) = (f \circ g)(x) = x$.

From 16 and 17, we observe that $(g \circ f)(x) = (f \circ g)(x) = x$ which tells us that f and g are inverses of each other.

~~(given f and g are one to one which is not necessary for this particular problem. Because f and g are one to one)~~.

⑲ 19. Given $f(x) = \sqrt{x-2}$, find $f^{-1}(x)$ and the domain of $f^{-1}(x)$.

$$y = \sqrt{x-2}$$

$$\Rightarrow x = \sqrt{y-2}$$

$$\Rightarrow x^2 = y - 2$$

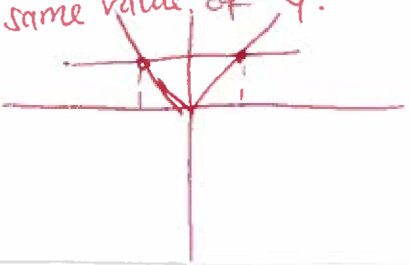
$$\Rightarrow y = x^2 + 2.$$

$$\Rightarrow \underline{f^{-1}(x) = x^2 + 2}$$

Domain $f^{-1}(x)$ should satisfy the domain of $f(x)$ in order to be one-to-one $\Rightarrow [2, \infty)$.

⑳ 20. Determine if $h(x) = |x|$ is a one-to-one function and explain.

<u>using horizontal line test</u>, $h(x)$ fails to be one to one. We observe two ~~values of~~ different values of x producing same value of y.



<u>using the definition</u>; we know that if $f(a) = f(b)$ then $a = b$ for $f(x)$ to be one to one.

$$|a| = |b| \Rightarrow a = b \text{ or } a = -b$$

Thus failing to be one-one by definition.