Math 1200 (Fall 2015) Exam 3A Name: Solutions

Show ALL your work to get full/partial credit. Each problem is worth 5 points.

1) Determine if the function f(x) = $x^3 + |x| - 4$ is even, odd, or neither.

$f(-x) = (-x)^3 + |-x| - 4$
$= -x^3 + |x| - 4$
$\neq f(x)$

So not even

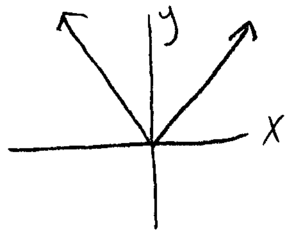
$-f(x) = -(x^3 + |x| - 4)$
$= -x^3 - |x| + 4$
$\neq f(-x)$

So not odd

$\boxed{\text{Neither even or odd}}$

2) Is y = $|x|$ symmetric with respect to x-axis, y-axis or origin?

graph of $y = |x|$ is:



$\boxed{\text{symmetric to } y\text{-axis}}$

For h(x) = $3x^4 - 9x^2$, determine

3) The zeros

$h(x) = 3x^2(x^2 - 3)$

$3x^2 = 0$ or $x^2 - 3 = 0$

$x = 0$ or $x = \pm\sqrt{3}$

$\boxed{\text{So zeros are: } 0, \sqrt{3}, -\sqrt{3}}$

4) The multiplicities for each zero.

0 has multiplicity 2

$\sqrt{3}$ has multiplicity 1

$-\sqrt{3}$ has multiplicity 1

5) Use the leading term to determine the end behavior of the graph of the function, g(x) = $\frac{1}{2}x(x-3)^3(x+2)^2$

leading term: $a_n x^n = \frac{1}{2} x \cdot x^3 \cdot x^2 = \frac{1}{2} x^{1+3+2} = \frac{1}{2} x^6$

So $a_n = \frac{1}{2} > 0$, $n = 6$ is even, $\Rightarrow$ End Behavior is $\boxed{\text{up left up Right}}$

Given f(x) = $x^2 + 1$, find

6) f(x+ h)

$f(x+h) = (x+h)^2 + 1$
$= \boxed{x^2 + 2xh + h^2 + 1}$

7) The difference quotient, $\frac{f(x+h)-f(x)}{h}$.

$$\frac{x^2 + 2xh + h^2 + 1 - (x^2 + 1)}{h}$$

$$\frac{\cancel{x^2} + 2xh + h^2 \cancel{+1} - \cancel{x^2} \cancel{-1}}{h} = \frac{2xh + h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}} = \boxed{2x+h}$$

Identify the basic parent functions and transformations for the following functions

8) $g(x) = 2|x-3| + 1$

$\boxed{\text{Parent}: |x|}$

Transformations: Right shift 3

Up shift 1

Vertical Stretch of 2

9) $f(x) = \sqrt{-x-5} = \sqrt{-(x+5)}$

$\boxed{\text{Parent}: \sqrt{x}}$

Transformations: Reflect over y-axis

Left shift 5

Given $g(x) = \sqrt{x}$ and $f(x) = -x^2 - 4$, find

10) $(f \circ g)(x)$

$= f(g(x)) = f(\sqrt{x}) =$

$= -(\sqrt{x})^2 - 4$

$= \boxed{-x-4}$

11) The domain of $(f \circ g)(x)$

Domain of f:

$(-\infty, \infty)$

Domain of g:

$[0, \infty)$

Intersection of domains gives domain of $(f \circ g)(x)$

So Domain of $(f \circ g)(x)$ is: $\boxed{[0, \infty)}$

12) $(f \circ g)(4)$

$= -(4) - 4 = \boxed{-8}$

13) Using Intermediate value theorem show that f(x) has a zero on the given interval

$f(x) = 2x^3 - 7x^2 - 14x + 30$ on $[1,2]$

$f(1) = 2(1)^3 - 7(1)^2 - 14(1) + 30$
$= 2 - 7 - 14 + 30 = 11$

$f(2) = 2(2)^3 - 7(2)^2 - 14(2) + 30$
$= 16 - 28 - 28 + 30 = -10$

Thus since $f(1) = 11$ and $f(2) = -10$ (they have opposite signs) so the intermediate value theorem guarantees that $f(x)$ has a zero on $[1,2]$.

Given $f(x) = x^2 + 4x + 3$

14) Determine the vertex of the parabola.

Vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$\frac{-b}{2a} = \frac{-4}{2(1)} = -2$

$f(-2) = (-2)^2 + 4(-2) + 3$
$= 4 - 8 + 3 = -1$

$\boxed{(-2, -1)}$

15) Identify the x- and y- intercepts.

x-intercepts: $f(x) = 0 = x^2 + 4x + 3$
$= (x+1)(x+3)$

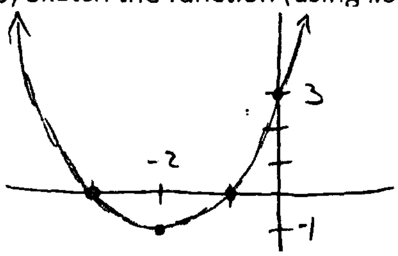
$\Rightarrow x = -1$ and $x = -3$

$\boxed{(-1, 0) \text{ and } (-3, 0)}$

y-intercept: $f(0) = y = (0)^2 + 4(0) + 3 = 3$
$y = 3$

$\boxed{(0, 3)}$

16) Sketch the function (using #s 14 & 15)



17) Determine its domain and range.

Domain : $(-\infty, \infty)$

Range : $[-1, \infty)$

18) Decompose $k(x) = \sqrt[7]{x^5 - 9x}$ in two functions m & n such that $k(x) = (m(n(x))$

$m(x) = \sqrt[7]{x}$

$n(x) = x^5 - 9x$

then $m(n(x)) = m(x^5 - 9x)$

$= \boxed{\sqrt[7]{x^5 - 9x}}$

If $g(x) = \begin{cases} x^3 & for \quad x < 2 \\ |x-2| & for \quad x > 2 \end{cases}$ find

19) g(-3)

$-3 < 2$

So, $g(-3) = (-3)^3$

$= \boxed{-27}$

20) g(5)

$5 > 2$

So, $g(5) = |5-2|$

$= |3|$

$= \boxed{3}$