

# MATH 1200 -EXAM 4A

SPRING 2014

NAME: Solution

Show ALL of your work for full credit. Simplify your answers as much as possible. Each problem is worth 7 points unless otherwise specified.

B6 1. Find the difference quotient  $\frac{f(x+h) - f(x)}{h}$  if  $f(x) = 2x + 4$ .

$$\begin{aligned} f(x) &= 2x + 4 \\ f(x+h) &= 2(x+h) + 4 \\ f(x+h) - f(x) &= 2x + 2h + 4 - 2x - 4 \\ \frac{f(x+h) - f(x)}{h} &= \frac{2h}{h} \\ &= 2 \end{aligned}$$

B3 2. Given  $f(x) = x^2 + 2$  and  $g(x) = x + 1$  where  $g(x) \neq 0$ , find the following functions and determine its domain.

A.  $(f + g)(x)$

$$\begin{aligned} (f+g)(x) &= f(x) + g(x) \\ &= (x^2 + 2) + (x + 1) \\ &= x^2 + x + 3 \end{aligned}$$

Domain:  $(-\infty, \infty)$

B.  $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$$= \frac{x^2 + 2}{x + 1} \quad \begin{array}{l} x + 1 \neq 0 \\ x \neq -1 \end{array}$$

Domain  $(-\infty, -1) \cup (-1, \infty)$

B4 3. Given  $f(x) = -3x + 1$  and  $g(x) = \frac{1}{x}$ , find the function  $(f \circ g)(x)$  and write the domain of  $(f \circ g)(x)$  using set-builder notation.

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= f\left(\frac{1}{x}\right) \\ &= -3\left(\frac{1}{x}\right) + 1 \quad x \neq 0 \\ &= \frac{-3 + x}{x} \end{aligned}$$

Domain  $(-\infty, 0) \cup (0, \infty)$

B5 4. Given  $h(x) = \sqrt[3]{2x + 1}$ , find two function  $f$  and  $g$ , such that  $h(x) = (f \circ g)(x)$ .

$$g(x) = 2x + 1$$

$$f(x) = \sqrt[3]{x}$$

B1

5. Write  $f(x) = 2x^2 - 4x - 3$  in vertex form using any method. Also identify the vertex and axis of symmetry.

$$\begin{aligned} f(x) &= 2x^2 - 4x - 3 \\ &= 2(x^2 - 2x) - 3 \\ &= 2[x^2 - 2x + 1 - 1] - 3 \\ &= 2(x-1)^2 - 2 - 3 \\ &= 2(x-1)^2 - 5 \end{aligned}$$

vertex (1, -5)  
axis:  $x-1=0$   
 $x=1$

B2

6. Given  $f(x) = x^4 + 8x^3 + 12x^2$ ,

- A. Determine the end behavior of  $f(x)$  [5pt]

Since the degree of  $f(x)$  is even  
and leading coefficient is positive  
both ends go up  $\cup$

- B. Find the zeros of  $f(x)$  and state the multiplicities of each zero.

$$\begin{aligned} f(x) &= 0 \\ x^4 + 8x^3 + 12x^2 &= 0 \\ x^2(x^2 + 8x + 12) &= 0 \\ x^2(x+6)(x+2) &= 0 \\ x=0 ; x=-6 ; x=-2 \end{aligned}$$

Multiplicity: 2 ; 1 ; 1

B8

7. Given  $f(x) = x^4 - 8x^3 + 3x^2 + 5x - 82$ , evaluate  $f(-2)$  using the Remainder Theorem, and determine if  $(x+2)$  is a factor of  $f(x)$ .

$$\begin{aligned} f(-2) &= (-2)^4 - 8(-2)^3 + 3(-2)^2 + 5(-2) - 82 \\ &= 16 + 8(-8) + 3(4) - 12 - 82 \\ &= (16 + 64 + 12) - 12 - 82 \\ &= 92 - 92 \end{aligned}$$

Since  $f(-2) = 0$ , so  $(x+2)$  is a factor of  $f(x)$ .

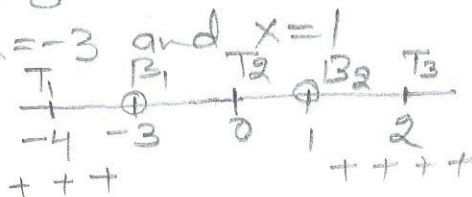
B9

8. Solve the inequality  $2x^2 - 3 > x^2 - 2x$ , and write your answer using interval notation.

$$\begin{aligned} 2x^2 - 3 - x^2 + 2x &> 0 \\ x^2 + 2x - 3 &> 0 \\ (x+3)(x-1) &> 0 \end{aligned}$$

Boundary values  $x = -3$  and  $x = 1$

Test values:  $-4, 0, 2$



$(-\infty, -3) \cup (1, \infty)$

B7 9. A coin is dropped from the top of Morton Hall. Its height above the ground in feet is given by  $h(t) = 100 - 16t^2$  at  $t$  seconds after it is dropped.

A. From what height is the coin dropped? at  $t=0$

$$h(0) = 100 - 16(0)$$

$$h(0) = 100 \text{ feet}$$

B. At what time does the coin reach the ground?

Coin reaches ground when  $h=0$

$$0 = 100 - 16t^2$$

$$16t^2 = 100$$

$$t^2 = \frac{100}{16}$$

$$t = \frac{10}{4} = \frac{5}{2} \text{ secs.}$$

B10 10. Given  $f(x) = \frac{x-2}{(2x+3)(x+4)}$ ,

A. Find the domain of  $f(x)$  [3pt].

Since  $2x+3 \neq 0$  and  $x+4 \neq 0$   
 $x \neq -\frac{3}{2}$  and  $x \neq -4$

So domain of  $f(x)$  is

$$(-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, -4) \cup (-4, \infty)$$

B. Find the  $x$ -intercept [3pt]

$x$ -intercept:  $y=0$

$$\frac{x-2}{(2x+3)(x+4)} = 0 \text{ only if}$$

numerator is zero

$$x-2 = 0$$

$$x = 2$$

C. Find the  $y$ -intercept [3pt]

$y$ -intercept:  $x=0$

$$f(x) = \frac{-2}{(0+3)(0+4)}$$

$$= -\frac{2}{12}$$

$$= -\frac{1}{6}$$

$$(0, -\frac{1}{6})$$

D. Find the Vertical asymptote [3pt]

Denominator = 0

$$2x+3=0 \text{ and } x+4=0$$

$$x = -\frac{3}{2} \text{ and } x = -4$$

Vertical Asymptotes are

$$x = -\frac{3}{2} \text{ and } x = -4$$

E. Find the Horizontal asymptote [3pt]

$f(x)$  behaves like

$$\frac{x}{2x^2} = \frac{1}{2x}$$

as  $x$  approaches to a very large or very small value  
 $f(x)$  approaches 0  
 $y=0$  is H. Asymptote

F. Sketch the graph of  $f(x)$  [3pt]

