Solution

Math 1200   Exam 3B (Fall 2014)   Name: _____

Show all your work in order to get full/partial credit. Each question (or sub-question) is worth 5 points.

1. Write the equation of the line passing through $(4, -6)$ with slope $m = 3$. (x_1, y_1)

$$y + 6 = 3(x - 4) = 3x - 12.$$

Thus,

$$y + 6 = 3x - 12$$
$$y = 3x - 12 - 6 = 3x - 18$$
$$y = 3x - 18$$

2. Determine if the lines $y = \frac{4}{3}x - 1$ and $4y = -3x + 20$ are parallel, perpendicular, or neither. Show work.

$$y = \frac{4}{3}x - 1, \quad m_1 = \frac{4}{3}, \quad y = -\frac{3}{4}x + 5, \quad m_2 = -\frac{3}{4}$$

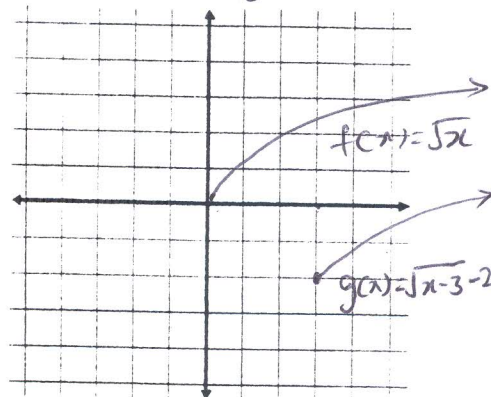$$m_1 m_2 = \frac{4}{3} \cdot -\frac{3}{4} = -1 \quad (m_1 m_2 = -1), \quad \text{Perpendicular}$$

3. A sales person makes a base salary of \$400 per week plus 12% commission on sales.
Write a linear function to model the sales person's weekly salary $S(x)$ for x dollars in sales.

$$S(x) = 400 + \frac{12}{100}x = 400 + 0.12x.$$

4. For $g(x) = \sqrt{x-3} - 2$,

   a. Describe the sequence of transformations from the parent function $f(x)$ to $g(x)$.

The parent function $f(x) = \sqrt{x}$ is shifted to the right 3 units and downward 2 units

   b. Graph $g(x)$.

$f(x) = \sqrt{x}$

$g(x) = \sqrt{x-3} - 2$

5. Given $h(x) = 4x^3 - 2x$,   a. find $h(-x) = 4(-x)^3 - 2(-x)$
$$= -4x^3 + 2x$$
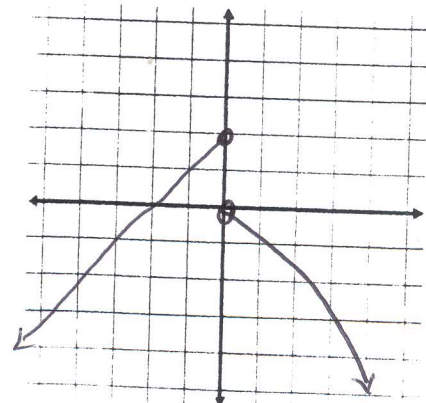
   b. find $-h(x) = -(4x^3 - 2x)$
$$= -4x^3 + 2x.$$

   c. Is this function even or odd? Show your work.

$$h(-x) = -h(x)$$

h is Odd

6. Graph $f(x)$ if $f(x)$ is defined by the piece-wise function

$$f(x) = \begin{cases} x + 2 & for \quad x \le 0 \\ -x^2 & for \quad x > 0 \end{cases}$$

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7. Given $f(x) = \sqrt{25 - x^2}$ and $g(x) = 5$, find $(f + g)(x)$.

$$(f+g)(x) = f(x) + g(x) = \sqrt{25-x^2} + 5$$

8. If $f(x) = x^2 - 3x$
   a. Find $f(x + h)$.

$f(x+h) = (x+h)^2 - 3(x+h)$
$\qquad = x^2 + 2xh + h^2 - 3x - 3h$

   b. Find the difference quotient $\frac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$$
$$= \frac{2xh + h^2 - 3h}{h} = \boxed{2x+h-3}$$

9. Given $f(x) = 2x - 6$ and $g(x) = \frac{1}{x+4}$
   a. Find $(f \circ g)(x)$
   b. Write the domain of $(f \circ g)(x)$ in interval form.

$(f \circ g)(x) = f(g(x))$
$\qquad = 2\left(\frac{1}{x+4}\right) - 6$

$D(f \circ g) \subset D(g)$
$D(f \circ g) = (-\infty, -4) \cup (-4, \infty)$

10. Given $f(x) = -x^2 + 4x - 5$
   a. Explain whether the graph of the parabola opens upward or downward.

$a = -1$
$b = 4$
$c = -5$

Since $a = -1 < 0$, Thus the graph of the parabola opens downward.

   b. Determine the vertex of the parabola by using the vertex formula.

$h = -\frac{b}{2a} = \frac{-4}{2(-1)} = 2$, $k = f(2) = -(2)^2 + 4(2) - 5 = -1$
$\qquad\qquad\qquad\qquad$ Vertex $(2, -1)$

   c. Determine the minimum/maximum value of $f(x)$ and where it is achieved?
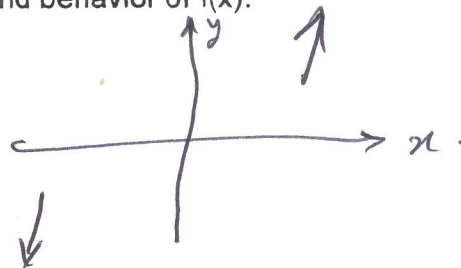
Maximum value of $f(x)$ is -1
Achieved at $(2, -1)$.

11. If $f(x) = x^3 - 9x$, find   a. The zeros of $f(x)$.   b. The end behavior of $f(x)$.

$f(x) = x^3 - 9x = 0$
$x(x^2 - 9) = 0$
$x = 0 \quad x^2 - 9 = 0$

$x = 0, x^2 = 9$
$x = 0, x = \pm\sqrt{9} = \pm 3$
$x = \{0, \pm 3\}$.

   c. The maximum number of turning points $f(x)$ can have.

The maximum number of turning points of $f(x)$
is $3 - 1 = \boxed{2}$

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