Math 1300–Precalculus Final Exam

Your Name: _______________________________________________________

Your Instructor: ___________________________________________________

Instructions: Please solve all ten of the boxed problems. Please solve any two of the remaining four problems. If you solve all four of the remaining problems you will receive credit for the highest scoring two. Please justify all your answers. No partial credit will be awarded for answers without calculations or explanations. You may Not use any electronic technology. Please sign your name below signifying that you have read, understood and will abide by these directions and will not cheat.

Your Signature: ___________________________________________________

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Problem 1: (12 points) Find the exact value of

\[ \cos \left( -\frac{\pi}{4} \right) \]

\[ \tan \left( \frac{3\pi}{2} \right) \]

\[ \sin \left( \frac{4\pi}{3} \right) \]

\[ \sin \left( \frac{\pi}{8} \right) \]
Problem 2: (14 points) \( p(s) = s^3 - 3s^2 - 2s + 6 \)

(1) Find the real zeros of \( p \) using the rational roots theorem.

(2) State the multiplicity of each root.

(3) Describe the end behavior of \( p \)

(4) Graph the equation on the grid below.
Problem 3: (12 points) Verify the identity:

\[ \sin(x) + \cot(x) \cos(x) = \csc(x) \]
Problem 4: (12 points) Consider the ellipse

\[ x^2 + 4y^2 + 10x - 8y + 13 = 0 \]

(1) Put the equation in standard form by completing the square for \( x \) and \( y \).

(2) Graph the equation on the grid below, label the foci, vertices and center.
Problem 5: (12 points) Find the exact values of

\[
\sin \left( \arccos \left( \frac{3}{5} \right) \right)
\]

\[
\cos \left( \arcsin \left( \frac{5}{13} \right) \right)
\]

\[
\tan \left( \arcsin \left( \frac{1}{4} \right) \right)
\]

\[
\arccos \left( \sin \left( \frac{5\pi}{3} \right) \right)
\]
Problem 6: (12 points) Solve the following equation for \( x \).

\[
\log_2(x + 2) - 2 \log_2(x) = 0
\]
Problem 7: \( f(x) = x^3 - 1 \)

1. (3 points) Demonstrate that \( f(x) \) is one to one.

2. (3 points) Find \( f^{-1}(x) \)

3. (4 points) Express the domain and range of \( f \) and \( f^{-1} \) in interval notation.

4. (3 points) Compute \( f \circ f^{-1} \)

5. (4 points) Graph \( f \) and \( f^{-1} \) on the next page.
Problem 7: Continued
Problem 8: (12 points) Let $f(x) = x^2 - 2x - 8$.

(1) Put $f(x)$ in standard form.

(2) Write the coords of the vertex and the equation of the axis of symmetry.

(3) Find the $x$ and $y$ intercepts.

(4) Graph $f$ on the grid below. Label the axis of symmetry.
Problem 9: (20 points) For \( f(x) = \frac{x^2 - x - 12}{x^2 + x - 6} \)

(1) Express the domain of \( f \) in interval notation.
(2) Find the \( x \) and \( y \) intercepts of \( f \).
(3) Find all vertical and horizontal asymptotes and identify and holes.
(4) Sketch a detailed graph of \( f \) on the grid below.
Problem 10: From the graph of \( y = f(x) \) shown

(1) (6 points) Find the equations of the lines that comprise the graph of \( f \).

(2) (8 points) Draw the graph of \( g(x) = 2f(-x) + 1 \) on the grid provided.

(3) (6 points) Write the coordinates of the three reference points before and after the transformation in the table to the right.

Figure 1: The graph of \( y = f(x) \)

\[
\begin{array}{|c|c|}
\hline
| f(x) | g(x) | \\
\hline
\text{Point A} & & \\
\text{Point B} & & \\
\text{Point C} & & \\
\hline
\end{array}
\]
Problem 11: (10 points) Consider the first few terms of the sequence:

5, −1, −7, −13, −19 . . .

(1) Assuming that the pattern established by the terms shown continues, find an expression for the nth term of the sequence.

(2) Find the 25th term of the sequence.

(3) Find the sum of the first 25 terms of the sequence.
Problem 12: (10 points) If the following system of equations has a solution then find it. If it does not then demonstrate that the system is inconsistent.

\[
\begin{align*}
2x - z &= -1 \\
x - 4y - z &= 0 \\
-2x - 2y + z &= 2
\end{align*}
\]
**Problem 13:** (10 points) An 8 meter tall ladder is leaning against a vertical wall. The ladder makes an angle of $\frac{\pi}{3}$ with the horizontal ground. How high on the wall does the ladder reach?
Problem 14: (10 points) Express the domain of the function \( f(x) = \frac{3}{\sqrt{x^2 - 2x - 3}} \) in interval notation.