Fall 2014

Math 1300–Precalculus Final Exam

Your Name: ________________________________________________

Instructions: Please justify all your answers. No partial credit will be awarded for answers without calculations or explanations. You may **Not** use any electronic technology. Please sign your name below signifying that you have read, understood and will abide by these directions and will not cheat.

Your Signature: ____________________________________________

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**Problem 1:** (6 points) Evaluate the following functions at the specified point. Find the exact values.

(1) \( \sin \left( -\frac{5\pi}{6} \right) \)

(2) \( \tan(\pi) \)

**Problem 2:** (10 points) Consider the first few terms of the sequence:

\[
\frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \frac{2}{81}, \cdots
\]

(1) Assuming that the pattern established by the terms shown continues, can this sequence be classified as arithmetic, geometric or neither? (Justify your answer).

(2) Find a general formula for the n’th term of the sequence.
Problem 3: (10 points) Find the quotient and remainder when the polynomial $3x^3 - 5x + 3$ is divided by $x - 2$. 
Problem 4: (10 points) Verify the identity:

\[ \frac{1}{1 - \cos(\theta)} + \frac{1}{1 + \cos(\theta)} = 2 \csc^2(\theta) \]
Problem 5: (15 points) Consider the equation

\[ 2x^2 + 3y^2 - 4x + 24y + 32 = 0 \]

(1) Put the equation in standard form and identify as a type of conic section.
(2) Graph the equation, label the foci, vertices and center.
Problem 6: (10 points) Rewrite the following expression as an algebraic function of $x$.

$$\sin\left(\arccos\left(\frac{x}{2}\right)\right)$$
Problem 7: (10 points) Solve the following equation for \( x \).

\[ \log_3(x - 4) + \log_3(x + 4) = 2 \]
Problem 8: (15 points) Graph each of the equations in the following system on a single graph. Then solve the system algebraically. Describe the connection between your algebra and your graph. Provide ticks and labels for your graph.

\[2x + y = 3\]
\[x - 3y = 0\]
Problem 9: (10 points) Elevation sightings are made from points $A$ and $B$ respectively. The angles of inclination, $\frac{\pi}{6}$ and $\frac{\pi}{4}$, are marked along with the distance between the sightings. Determine the height $h$ from the given data.
Problem 10: (10 points) Let $f(x) = x(10 - 7x + x^2)$.

(1) Determine the values of $x$ for which $f(x) \leq 0$ and express your answer in interval notation.

(2) Express the domain of the function $g(x) = \frac{1}{\sqrt{f(x)}}$ in interval notation.
Problem 11: (10 points) The function $f(x) = -7 + \sqrt{4x - 5}$ is one to one on its domain.

(1) Find a formula for its inverse, $f^{-1}(x)$.

(2) Verify your formula is correct by computing and simplifying $f \circ f^{-1}(x)$. 
**Problem 12:** (10 points) Let \( f(x) = 2x^2 - 4x - 1 \).

(1) Put \( f(x) \) in standard form and draw its graph.

(2) Identify the vertex and axis of symmetry in your graph of \( f \).

(3) Identify the interval(s) on which \( f \) is increasing and separately decreasing.

(4) Is the vertex an absolute maximum or minimum? Explain.
**Problem 13:** (20 points) Consider the rational function

\[ f(x) = \frac{3x^2 - 3x}{x^2 - 5x + 4} \]

(1) Express the domain of \( f \) in interval notation.

(2) Find the \( x \) and \( y \) intercepts of \( f \).

(3) Find all vertical and horizontal asymptotes.

(4) Sketch a detailed graph of \( f \).
**Problem 14:** (20 points) Use the method of transformations to sketch a detailed graph of each of following functions. First sketch the basic function and then sketch its transformation. Label all asymptotes and intercepts.

(1) Basic: $y = \log(x)$ and Transformed: $y = 4 - \log(2x + 1)$.

(2) Basic: $y = \sqrt{x}$ and Transformed: $y = 1 - \sqrt{1 - x}$.