Math 1200 - Exam 1B

Name: ____________________________

Show ALL of your work for full credit. Simplify your answers as much as possible. Each problem is worth 6 points unless otherwise specified.

10. Completely factor the following polynomials. [5 pts. each]
   A. \(7xy - x^2 + 14y - 2x\)
      \[= x(7y - x) + 2(7y - x)\]
      \[= (7y - x)(x + 2)\]
   B. \(x^2 + 10x + 25\)
      \[= (x + 5)^2\]

12. Solve each equation. Identify any restricted values in each equation. [6 pts. each]
   A. \(-6(x - 2) + 3 = 9 - (x + 4)\)
      \[-6x + 12 + 3 = 9 - x - 4\]
      \[-6x + 15 = 5 - x\]
      \[-6x + x = 5 - 15\]
      \[-5x = -10\]
      \[x = 2\]
   B. \(\frac{1}{2} - \frac{7}{2x} = \frac{5}{x}\)
      \[2 \times \left[\frac{1}{2} - \frac{7}{2x}\right] = 2 \times \frac{5}{x}\]
      \[2 - 7 = 10\]
      \[x = 17\]

3. A 5 in. by 7 in. rectangular photograph is in a frame that adds a border of \(x\) inches on all sides. The perimeter of the frame is 40 in. Find \(x\), the width of the frame border. \(P = 2L + 2W\) [8 pts.]
   \[L = 7 + 2x\]
   \[W = 5 + 2x\]
   \[P = 2(L + 2x) + 2(5 + 2x)\]
   \[40 = 14 + 4x + 10 + 4x\]
   \[40 = 24 + 8x\]
   \[8x = 16\]
   \[x = 2\] inches

4. Multiply and simplify \(\frac{x^2 - 9}{x - 1} \cdot \frac{2x + 4}{x^2 + 5x + 6}\)
   \[= \frac{(x - 3)(x + 3)}{x - 1} \cdot \frac{2(x + 2)}{(x + 3)(x + 2)}\]
   \[= \frac{x - 3}{x - 1} \cdot 2 = \frac{2(x - 3)}{x - 1}\]

5. Your cell phone plan costs $60.00 a month plus $0.10 for each additional text sent over the limit.

4. A. Write a model for the cost \(C\) of the monthly bill for \(t\) additional texts sent over the limit. [4 pts.]
   \[C(t) = 60 + 0.10t\]

4. B. Use this model to compute the cost of the monthly bill if you send 150 texts over the limit. [4 pts.]
   \[C(150) = 60 + 0.10(150)\]
   \[= 60 + 15\]
   \[= $75\]
6. Find the LCD of \( \frac{7}{15x^2} \) and \( \frac{1}{3x^5} \).
\[
\text{LCD of } 15x^2 \text{ and } 3x^5 \text{ is } 15x^5.
\]
So LCD of \( 15x^2 \) and \( 3x^5 \) is \( 15x^5 \).

7. Simplify \( \frac{15c^{13}d^4}{20c^{19}d} \). Write your answer using positive exponents.
\[
\frac{15c^{13}d^4}{20c^{19}d} = \frac{15}{20} \cdot \frac{c^{13}}{c^{19}} \cdot d^4 \cdot d^{-1} = \frac{3}{4} \cdot \frac{c^{13-19}}{d} = \frac{3}{4} \cdot \frac{1}{c^6}d^3.
\]

8. Simplify each expression. All variables represent positive real numbers.

A. \(-25^0 \) [4 pts.]
\[
= \left(-\frac{25}{5}\right)^0 = 1
\]

B. \((16)^{3/4} \) [4 pts.]
\[
= \left((16^{1/4})^3\right) = 2^3 = 8
\]

C. \(2\sqrt{6x^2} \cdot \sqrt{3x^3}y \) [6 pts.]
\[
= 2 \cdot \sqrt[4]{6x^2 \cdot 3x^3} \cdot y
= 2 \cdot \sqrt[4]{18x^5} \cdot y
= 2 \cdot \sqrt[4]{18} \cdot x^{5/4} \cdot y
= 2 \sqrt[4]{18x^{5/4} y}
= 6 \sqrt[4]{3x^{5/4} y}
\]

D. \((3\sqrt{x} + 5)(2\sqrt{x} - 7) \) [6 pts.]
\[
= 3\sqrt{x}(2\sqrt{x} - 7) + 5(2\sqrt{x} - 7)
= 6x - 21\sqrt{x} + 10\sqrt{x} - 35
= 6x - 11\sqrt{x} - 35
\]

9. Subtract the polynomials.
\[
(7x^3 + 3x^2 + 2x) - (2x^3 + 3x^2 - 5x - 4)
= 7x^3 + 3x^2 + 2x - 2x^3 - 3x^2 + 5x + 4
= (7x^3 - 2x^3) + 2x + 5x + 4
= 5x^3 + 7x + 4
\]

10. Use the Order of Operations to simplify.
\[
6 - \left[-12 + 3((1 - 6)^2 - 18)\right]
= 6 - \left[-12 + 3(25 - 18)\right]
= 6 - \left[-12 + 3(7)\right]
= 6 - [-12 + 21]
= 6 - 9 = -3
\]

11. Write the inequality \( x \geq 3 \) using a number line, intervals, and set notation.

\[
\text{Number Line} \quad \text{Interval Notation} \quad \text{Set Notation}
\]
\[
\begin{align*}
\text{Number Line} & \quad \text{Interval Notation} & \quad \text{Set Notation} \\
\_ & \quad [x, \infty) & \quad \{x \mid x \geq 3\}
\end{align*}
\]

12. Perform the following conversions.

A. \( 2 \times 10^{-3} \) to standard decimal notation.
\[
2 \times 10^{-3} = 2 \times \frac{1}{1000} = 2.000 \times 10^{-3}
\]

B. \( 470000 \) to scientific notation.
\[
(4.7) \times 10^5
\]