

Error Type, Power, Assumptions

Parametric vs. Nonparametric tests

Type-I & -II Error

Power Revisited

Meeting the Normality Assumption

- Outliers, Winsorizing, Trimming
- Data Transformation



Parametric Tests

Parametric tests assume that the variable in question has a known underlying mathematical distribution that can be described (normal, binomial, poisson, etc.).



This underlying distribution is the fundamental basis for all of sample-to-population inference.

Parametric vs. Nonparametric Tests

Nonparametric tests are considered *distribution-free* methods because they do not rely on any underlying mathematical distribution.

Q. So why even worry about what the distribution is or is not?

Why not just use nonparametric tests all the time?

A. Nonparametric tests usually result in loss of efficiency (the ability to detect a false hypothesis). Efficiency is tied to error type.

Error Type

- Truth Table -

Ho	Accepted	Rejected
True	Correct	Type - I
False	Type - II	Correct

4

Type-I Error

Before you apply a statistical test, you must specify an acceptable level of Type-I error.

Usually, one accepts that there will always be some deviant observations by chance alone and that 5% error is acceptable.

Recall, Type-I error is expressed as a probability and is symbolized by α .

Thus, a Type-I error of $\alpha = 0.05$ corresponds to a 5% error level and specifies the rejection region or critical region of a statistical test.

5

Error Type

Q. So why not specify a very small error rate such as 0.01 or 0.001?

A. Because as your Type-I error rate diminishes, Type-II error increases!

Unfortunately, while Type-II error is important, it is difficult to evaluate in many biological applications.

6

Type-II Error

Type-II error is the probability of accepting a false H_0 .

Type-II error is also referred to as a probability and symbolized as β .

β is harder to specify because it requires knowledge of the alternate hypothesis (which is unknown in most circumstances).

β is not fixed, but may increase to a maximum of $1 - \alpha$.

7

Power

Important Concept:



$$\text{Power} = 1 - \beta$$



Power and β are complements.

Thus, for any given test, we would like power to be as high as possible and β to be as low as possible.

8

Power

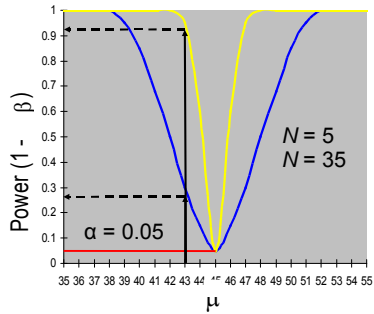
Since we can not generally provide an alternative hypothesis, we must describe β or $1 - \beta$ as a continuum of alternative values.

This is known as a **Power Curve**.

To improve the power of a test (i.e., decrease β) while keeping α fixed, we vary N .

9

Power Curve



Power Curves
for testing:

$H_0: \mu = 45$
 $H_a: \mu \neq 45$

at $\alpha = 0.05$
and $N = 5, 35$

10

Nonparametric Tests



Q. Well then, doesn't this mean
nonparametric tests are
undesirable or inferior?

A. No ! They just have less
efficiency. They are the
appropriate test to use when the
conditions warrant.

11

Parametric vs. Nonparametric Tests

In general,

Parametric tests are more "conservative"
(i.e., less likely to make a Type-I Error).



Nonparametric tests are more "liberal"
(i.e., more likely to make a Type-I Error).



Thus, in most biological applications, one should
always attempt to use a parametric test first.

12

Meeting the Normality Assumption

Q. What if you are unable to meet the assumption of normality? You can not continue to do parametric statistics if this has not been met, correct?

A. The best strategy is to first try a simple manipulation or **re-arrangement of the data**. This may allow you to meet the normality assumption and continue with parametric statistics.

13

Data Manipulations

Options for Data Manipulation:

Delete outliers
Winsorize data
Trim data

These procedures are "legal" as long as:

- (1) they are exercised judiciously
- (2) never used to adjust a P -value

14

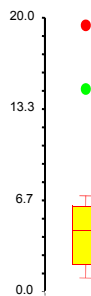
Outliers

Handling outliers is tricky business.

Do these values represent natural biological variability, or are they fluke values, or are they a mistake in data collection or recording?

During EDA, use box-plots to help identify outliers. Carefully examine outliers.

- Mild outliers are usually biologically possible.
- Severe outliers are often mistakes.



15

Outliers

Data can be normalized if there are mild outliers usually by winsorization, trimming, or transformation.

Generally, severe outliers must be deleted from the data to achieve normality.

★ CAUTION: ★
Do not ever delete more than 5% of your data.

Severe outliers can legitimately fall within this range. However, if there are more than 5% severe outliers, usually something else is going on.

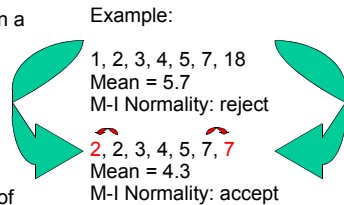
16

Winsorizing Data

Usually, but not necessarily, performed in a symmetrical fashion.

Rank data, then give extremes the same value as adjacent rank.

Recompute stats & test of normality.



17

Trimming Data

Alternatively, data can be trimmed from the tails.

Usually, drop X_{\min} & X_{\max}

This reduces N and may affect Power.

Example:

1, 2, 3, 4, 5, 7, 18
Mean = 5.7
M-I Normality: reject

2, 3, 4, 5, 7
Mean = 4.2
M-I Normality: accept

18

Winsorization vs. Trimming

Note that, in our example, there was very little difference in the effect of trimming vs. winsorization.

There are no hard and fast rules as to when to apply one and not the other.

Winsorization is probably more appropriate when sample sizes are small and you need to protect your power.

19

Data Transformations

The necessity to transform data may arise under the conditions of non-independence or non-normality.

Data transformation seems like a lot of manipulation at first glance, but it just involves placing your data on another scale.

Data from a linear scale can be transformed on to a \log_{10} scale (or any other). This often corrects a variety of problems. Different transformations are available to correct different problems.

20

Effect of Transformation (ln)

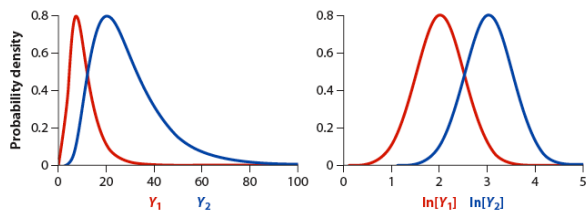


Figure 13.3-1 Left panel: Two right-skewed probability distributions having different standard deviations. In this case, the distribution with the highest standard deviation also has the highest mean. Right panel: The log transformations of the same variables. On the log scale, these two distributions have normal distributions with the same standard deviation.

21

Data Transformations

Typical Transformations:

- Logarithmic
- Square Root
- Angular
- Box-Cox
- Reciprocal
- Power

Logarithmic Transformations


Logarithmic transformations are useful starting points when:

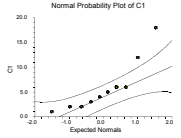
- (1) mean is correlated with variance
- (2) data are skewed to the right

Can take a variety of forms:

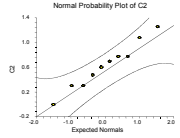
- $Y' = \log_{10}(Y)$
- $Y' = \log_{10}(Y+1)$
- $Y' = \ln(Y)$
- $Y' = \ln(Y+1)$

Transformation Example

Y		$Y' = \log_{10}(Y)$	
1		0	
2	K ² Omnibus:	0.301	K ² Omnibus:
2	<u>Reject</u> Normality	0.301	<u>Accept</u> Normality
3		0.477	
4		0.602	
5		0.699	
6		0.778	
6		0.778	
12		1.079	
18		1.255	



Normal Probability Plot of C1



Normal Probability Plot of C2

```

> Y<-c(1,2,2,3,4,5,6,6,12,18)
> shapiro.test(Y)

      Shapiro-Wilk normality test

data:  Y
W = 0.816, p-value = 0.02268

> Ylog<-log(Y)
> Ylog
[1] 0.0000000 0.6931472 0.6931472 1.0986123 1.3862944
[6] 1.6094379 1.7917595 1.7917595 2.4849066 2.8903718

> shapiro.test(Ylog)

      Shapiro-Wilk normality test

data:  Ylog
W = 0.9797, p-value = 0.9633

```



Square Root Transformations

Most appropriate when data are counts (e.g., number of leaves, number of flowers, etc.).

Count data tend to more closely follow a Poisson distribution. A square root transformation brings closer to normal.

$$Y' = \sqrt{Y + 0.05}$$

$$Y' = \sqrt{Y} + \sqrt{Y + 1}$$

Variety of forms:

$$Y' = \sqrt{Y + \frac{3}{8}}$$

$$Y' = \sqrt[3]{Y}$$

26

Angular Transformations

Whenever the data are proportions or percentages, you should consider an angular transformation.

Percentages tend to usually follow a binomial distribution.

Typical transforms:

$$\theta = \arcsin \sqrt{p} \text{ (where } p \text{ ranges } 0-1)$$

$$\theta = \arcsin \sqrt{\frac{Y + \frac{3}{8}}{N + \frac{3}{4}}}$$

27

Box-Cox Transformation

When there is no *a priori* reason for choosing one transformation over another, the Box-Cox transformation might be an appropriate place to start.

Iterate through a series of power functions until normality is maximized:

$$Y' = (Y^\lambda - 1) / \lambda \quad (\text{for } \lambda \neq 0)$$

28

Reciprocal Transformation

Reciprocal transforms often prove useful when the standard deviations of the groups of data are proportional to the square of the means of the groups.

Iterate through a series of \square until normality is maximized:

$$Y' = \frac{1}{Y^\lambda}$$

29

Power Transformation

A power transformation is often effective in dealing with two situations:

- (1) if S decreases with increasing \bar{y}
- (2) if the distribution is skewed to the left

Iterate through a series of λ until normality is maximized:

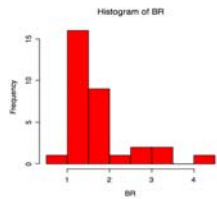
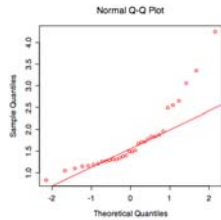
$$Y' = Y^\lambda$$

30

Example 13.1 (p. 323)

```
> BR
[1] 1.34 1.96 2.49 1.27 1.19 1.15 1.29 1.05 1.10 1.21 1.31
[12] 1.26 1.38 1.49 1.84 1.84 3.06 2.65 4.25 3.35 2.55 1.72
[23] 1.52 1.49 1.67 1.78 1.71 1.88 0.83 1.16 1.31 1.40

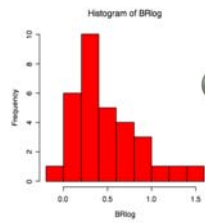
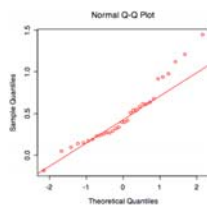
> qqnorm(BR,col="red")
> qqline(BR,col="red")
> hist(BR,col="red")
```



31

```
> BRlog
[1] 0.29266961 0.67294447 0.91228271 0.23901690
[5] 0.17395331 0.13976194 0.25464222 0.04879016
[9] 0.09531018 0.19062036 0.27002714 0.23111172
[13] 0.32208350 0.39877612 0.60976557 0.60976557
[17] 1.11841492 0.97455964 1.44691898 1.20896035
[21] 0.93609336 0.54232429 0.41871033 0.39877612
[25] 0.51282363 0.57661336 0.53649337 0.63127178
[29] -0.18632958 0.14842001 0.27002714 0.33647224

> qqnorm(BRlog,col="red")
> qqline(BRlog,col="red")
> hist(BRlog,col="red")
```



32

```
> shapiro.test(BR)
```

Shapiro-Wilk normality test

```
data: BR
W = 0.8175, p-value = 8.851e-05
```

```
> shapiro.test(BRlog)
```

Shapiro-Wilk normality test

```
data: BRlog
W = 0.938, p-value = 0.06551
```



33
