# Cooking with Numbers

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<th>Students will recall prior knowledge of working with fractions and will apply measurement scales.</th>
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**Student/Class Goal**
Students want to be able to take recipes and scale them.

**Standard** Use Math to Solve Problems and Communicate
(Primary benchmarks in bold.)

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**Materials**
Problem Solving Steps—Handout
SmartPal kit (SmartPal sleeves, wipe off cloths, dry erase markers) – inserting a blank sheet of paper into the sleeves will give students a reusable sheet of paper that they can quickly try answers out on and erase without using up a pencil eraser. It’s quicker as well.
(Optional) Calculators
Fraction circles
Fraction Procedures—Handout
Recipes: Can be the ones given on the handout or, for additional student integration, have them bring their favorites to class as well

**Learner Prior Knowledge**
Students should be familiar with applying all four arithmetic operations on whole numbers.
While not necessary, it may be useful for application problems if students are able to distinguish between metric and U.S. measurement terms as well as tell whether a term is for distance, volume, or weight.

**Vocabulary**
Fraction - a part of a whole represented as one number (the part) over another number (the whole).
Numerator - the top number (the part) in a fraction.
Denominator - the bottom number (the whole) in a fraction.
Least Common Denominator (LCD) - when comparing two fractions, this is the least common multiple of the two denominators. In other words, this is the smallest multiple that the two denominators have in common.
Reciprocal - the multiplicative inverse of a number. For a fraction, we can simplify finding this by just swapping the positions of the numerator and denominator. For a mixed number, we will first need to turn it into an improper fraction.

**Instructional Activities**
Part 1: Give them the handout on Polya’s 4 step problem solving process.
We want our students to be able to follow a sequence of steps when solving problems. Whether they know it or not, they probably already do this. We want them to follow Polya’s four step process:

1. Understand the problem (What is the unknown? The data? The conditions?)
2. Pick a strategy to solve the problem (Have you seen a similar problem? One with a similar unknown?)
3. Implement that strategy to come to a solution
4. Review the work and the solution to make sure the solution makes sense in the given context.

After step 4, if there seems to be an error with the solution, students should go back to step 1 and repeat the process until they come to a solution that makes sense.
(For the first few lessons, these steps should be discussed and written down so that students can refer to them as a guide when solving problems. During the I do steps, your thinking aloud should show you going through all four steps in the process. Though they are not explicitly stated in the lesson plan, you should be going through them as you solve the problem. While you begin the problem, make sure to state what you need to pull out of the problem to show you are understanding what is being asked. Then be sure to describe the strategy you are going to use to solve the problem. This could be as simple as stating that there is a given procedure for adding fractions. Then go through the implementation. Finally, make sure to go back and review the answer so students see that the solution makes sense given the context.)

Note: Keep in mind that your class may not need to go through each of the parts below. Please pick and choose which elements to incorporate into your actual lesson based on what you know of your students. In addition, extra sample problems may need to be incorporated based upon your particular class. See the technology integration portion for a website that gives many example problems.

Part 2: Give them the SmartPal kit (optional), calculators (also optional), the Fraction Procedures handout, and the Fraction Circles to use. Since your students should have a firm grasp of arithmetic with whole numbers, we should be able to jump right into introducing the concept of a fraction. A fraction is a part of a whole. Since we’ll be dealing with food in this lesson anyway, a good way to think of this is with a pizza or a pie. If a family were to order a pizza for dinner, the entire pizza would be considered the “whole.” When you take a slice out of the box, you are eating a portion, or a part, of that whole. As an example, let’s think about a pizza that has been cut into 8 slices. (For simplicity, let’s assume in all the examples that the pizzas have slices of equal size. So all 8 slices are the same size as one another.) If we take one of these slices to eat, we have taken 1 out of 8 possible slices. To put this into a fraction, we would have: \[ \frac{1}{8} \]. In this fraction, the 1 is known as the numerator, which is the portion or part of the whole that we are currently working with. The 8 is the denominator, which is the whole, or how many of our parts would make that whole. In our case, it would take 8 of our slices to comprise the whole pizza, so 8 is our denominator.

Part 3: Now that we know what a fraction is, we see it is made up of whole numbers. This means we should be able to do arithmetic operations with fractions. Let’s start with addition.

- **(I do)**
  1. **(Understand the problem)** Working with our pizza with 8 slices, let’s say Jane eats 2 slices and Bob eats 4. If we want to know what fraction of the entire pizza has been eaten, we would want to add their two portions together.
  2. **(Pick a strategy)** Jane ate \[ \frac{2}{8} \] of the pizza and Bob ate \[ \frac{4}{8} \] of it. To find the portion eaten, we must solve \[ \frac{2}{8} + \frac{4}{8} \]. In our problem, we see the denominators are the same. When adding fractions with a common denominator, the procedure is to add the numerators together and put that sum over the common denominator. The resulting fraction is our answer.
  3. **(Implement the strategy)** In this case, our answer will have that same denominator, and we just add the numerators: \[ \frac{2+4}{8} = \frac{6}{8} \]. Now might be a good time to discuss reducing fractions. Students may recognize that the whole numbers 6 and 8 are both divisible by 2. When we have a fraction where that is the case, the numerator and denominator are both divisible by the same number, then the fraction can be reduced. In this case, we would do the following: \[ \frac{6}{8} = \frac{3}{4} \]. When we reduce a fraction, we find a new fraction that is actually equal to our previous fraction. Thus, \[ \frac{6}{8} \] is the same as \[ \frac{3}{4} \]. This can be seen/explored further using the fraction circles. This situation worked because we had a common denominator between the two fractions. (You can also add whole numbers to fractions. Remember: The denominator for a whole number would be 1, so to write 3 as a fraction, we would have \[ \frac{3}{1} \].
  4. **(Review)** With the Fraction Circles, it would be easy to add up that six total slices were eaten out of a possible eight. Thus, we have six (the part) over eight (the whole). Therefore, our answer makes sense.

  - **(We do)** Give the class a second scenario in which Jane and Bob both eat \[ \frac{4}{8} \] of the pizza and have the class as a whole walk you through Polya’s steps, solving the problem as before. In this case, we have:
    \[ \frac{4}{8} + \frac{4}{8} = \frac{4+4}{8} = \frac{8}{8} = 1 \].
• (You do) And finally, give the class the scenario in which two pizzas are bought, Jane eats $\frac{2}{8}$, Bob eats $\frac{5}{8}$, and they invite their friend Dave over who eats $\frac{3}{8}$. In this case, we have:

$$\frac{2}{8} + \frac{5}{8} + \frac{3}{8} = \frac{2 + 5 + 3}{8} = \frac{10}{8} = \frac{10}{8} \div 2 = \frac{5}{4}.$$ 

This is called an improper fraction since the numerator is bigger than the denominator. Walk them through changing this to a mixed number (a number that has both a whole number and a fractional part). We would figure out how many times the denominator goes into the numerator evenly, in our case 4 can go into 5 one time. Thus, our whole part is 1. From our division of the numerator and denominator, we have a remainder of 1. We put that remainder over the denominator to get $\frac{1}{4}$. Thus, $\frac{5}{4} = 1 \frac{1}{4}$. During this step, you should be walking around the room to check student work and provide help/feedback as necessary. This also lets you gauge whether it is time to move on or do more problems.

• (I do)

1. (Understand the problem) What if we do not start with a common denominator? Let’s say that in addition to our pizza with 8 slices, we have another pizza that is the same size, but this pizza was cut into 6 slices. If Mike ate a slice of each pizza, how much of a whole pizza did he eat? In this case, we have $\frac{1}{8} + \frac{1}{6}$.

2. (Pick a strategy) The key to remember when adding (or subtracting!) fractions is that the denominators must match. So in order for us to add here, we must find a common denominator. When we reduced the last fraction, we saw that we could divide by the same number on the top and bottom of a fraction to get an equal fraction. We could do the same thing by multiplying $\frac{3 \cdot 2}{4 \cdot 2} = \frac{6}{8}$. So we want to change each fraction (by multiplying) into an equal fraction so that we have the same denominator. We are looking for the LCD (least common denominator). This means I want the smallest possible multiple that 6 and 8 share. The first few multiples of 6 are 6, 12, 18, 24, 30, 36, .... The first few multiples of 8 are 8, 16, 24, 32, 40, .... Since the first multiple they have in common is 24, that is their LCD.

3. (Implement the strategy) I want to change each fraction to an equal fraction with a denominator of 24. This will give us $\frac{1 \cdot 3}{8 \cdot 3} = \frac{3}{24}$ and $\frac{1 \cdot 4}{6 \cdot 4} = \frac{4}{24}$. Our addition problem has now become $\frac{3}{24} + \frac{4}{24} = \frac{7}{24}$. This fraction cannot be reduced as there is no number (other than 1) that can go into both 7 and 24.

4. (Review) We could again use the Fraction Circles to convince ourselves of this, but we do not have a denominator of 24. Instead, you could line up the $\frac{1}{8}$ tile and the $\frac{1}{6}$ tile and see that it is between the $\frac{1}{3}$ tile and the $\frac{1}{4}$ tile. If we divide 7 by 24, we will get an answer between those two numbers, so our answer does make sense.

• (We do) This time, having the class walk you through Polya’s process, pose a new problem. Let’s say that Mike and his friend Carlos ordered a pepperoni pizza cut into 8 slices and a cheese pizza cut into 10. If Mike ate $\frac{5}{8}$ of the pepperoni pizza and Carlos ate $\frac{4}{10}$ of the cheese pizza, how much pizza was consumed in total? Our problem is:

$$\frac{5}{8} + \frac{4}{10} = \frac{5 \cdot 5}{8 \cdot 5} + \frac{4 \cdot 4}{10 \cdot 4} = \frac{25}{40} + \frac{16}{40} = \frac{25 + 16}{40} = \frac{41}{40} = 1 \frac{1}{40}.$$ 

• (You do) Having students work on their own, pose the following problem: Mike and Carlos order the same pepperoni and cheese pizzas the next time they hang out. This time, Carlos eats $\frac{3}{8}$ of the pepperoni pizza and Mike eats $\frac{6}{10}$ of the cheese. How much pizza was eaten? Our solution is:

$$\frac{3}{8} + \frac{6}{10} = \frac{3 \cdot 5}{8 \cdot 5} + \frac{6 \cdot 4}{10 \cdot 4} = \frac{15}{40} + \frac{24}{40} = \frac{15 + 24}{40} = \frac{39}{40}.$$
• (I do)

1. (Understand the problem) Our next step in working with fractions would be subtraction. As subtraction is just the inverse of addition, it follows the same basic rule: the denominators must match. Let’s say we started with two whole pizzas, one with pepperonis and cut into 6 slices and one with mushrooms cut into 8 slices. If Ben ate two slices of pepperoni pizza and Marcus ate one slice of mushroom, how much more pizza did Ben eat than Marcus? In this case we want to solve $\frac{2}{6} - \frac{1}{8}$.

2. (Pick a strategy) Again, we must find a common denominator. From the last problem, we know it to be 24. We must change our fractions into ones with common denominators before subtracting.

3. (Implement the strategy) We can change this problem into this equal problem:

4. (Review) We could explore this with the Fraction Circles. Instead of adding the single eighth tile to the two sixths tiles, we would want to subtract. We can mimic this by setting the two sixths tiles side-by-side and then placing the eighth tile on top of one of the sixth tiles. The portion of the sixth tiles that is still visible is the difference. If we divide 5 by 24, what number is it near? How close are we to other fractions that can be reduced? Namely $\frac{4}{24}$ and $\frac{6}{24}$. This puts our answer between $\frac{1}{6}$ and $\frac{1}{4}$, which we can see based on our Fraction Circles.

• (We do) This time, have the students walk you through the problem solving process. Suppose a 10-slice pizza is ordered. If 3 slices were eaten, what fraction of the pizza is remaining? In this problem, we need to subtract a fraction from a whole number. Our problem is:

• (You do) A 12-slice pizza is ordered by Sally and Joe. Sally’s sister Janet calls and asks them to leave her some pizza. Sally thinks it’s only right to leave Janet a third of the pizza, or four slices. If Joe eats 5 slices, how many can Sally eat to leave Janet a third of the pizza? Here, we have a multi-step problem. We need to first find out how many slices Joe eats and Janet is left. If we leave Janet with 4 slices and Joe ate 5, then $\frac{9}{12}$ of the pizza is gone. This leaves Sally with:

or 3 slices. Notice that there is no need to reduce because we want to know the number of slices, not the fraction she is left with.

Part 4:

• (I do)

1. (Understand the problem) Our next step would be to introduce multiplication/division with fractions. This time, it does not matter whether or not our denominators match. If we stick with our pizza idea, let’s say that Danny and Rebecca want to split a pizza. Each one gets to pick the toppings for their half of the pizza. If Danny wants his entire half to be all veggies and Rebecca wants $\frac{2}{3}$ of hers to be just cheese and the other $\frac{1}{3}$ of hers to be Hawaiian, how much of the entire pizza is just cheese? In this case, we have $\frac{2}{3} \times \frac{1}{2}$ is just cheese. “Of” clues us in that this is a multiplication problem, so we have $\frac{2}{3} \times \frac{1}{2}$.

2. (Pick a strategy) One way to solve this would be to multiply straight across: the numerators multiplied together with the product being our new numerator and the denominators multiplied together with the product being our new denominator.

3. (Implement the strategy) This would give us: $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$. Based on our earlier discussion of reducing fractions, this is the same as $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$. When multiplying fractions, we can reduce before multiplying. Of course, we can reduce
single fractions before performing an arithmetic operation. If one of our fractions was \( \frac{2}{8} \) we could immediately reduce that to \( \frac{1}{4} \) by dividing numerator and denominator by 2. However, in our problem, we cannot immediately reduce either fraction. However, since what we really have is \( \frac{2}{3} \cdot \frac{1}{2} \) we can reduce the numerator and denominator by cancelling out common factors. Here, we can cancel out a factor of 2 in both the numerator and denominator even though they were from separate fractions in the beginning. Thus, we get \( \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3} \). Thus, \( \frac{1}{3} \) of the entire pizza will be cheese only.

4. \( \text{Review} \) Once again, we could explore this using Fraction Circles. If we take the tile representing a half, what tiles would we need to use to divide that space into three equal sections, or thirds? We would need the tiles that are sixths overall. Since it takes two of those to cover a third of the half tile, then our answer would be \( \frac{2}{6} \) or \( \frac{1}{3} \).

- \( \text{We do} \) Juan finds a recipe for calzones that calls for \( \frac{2}{3} \) cup of flour per calzone. If each calzone feeds one person, and Juan wants to make enough to feed his 4 person family, how much flour will he need to use? The solution can be found by:

\[
\frac{2}{3} \cdot 4 = \frac{2}{3} \cdot \frac{4}{1} = \frac{8}{3} = 2\frac{2}{3} \text{ cups of flour.}
\]

- \( \text{You do} \) Let your students work on the following problem: Robert orders a pizza one night for dinner and eats half of it. The following day, he decides to eat half of what is remaining for lunch. How much of the whole pizza is remaining?

\[
\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}
\]

which is what he ate for lunch, plus the \( \frac{1}{2} \) he ate for dinner means he has eaten \( \frac{3}{4} \).

\[
1 - \frac{3}{4} = \frac{4}{4} - \frac{3}{4} = \frac{4 - 3}{4} = \frac{1}{4}.
\]

- \( \text{I do} \)

1. \( \text{Understand the problem} \) To divide, we have to do something a little different. If Sheila knows that the serving size of a pizza is \( \frac{1}{8} \) of the entire pizza, and she chooses to get \( \frac{1}{2} \) of the pizza as a supreme, how many servings will the supreme part of the pizza have? In this case we are taking half of the pizza and dividing it into slices that are \( \frac{1}{8} \) of the entire pizza.

2. \( \text{Pick a strategy} \) Our problem is to solve \( \frac{1}{2} \div \frac{1}{8} \). The rule for dividing fractions is that we take the divisor (in this case it is \( \frac{1}{8} \)) and invert it before multiplying. To invert it means that we swap the numerator and denominator.

3. \( \text{Implement the strategy} \) So inverting \( \frac{1}{8} \) would give us \( \frac{8}{1} \). We take this new fraction and multiply. Thus \( \frac{1}{2} \div \frac{1}{8} \) becomes:

\[
\frac{1}{2} \cdot \frac{8}{1} = \frac{1 \cdot 8}{2 \cdot 1} = \frac{1 \cdot 4}{1 \cdot 1} = \frac{4}{1} = 4.
\]

Thus, there are 4 servings of the supreme pizza.

4. \( \text{Review} \) It may seem weird that we get a number larger than we started with since we are dividing. However, if we think about this intuitively, we will find that a larger number makes sense. We know there are 8 servings in the entire pizza, and we are basically just asking how many servings there are in half of that (since half the pizza is a supreme).

- \( \text{We do} \) Having the class walk you through the problem solving process, solve the following problem. Jack is having two friends over tonight to watch a movie. He finds exactly half of a leftover pizza in the fridge from the previous night. If
the three friends split the remaining pizza evenly, what fraction of the whole pizza would each person get? Here our solution would be:

\[
\frac{1}{2} \div 3 = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}.
\]

1. \((You \ do)\) Have the students solve the following problem: Nancy has \(3 \frac{1}{2}\) cups of mozzarella cheese in her fridge and wants to make personal pizzas from a recipe that calls for \(\frac{1}{2}\) cup of cheese per pizza. How many pizzas can she make? You may need to talk about converting from a mixed number to a fraction. To do so, the procedure is to multiply the whole part (3) by the denominator (2). This gives us 6. We take this number and add it to the numerator (1), giving us 7. This is our new numerator and it goes over our previous denominator. Thus \(3 \frac{1}{2}\) becomes \(\frac{7}{2}\). To solve our problem:

\[
3 \frac{1}{2} \div \frac{1}{2} = \frac{7}{2} \div \frac{1}{2} = \frac{7}{2} \div \frac{1}{2} = \frac{7}{1} = 7.
\]

Part 5:

1. \((I \ do)\) \(\text{(Understand the problem)}\) Give each student the list of recipe ingredients, or pass out some that the students have brought in. Recipes often contain many fractions in the ingredients list. If following the exact recipe, then it isn’t too hard to find the \(\frac{1}{2}\) cup line on a measuring cup or to find the correct spoon to measure \(\frac{1}{2}\) teaspoon. However, if scaling the recipe, or making more or less than what the written recipe calls for, we must use math to find out how much of each ingredient we need. Let’s say the Scott family is planning their dinner and dessert for the next two nights. They want to make enough today to have leftovers tomorrow. There are four members in their family, so they will need to make 8 servings of each dish. The first thing they want to make is Chicken Noodle Soup. We will have to scale the recipe so that each ingredient would be enough for 8 servings instead of the 12 on the recipe.

2. \((Pick \ a \ strategy)\) The first thing we should recognize is that this will be a multiplication problem as we will only be using a certain fraction “of” each ingredient. Then, we need to find that fraction. Since we want only 8 servings out of 12, our fraction is \(\frac{8}{12} = \frac{8\div4}{12\div4} = \frac{2}{3}\). This is the fraction we will use to multiply.

3. \((Implement \ the \ strategy)\) We must now multiply each ingredient by this fraction. This will leave us with the following new recipe:

- **Makes 12 servings**
  \[
  \frac{2}{3} \cdot \frac{12}{1} = \frac{2}{3} \cdot \frac{12}{1} = \frac{2}{1} \cdot \frac{4}{1} = \frac{8}{1} = 8 \text{ servings}
  \]
- 3 medium carrots, sliced
  \[
  \frac{2}{3} \cdot \frac{3}{1} = \frac{2}{3} \cdot \frac{3}{1} = \frac{2}{1} \cdot \frac{1}{1} = \frac{2}{1} = 2 \text{ carrots, sliced}
  \]
- 3 stalks celery, sliced
  - We could do the math, but it would be the same as the carrots, so 2 stalks celery, sliced
- 3 onions, sliced
  - Again, same as the previous two, so 2 onions, sliced
- 12 cups chicken broth
  - We have already multiplied by 12, when we found the new servings. Thus, we need 8 cups of chicken broth.
- 3 cups cubed, cooked chicken or turkey
  - Again, we have multiplied by 3, so 2 cups cubed, cooked chicken or turkey
- 1 \(\frac{1}{2}\) cups uncooked, medium egg noodles
  - We again have multiplied by \(\frac{1}{2}\), so \(\frac{1}{2}\) cups uncooked, medium egg noodles

4. \((Review)\) We should have all positive amounts, and as \(\frac{2}{3}\) is less than one, we should have a bit less of each ingredient.
• (We do) Have the class walk you through scaling the recipe for mashed potatoes so that it makes 8 servings. Now we want to get 8 servings out of our current 6. Thus, we want to multiply by \( \frac{8}{6} = \frac{8+2}{6+2} = \frac{4}{3} \). We do not need to make this a mixed number as we will just need to convert it back to an improper fraction to be able to multiply later.

- **Makes 6 servings**
  \[
  \frac{4}{3} \cdot 6 = \frac{4}{3} \cdot \frac{6}{1} = \frac{4}{3} \cdot \frac{6}{1} = \frac{4 \cdot 6}{3 \cdot 1} = \frac{24}{3} = 8 \text{ servings}
  \]

- 1/2 quart warm milk
  \[
  \frac{4}{3} \cdot \frac{1}{2} = \frac{2}{3} \text{ quarts warm milk}
  \]

- 6 medium russet potatoes, peeled and cubed
  \[
  \frac{4}{3} \cdot \frac{6}{1} = \frac{4}{3} \cdot \frac{6}{1} = \frac{4 \cdot 6}{3 \cdot 1} = \frac{24}{3} = 8 \text{ potatoes}
  \]

- 1/4 cup butter or margarine
  \[
  \frac{4}{3} \cdot \frac{1}{4} = \frac{1}{3} \text{ cup butter or margarine}
  \]

- 3/4 teaspoon salt
  \[
  \frac{4}{3} \cdot \frac{3}{4} = \frac{1}{3} \text{ teaspoon salt}
  \]

- dash of pepper

• (You do) Finally, have the students figure out how to scale the brownie recipe so that there are only 8 made. To go from 16 to 8, we are halving the recipe, so our fraction is \( \frac{1}{2} \) \( \frac{8}{16} = \frac{8+8}{16+8} = \frac{1}{2} \).

- **Makes 16 brownies**
  \[
  \frac{1}{2} \cdot 16 = \frac{1}{2} \cdot \frac{16}{1} = \frac{1}{2} \cdot \frac{16}{1} = \frac{1 \cdot 16}{2 \cdot 1} = \frac{8}{1} = 8 \text{ servings}
  \]

- 1/2 cup butter
  \[
  \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \text{ cup butter}
  \]

- 1 cup white sugar
  \[
  \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \text{ cup white sugar}
  \]

- 2 eggs
  \[
  \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \text{ egg}
  \]

- 1 teaspoon vanilla extract
  \[
  \text{Same as the cup of sugar, so } \frac{1}{2} \text{ teaspoon vanilla extract}
  \]

- 1/3 cup unsweetened cocoa powder
  \[
  \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} \text{ cup unsweetened cocoa powder}
  \]

- 1/2 cup all-purpose flour
  \[
  \text{Same as the butter, so } \frac{1}{4} \text{ cup all-purpose flour}
  \]

- 1/4 teaspoon salt
  \[
  \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \text{ teaspoon salt}
  \]

- 1/4 teaspoon baking powder
  \[
  \text{Same as the salt, so } \frac{1}{8} \text{ teaspoon baking powder}
  \]

**Assessment/Evidence (based on outcome)**
Each of the *you do* steps will serve as assessment. The instructor should be able to gauge understanding by having different students provide their solutions and explanations of how they arrived at that solution. In addition, during the *we do* steps, instructors should be encouraging all students to participate in the discussion. The ability to provide input in these discussions...
will help the teacher gauge each student’s mastery of the concepts.

Have the students scale any of the remaining recipes to hand in. For further critical thinking, have them explain the reasoning for their scale. Did they need to make less cookies because their class only has so many people? Did they want to use the salad for lunch the next day so they made a double batch?

**Teacher Reflection/Lesson Evaluation**
*Not yet completed*

**Next Steps**
Though not in the number sense strand, conversions would be a nice follow-up. Conversions within and between US/Metric units often involve multiplication using fractions. Another good follow-up would be introducing and working with proportions.

**Technology Integration**
- [http://www.mathsisfun.com/fractions-menu.html](http://www.mathsisfun.com/fractions-menu.html)
  - This website gives a number of topics dealing with fractions including how to perform the four arithmetic operations as well as how to simplify, convert between improper fractions and mixed numbers, and how to compare/order fractions.
  - This website allows you to quickly generate example problems. While the problems are strictly computational instead of in a context, you could put them into a context (or have your students do so) or use them for extra practice.

**Purposeful/Transparent**
Students want to be able to use fractions in everyday problems. The teacher will model a practical use of fractions and conversions by modeling and guiding students through converting fractional ingredient amounts in common recipes.

**Contextual**
There are many things outside of the mathematics classroom that deal with fractions. From buying fabric, to manipulating recipes, to working with distances and money, fractions crop up all over the place. Being able to work with these fractions and add, subtract, multiply, and divide them will be useful to students in many applications.

**Building Expertise**
Students will become familiar with working with fractions. They are also using this knowledge in a real-life application as opposed to just applying the operations to numbers for a math class.
Problem Solving

One of the primary reasons people have trouble with problem solving is that there is no single procedure that works all the time — each problem is slightly different. Also, problem solving requires practical knowledge about the specific situation. If you misunderstand either the problem or the underlying situation you may make mistakes or incorrect assumptions. One of our main goals for this semester is to become better problem solvers.

To begin this task, we now discuss a framework for thinking about problem solving: Polyà’s four-step approach to problem solving.

Polyà's four-step approach to problem solving

1. Preparation: Understand the problem
   - Learn the necessary underlying mathematical concepts
   - Consider the terminology and notation used in the problem:
     1. What sort of a problem is it?
     2. What is being asked?
     3. What do the terms mean?
     4. Is there enough information or is more information needed?
     5. What is known or unknown?
   - Rephrase the problem in your own words.
   - Write down specific examples of the conditions given in the problem.

2. Thinking Time: Devise a plan
   - You must start somewhere so try something. How are you going to attack the problem?
   - Possible strategies: (i.e. reach into your bag of tricks.)
     1. Draw pictures
     2. Use a variable and choose helpful names for variables or unknowns.
     3. Be systematic.
     4. Solve a simpler version of the problem.
     5. Guess and check. Trial and error. Guess and test. (Guessing is OK.)
     6. Look for a pattern or patterns.
     7. Make a list.
   - Once you understand what the problem is, if you are stumped or stuck, set the problem aside for a while. Your subconscious mind may keep working on it.
   - Moving on to think about other things may help you stay relaxed, flexible, and creative rather than becoming tense, frustrated, and forced in your efforts to solve the problem.

3. Insight: Carry out the plan
   - Once you have an idea for a new approach, jot it down immediately. When you have time, try it out and see if it leads to a solution.
   - If the plan does not seem to be working, then start over and try another approach. Often the first approach does not work. Do not worry, just because an approach does not work, it does not mean you did it wrong. You actually accomplished something, knowing a way does not work is part of the process of elimination.
   - Once you have thought about a problem or returned to it enough times, you will often have a flash of insight: a new idea to try or a new perspective on how to approach solving the problem.
   - The key is to keep trying until something works.

4. Verification: Look back
   - Once you have a potential solution, check to see if it works.
     1. Did you answer the question?
     2. Is your result reasonable?
     3. Double check to make sure that all of the conditions related to the problem are satisfied.
     4. Double check any computations involved in finding your solution.
   - If you find that your solution does not work, there may only be a simple mistake. Try to fix or modify your current attempt before scrapping it. Remember what you tried—it is likely that at least part of it will end up being useful.
   - Is there another way of doing the problem which may be simpler? (You need to become flexible in your thinking. There usually is not one right way.)
   - Can the problem or method be generalized so as to be useful for future problems?
Fraction Procedures

A **fraction** is a part of a whole. We write it as: \(\frac{\text{numerator}}{\text{denominator}}\), where the **numerator** is the number of parts you have and the **denominator** is the number of equal parts the whole is divided into.

**Equivalent fractions** look different from each other, but are actually the same or equal. Some examples of equivalent fractions are:

\[
\frac{1}{2} = \frac{2}{4} = \frac{6}{12}
\]

Without using a visual representation, two fractions are equivalent if we can multiply or divide both the numerator and the denominator of one fraction by the same number and arrive at the other fraction. For example:

\[
\frac{1\cdot2}{2\cdot2} = \frac{2}{4} \quad \text{and} \quad \frac{6\div3}{12\div3} = \frac{2}{4}
\]

**Reducing (or simplifying)** a fraction means that we find an equivalent fraction that has the lowest possible whole number numerator. To find the simplest form of our fraction, we can continually divide the numerator and denominator by the same number until we no longer have a common factor between the two parts of our fraction. Or, to do it in one step, we would divide the top and the bottom by the **greatest common factor (GCF)**. One simple way (but not the only way!) to find the GCF is by listing all the factors of both numbers and then finding the largest that the two numbers have in common. To reduce \(\frac{6}{12}\), we would list the factors of 6 [1, 2, 3, and 6] and the factors of 12 [1, 2, 3, 4, 6, 12]. We see that the two numbers have the factors of [1, 2, 3, and 6] in common, and the greatest of these is 6. Therefore, to find the simplest form of our fraction, we would divide both the numerator and denominator by 6:

\[
\frac{6\div6}{12\div6} = \frac{1}{2}
\]
**Adding Fractions**

1. Make sure the denominators are the same. If they are not, find a common denominator and change your fractions into equivalent fractions with that denominator.

\[
\frac{2}{5} + \frac{3}{4}
\]

Here, our denominators are not the same, so we need to change our fractions so they are over a common denominator. To keep our problem simple, we want to find the **least common denominator (LCD)** or the **least common multiple (LCM)** of our two denominators. To do this, we will list the multiples of each number until we find the first occurrence of a common multiple. The multiples of 5 are (5, 10, 15, 20, 25, 30, 35, 40...). We could keep going, but the list would never end so we can stop when we think we will find one. Knowing that 40 is a multiple of 4, we can stop there. The multiples of 4 are (4, 8, 12, 16, 20, 24, 28...). Comparing the two lists, I see that the first number they have in common is 20. Therefore, my LCD is 20. Now I want to change my two fractions into equivalent fractions with a denominator of 20. *(Remember! You must do the same to the top and bottom of the fraction!)*

\[
\frac{2\cdot4}{5\cdot4} + \frac{3\cdot5}{4\cdot5} = \frac{8}{20} + \frac{15}{20}
\]

2. Add the numerators and put the answer over the denominator.

\[
\frac{8}{20} + \frac{15}{20} = \frac{23}{20}
\]

3. Simplify the fraction. (If needed)

We want our fraction to not be an **improper fraction** (where the numerator is bigger than the denominator), so we will change it to a **mixed number** (a number with a whole number part and a fraction part). To do this, we divide the numerator by the denominator and put the whole number portion of the answer as the whole number portion of our mixed number. Since 20 goes into 23 one time, we will have a 1 as our whole number part. Then, we put the remainder over the denominator. Thus, we have:

\[
\frac{8}{20} + \frac{15}{20} = \frac{23}{20} = 1\frac{3}{20}.
\]

**Subtracting Fractions** follows the same three step process, only we subtract in step two instead of add.

**Multiplying Fractions**

1. Multiply the numerators. The answer will be the numerator of our final answer.

\[
\frac{2}{5} \cdot \frac{3}{4} = \frac{2\cdot3}{5\cdot4} = \frac{6}{20}
\]

2. Multiply the denominators. The answer will be the denominator of our final answer.

\[
\frac{2}{5} \cdot \frac{3}{4} = \frac{2\cdot3}{5\cdot4} = \frac{6}{20}
\]
3. Simplify if necessary.

6 has factors of (1, 2, 3, and 6) while 20 has multiples of (1, 2, 4, 5, 10, and 20). From these lists, we see their GCF is 2, and can simplify.

\[
\frac{6}{20} = \frac{6\div2}{20\div2} = \frac{3}{10}.
\]

**Dividing Fractions**

1. Change the second fraction (the divisor or the one you want to divide by) into its reciprocal.

   This just means that we will flip it, or trade the numerator for the denominator and vice versa.

   If our problem is \(\frac{2}{5} \div \frac{3}{4}\), then \(\frac{3}{4}\) will become \(\frac{4}{3}\).

2. Multiply the first fraction by that reciprocal.

   \[
   \frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \cdot \frac{4}{3} = \frac{2\cdot4}{5\cdot3} = \frac{8}{15}.
   \]

3. Simplify (if necessary). In our case, the GCF of 8 and 15 is 1, so we cannot reduce this fraction any further.

**Note**

When working with mixed numbers, it always a good idea to change these into improper fractions to begin with. You *cannot* perform the procedures above for multiplication and division if you are using mixed numbers. To convert a mixed number to an improper fraction:

1. Multiply the whole number part by the fraction’s denominator.
2. Add the result to the current numerator.
3. Write the result as the new numerator and put it over the initial denominator.

\[3 \frac{2}{5} = 3 \cdot \frac{2}{5} = \frac{15 + 2}{5} = \frac{17}{5}\]
Recipe Ingredients

Cookies
- Makes 6 dozen cookies
- 4 1/2 cups all-purpose flour
- 2 teaspoon baking soda
- 2 cups butter or margarine, softened
- 1 1/2 cups packed brown sugar
- 1/2 cup white sugar
- 2 (3.4 ounce) packages instant vanilla pudding mix
- 4 eggs
- 2 teaspoons vanilla extract
- 4 cups semi-sweet chocolate chips
- 2 cups chopped walnuts (optional)

Chicken Noodle Soup
- Makes 12 servings
- 3 medium carrots, sliced
- 3 stalks celery, sliced
- 3 onions, sliced
- 12 cups chicken broth
- 3 cups cubed, cooked chicken or turkey
- 1 1/2 cups uncooked, medium egg noodles

Mashed Potatoes
- Makes 6 servings
- 1/2 quart warm milk
- 6 medium russet potatoes, peeled and cubed
- 1/4 cup butter or margarine
- 3/4 teaspoon salt
- dash of pepper

Brownies
- Makes 16 brownies
- 1/2 cup butter
- 1 cup white sugar
- 2 eggs
- 1 teaspoon vanilla extract
- 1/3 cup unsweetened cocoa powder
- 1/2 cup all-purpose flour
- 1/4 teaspoon salt
- 1/4 teaspoon baking powder
**Salad**
- Makes 4 servings
- 1 head of lettuce
- 1 large tomato, sliced
- 1 small cucumber, sliced
- 1/4 cup olive oil
- 2 tablespoons lemon juice
- 1/4 teaspoon salt
- 1/8 teaspoon pepper

**Banana Bread**
- Makes 12 servings
- 2 cups all-purpose flour
- 1 teaspoon baking soda
- 1/4 teaspoon salt
- 1/2 cup butter
- 3/4 cup brown sugar
- 2 eggs, beaten
- 2 1/3 cups mashed, overripe bananas

**Quiche**
- Makes 6 servings
- 1/2 cup (light) mayonnaise
- 1/2 cup milk
- 4 eggs, lightly beaten
- 8 ounces shredded cheddar cheese
- 1 (10 ounce) package frozen chopped spinach, thawed and squeezed dry
- 1/4 cup chopped onion
- 1 (9 inch) unbaked pie shell

**Rolls**
- Makes 16 rolls
- 1/2 cup warm water
- 1/2 cup warm milk
- 1 eggs
- 1/3 cup butter, softened
- 1/3 cup white sugar
- 3 3/4 cup all-purpose flour
- 1 teaspoon salt
- 1 (.25 ounce) package active dry yeast
PERCENTAGES AND PRICES

Student/Class Goal
Students want to be able to calculate the final prices for items based on percentage off discounts.

Outcome (lesson objective)
Students will be able to convert fluently among fractions, decimals, and percentages as well as solve problems involving percentages.

Time Frame
2 hours

Standard  Use Math to Solve Problems and Communicate
(Primary benchmarks in bold.)

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<tr>
<th>Number Sense</th>
<th>Benchmarks</th>
<th>Geometry &amp; Measurement</th>
<th>Benchmarks</th>
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<th>Benchmarks</th>
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<tr>
<td>Connect number words</td>
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<tr>
<td>Solve problems using computations</td>
<td>3.2, 4.2, 5.2, 6.2</td>
<td>Connect graphical and algebraic representations</td>
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<td>Evaluate/solve expressions/equations</td>
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<tr>
<td>Order of operations</td>
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<tr>
<td>Compare/order numbers</td>
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<td>Graphing</td>
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</tr>
<tr>
<td>Estimate &amp; compute to solve problems</td>
<td>4.5</td>
<td>Use of correct units</td>
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<td>Evaluate using roots and exponents</td>
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Data Analysis & Probability

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<td>Create and display data</td>
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<td>Mathematical performance</td>
<td>5.36</td>
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</tbody>
</table>

Materials
(Optional) SmartPal kit – inserting a blank sheet of paper into the sleeves will give students a reusable sheet of paper that they can quickly try answers out on and erase without using up a pencil eraser. It’s quicker as well.
(Optional) Calculators
Fraction Circles
Fractions, Decimals, and Percentages: Conversions—Handout
Going, Going, Gone!—Handout

Learner Prior Knowledge
Working with fractions (Can be obtained by first doing the Cooking with Numbers lesson)
Ability to add, subtract, multiply, and divide whole numbers
Recognition of percentages and decimals

Vocabulary

Instructional Activities

Note: Keep in mind that your class may not need to go through each of the parts below. Please pick and choose which elements to incorporate into your actual lesson based on what you know of your students. In addition, extra sample problems may need to be incorporated based upon your particular class. See the technology integration portion for a website that gives many example problems.

Part 1: Give everyone the SmartPals, and the Fraction Circles. It is probably a good idea to have completed the Cooking with Numbers lesson, or some other work with fractions, prior to this lesson. Otherwise, the conversions may take longer as arithmetic with fractions will need to be explained. The idea behind this lesson is to be able to take percentages of prices off the total price to figure out a sale price.

• (I do) To introduce the context, you might start with a sample problem such as having a Buy One, Get One Half-Off coupon for shampoo. The store will have a price tag in front of the item stating its price. That’s great for the first item we buy, but how do we know how much the second item is? The one we are getting for “half-off”?  
  1. (Understand the problem) Thinking aloud about the problem, what do we know? We know that we are getting half-off our second item. Let’s say the item is normally sold for $2. (You could look up an actual item and give them more concrete prices. But for this first example, it’s less important.) Based on our work with fractions, we know that “a half” is equal to the fraction $\frac{1}{2}$. We want to find a reduced price, so the answer should be less than our $2$ original price.
2. **(Pick a strategy)** We also know that we are taking half “of” something. That clues us in that we need to multiply.
   So we are going to multiply our fraction by our original price to find our sale price.

3. **(Implement the strategy)**

   \[
   \frac{1}{2} \cdot \$2 = \frac{1}{2} \cdot \frac{\$2}{1} = \frac{1 \cdot \$1}{1} = \frac{\$1}{1} = \$1.
   \]

4. **(Review)** This answer does make sense as it is less than the original price. We can check this by thinking about
   having two $1 bills and splitting them among two people. Each person would get $1.

   - **(We do)** Have the students walk you through a similar problem. Let’s say that this time, we are looking to buy some
     apples. Normally, they are $4 for a bag. However, the store is selling them for one-fifth off of that normal price. In this
     case, our solution is:
     \[
     \frac{1}{5} \cdot \$4 = \frac{1}{5} \cdot \frac{\$4}{1} = \frac{\$4.00}{5} = \frac{\$0.80}{1} = \$0.80.
     \]
     However, we are not done. This is just the discount. To get the final price, we would need to subtract this from $4 to get
     a cost of $3.20.

   - **(You do)** Finally, have the students figure out how much a used car would be if it’s current price is one-quarter off of the
     original new price of $16,000.

     \[
     \frac{1}{4} \cdot \$16,000 = \frac{1}{4} \cdot \frac{\$16,000}{1} = \frac{\$4,000}{1} = \$4,000.
     \]
     Again, we must subtract this from the original $16,000 to get a final price of $12,000.

Part 2: Though it may be easy to multiply by fractions easily with pencil and paper when you can see cancellations easily and do
any necessary long division, this may not be the preferred choice for everyone. This is especially true if calculators are involved.
Because of this, students should be able to interchange their fractions, decimals, and percentages. Give them the Fractions,
Decimals, and Percentages: Conversions handout. This lesson plan only covers switching from percentages as that is how sales
are commonly advertised. However, if there is time, you can go over changing into a percentage as well.

   - **(I do)** For our first problem, we will be changing from a percentage into a decimal. This makes it especially easy to
     multiply using a calculator, as it takes extra time to insert fractions into calculators, if it is even a function the calculator
     has. In addition, it is simpler to insert a decimal than a percentage into a calculator. Let’s say that a big name clothing
     chain is offering 30% off of an entire purchase. If we purchase enough that our total would be $90 without the discount,
     what would be our total after the discount (but before tax)?

     1. **(Understand the problem)** We are taking our total price, $90, and reducing it by 30%. We’re taking out any
        extra obstacles like tax, so the idea is that we just need to find a way to quickly multiply our two numbers
        together.

     2. **(Pick a strategy)** We are going to assume that we cannot dig out a pencil and paper in the store. However, most
        people carry phones that have a calculator, so we could quickly use a decimal to multiply. So I want to convert
        the 30% to a decimal so we can do that.

     3. **(Implement the strategy)** Based on our conversions handout, we are going to drop the % symbol and divide by
        100, or just move the decimal two places to the left. Thus, 30% becomes 0.30. Then, we can multiply, and we
        get: $90 \cdot 0.30 = \$27$. As before, this is the amount discounted, so we must subtract this from our initial
        amount: $90 - \$27 = \$63$.

     4. **(Review)** We received a discount of 30%, which, being less than 100%, tells us we will pay less than our original
        $90 price. Our answer conveys that. In addition, it makes sense since 30% means we get about a third off,
        which would be $30 off.

   - **(We do)** In the fall, a sporting goods store puts their spring and summer sports gear on sale for 70% off to try to free up
     some space for new items. Have the class lead you through how to solve a problem where a person goes in and buys
     baseball gear that would normally cost $200. In this case, we would have to convert 70% to 0.70, and then we would
     have $200 \cdot 0.70 = \$140$, and finally $200 - \$140 = \$60$.

   - **(You do)** Pose that your students solve the following situation. A shoe store is having a Buy One Pair, Get the Second
     Pair 50% off sale. You and a friend go in and decide to buy the same pair of shoes priced at $60. Without worrying about
     tax, how much can you expect the price to be if you put both pairs on the same transaction? Here we would have a
     multi-step problem. We hold onto the $60 for the first pair, and take 50%, or 0.50 of the second pair. This means the
second pair would cost: $60 \cdot 0.50 = $30, and $60 - $30 = $30. So the second pair costs $30, and the first pair costs $60, so our total, before tax, would be $90, compared to $120 with no sale.

Part 3:

- **(I do)** If you wanted to figure out sales using mailers prior to actually going to the stores, then you would be able to use pencil and paper to figure out the costs. In this case, fractions may prove easier than dealing with decimal places.
  1. **(Understand the problem)** A clothing store offers customers a 15% discount on items if they order online. While at home shopping, Danielle finds $150 worth of clothing that she wants to purchase. If she uses the internet coupon, how much can she expect to pay (before tax)? Since Danielle is shopping at home, she can figure this out using fractions. This can be solved similarly to the earlier problems with decimals, but using a fraction in place of the decimal.
  2. **(Pick a strategy)** We will convert the percentage to a fraction as stated on the conversion handout. Then we will multiply to find the discount, and subtract that from the original price to find the new price.
  3. **(Implement the strategy)** Based on the handout, we are going to drop the % symbol and put the number over 100. So $15\% = \frac{15}{100}$. We can skip step two since we have a whole number in the numerator. Next, then, we need to reduce. We can list the factors of each, $15 [1, 3, 5, 15]$ and $100 [1, 2, 4, 5, 10, 20, 25, 50, 100]$. We can see that 5 is the GCF, so \[ \frac{15}{100} = \frac{15}{100} ÷ 5 = \frac{3}{20}. \] Now, we take that fraction, and multiply it by the $150. Thus:

\[
\frac{3}{20} \cdot \$150 = \frac{3}{20} \cdot \frac{150}{1} = \frac{3}{2} \cdot \frac{15}{1} = \frac{45}{2} = \$22\frac{1}{2}.
\]

When dealing with dollars, we don’t usually work with fractions, so I want to convert this to a decimal and get $22.50. That is our discount and so our total before tax will be $150 - $22.50 = $127.50.

- **(Review)** We reduced our price by taking off 15% and our price does show a slight reduction. As 15% is rather small, we knew we wouldn’t be subtracting much, so our answer does seem reasonable.

- **(We do)** Have the students walk you through solving this problem. While at home looking through Black Friday ads, a couple finds an ad for a store that has all their flannel pajama sets on sale for 60% off. The couple has 3 children and they decide to get a set of pajamas for each child and themselves (5 total sets). If each set of pajamas normally cost $20, how much can they expect to pay after the discount?

\[
60\% = \frac{60}{100} = \frac{60 ÷ 20}{100 ÷ 20} = \frac{3}{5}.
\]

And:

\[
\frac{3}{5} \cdot \$100 = \frac{3}{5} \cdot \frac{100}{1} = \frac{3}{1} \cdot \frac{20}{1} = \frac{60}{1} = $60.
\]

Thus, the total amount would be $100 - $60 = $40.

- **(You do)** A video game store is offering 25% off new games if a trade-in is brought in. If Paula wants to buy a new video game that is priced for $50, what would the price be if she brought in a trade-in?

\[
25\% = \frac{25}{100} = \frac{25 ÷ 25}{100 ÷ 25} = \frac{1}{4}.
\]

And:

\[
\frac{1}{4} \cdot \$50 = \frac{1}{4} \cdot \frac{50}{1} = \frac{1}{2} \cdot \frac{25}{1} = \frac{25}{2} = \$12\frac{1}{2} = $12.50.
\]

Thus, the total amount would be $50 - $12.50 = $37.50.

Part 4:

- **(I do)** Now that we have strategies to solve these problems when we have tools, a calculator or pencil and paper, let’s discuss using percentages alone to solve the problems using some mental math. As a class discussion, have everyone
think back to converting from a percentage to a decimal. There, we divided by 100 (or multiplied by \( \frac{1}{100} \)). Since \( \frac{1}{100} = 1\% \), this is the same as finding “1% of” something. This was also the same as moving the decimal two places to the left.

What if we were dividing by 10? This would be the same as multiplying by \( \frac{1}{10} \), finding 10% of something, and moving one decimal place to the left. Thus, sales of 10% mean we just take the current price and move the decimal one place to the left to find the discount. For multiples of 10% (i.e.: 20%, 50% off), we do the procedure for 10% - move the decimal one spot to the right – and then multiply. For 20% we would multiply by 2 and for 50% we would multiply by 5. For some percentages, it may be easier to think about halving or quartering instead of working with the 10% idea above. For 50% off, we are paying half price, so we can just halve our original price. For 25% off, we are getting a quarter off, and can just divide by four to find the discount. For 75% off, we are actually only paying for 25% of the original price, so we can divide by 4 like with 25% off, only this time our answer is the price paid instead of the discount we would be receiving. Finally, for amounts that are not as clean as the multiples of 10 or 25, we want to do two steps. We first use the 10% idea and then we do the same thing using 1%. So, for instance, for 15% off, I would find what 10% off would discount, and then I would either halve that number to find 5% off or I would find 1% (by moving the decimal two places to the left) and multiplying that by 5 to find 5% off. With these mental math strategies, we will be working on the final activity. Give everyone the Going, Going, Gone! handout. (You can do more examples than what is outlined below, but remember to keep a few back for assessment.)

1. (Understand the problem) If we look at our first problem on the handout, the one about the candy at Walmart, we can use the discussion about mental math strategies. We know that Maryann put what would normally be $25 worth of candy into her cart. However, based on the sale, she’ll only need to pay 50% of that price.
2. (Pick a strategy) Since we have a relatively simple percentage, we will use a strategy brought up during the class discussion.
3. (Implement the strategy) There are two possibilities here. We can use the idea that 50% is half. In that case, $25 ÷ 2 = $12.50. Our other option is to first take 10% and then multiply by 5. In this case, $25 · 10% = $2.50 · 5 = $12.50.
4. (Review) Our answer is definitely reasonable as we are paying less than the original $25. In addition, if we do the problem both ways and end up with the same result, we are likely correct.

- (We do) Together as a class, do the second problem about the MP3 player from Best Buy. Here, we will use the 10% shortcut of moving the decimal one to the right, and then subtracting that discount from our original price. Thus, $250 ÷ 10% = $25 and $250 − $25 = $225.
- (You do) Have the students do the Goody’s 30% off problem individually. Here we have: $110 · 10% = $11 · 3 = $33 and $110 − $33 = $77.

Assessment/Evidence (based on outcome)
Each of the you do steps will serve as assessment. The instructor should be able to gauge understanding by having different students provide their solutions and explanations of how they arrived at that solution. In addition, during the we do steps, instructors should be encouraging all students to participate in the discussion. The ability to provide input in these discussions will help the teacher gauge each student’s mastery of the concepts.

The last samples on the Going, Going, Gone! handout can be turned in for a summative assessment. This will allow instructors to see how well students grasped the concept.

Teacher Reflection/Lesson Evaluation
Not yet completed

Next Steps
Proportions would be one possible follow-up as it would allow students to do further exploration in the financial realm with finding unit costs. Another possible follow-up would be to work with percentages in probability. For instance, if you have a 50% chance of getting a tails on a coin flip and you perform 100 coin tosses, how many can you expect to be tails? Or, if 10% of the population is left-handed, how many people in the class would you expect to be left-handed? Or how many of your friends/acquaintances? Yet another option would be introducing variables for equations. Many of the problems would involve less steps if we set up an equation at the beginning. Instead of needing to subtract the percentage off at the end, we could set up an equation to set that up from the start. To help in reducing fractions, divisibility tests could be set up.

Technology Integration
http://www.mathsisfun.com/percentage-menu.html
Website with resources on percentages. This includes conversions as well as a place to generate worksheets/practice problems.
http://www.percentoffcalculator.com/index.php
Online calculator that will find the sale price if you insert an original price and a % discount. There is also a QR code to
An app that can be put on Apple products (iPhone, iPad) to calculate sales prices.

**Purposeful/Transparent**
Students want to be able to deal with percentages in an everyday setting. The context of finances is applicable to everyone. The instructor will provide examples in this context as well as give ways to solve problems using mental math and not just with paper/pencil or a calculator.

**Contextual**
Percentages are used in financial situations quite often. In addition to discounts, percentages are used to describe the amount of interest on savings accounts and loans. Percentages (or decimals or fractions) also appear in probability and statistics. This can be seen in sports such as free throw percentages, batting averages, and win-loss ratios.

**Building Expertise**
Students will have a better grasp of not only working with percentages but also knowing how they compare with decimals and fractions. By the end of this lesson, they should be more comfortable and confident working with all rational numbers (positive and negative whole numbers and fractions).
Fractions, Decimals, and Percentages: Conversions

As fractions, decimals, and percentages are all ways for us to describe parts of a whole, we can use the three interchangeably. This means we can also convert from one to another.

Decimals to Percentages
Decimals and percentages are very closely related. To go from a decimal to a percentage, you multiply the decimal by 100 and add the % symbol. Note: Multiplying by 100 has the same result as moving the decimal two places to the right.

Thus, $0.375 \cdot 100 = 37.5\%$.

Percentages to Decimals
To go from a percentage to a decimal, we do the opposite of the above procedure. We drop the % symbol and divide by 100. Note: Dividing by 100 has the same result as moving the decimal to places to the left.

Thus, $37.5\% \div 100 = 0.375$.

Fractions to Percentages
Here, we have two options for how to solve this. If we have a calculator, the first way will be easier.

1. Divide the numerator by the denominator.
   
   Say we have the fraction $\frac{3}{8}$. Then we take $3 \div 8 = 0.375$.

2. Multiply the result by 100 and tack the % symbol to the end.
   
   $0.375 \cdot 100 = 37.5\%$

The other method can still utilize a calculator, but is easier to use if a calculator is not handy. The idea is that percent means “per 100,” so we are looking for an equivalent fraction with a denominator of 100.

1. Find a number you can multiply the denominator of your fraction by to get 100. This may not be a whole number!
   
   In $\frac{3}{8}$, our denominator is 8. We want to know what to multiply 8 by to get 100. If we divide 100 by 8, we will find this number. $100 \div 8 = 12 \frac{4}{8} = 12 \frac{1}{2} = 12.5$.

2. Multiply both the numerator and denominator of our fraction by this number.
   
   $3 \cdot 12.5 = \frac{37.5}{8}$
   
   $8 \cdot 12.5 = \frac{100}{100}$

3. Write down just the top number with the % symbol tacked on.
   
   $\frac{3}{8} = \frac{37.5}{100} = 37.5\%$

Percentages to Fractions

1. Write down the percent as a fraction. The numerator is the percent (without the % symbol) and the denominator is 100.
   
   $37.5\% = \frac{37.5}{100}$
2. If the numerator is not a whole number, multiply both the numerator and denominator by 10 for every digit to the right of the decimal. (If there is one digit to the right of the decimal, multiply by 10 once. If there are two digits to the right of the decimal, multiply by 10 twice, or 100. If there are three digits to the right of the decimal, multiply by 10 three times, or 1000. The goal is to get a whole number in the numerator.)

\[
\frac{37.5}{100} = \frac{37.5 \cdot 10}{100 \cdot 10} = \frac{375}{1000}
\]

3. Simplify the fraction.

We need to find the GCF of 375 and 1000. Instead of listing all the factors (as there are many) I am just going to divide both by a number I know goes into both, namely 5.

\[
\frac{375}{1000} = \frac{75}{200}
\]

I can repeat the previous step again.

\[
\frac{75}{200} = \frac{15}{40}
\]

And I can do it one more time.

\[
\frac{15}{40} = \frac{3}{8}
\]

Decimals to Fractions

1. Put the decimal over 1.

\[
0.375 = \frac{0.375}{1}
\]

2. Multiply both the numerator and denominator by 10 for every digit to the right of the decimal. (If there is one digit to the right of the decimal, multiply by 10 once. If there are two digits to the right of the decimal, multiply by 10 twice, or 100. If there are three digits to the right of the decimal, multiply by 10 three times, or 1000. The goal is to get a whole number in the numerator.)

\[
\frac{0.375}{1} = \frac{0.375 \cdot 1000}{1 \cdot 1000} = \frac{375}{1000}
\]

3. Simplify if necessary.

Based on the fraction we had in the previous section, we see that this reduces to:

\[
\frac{375}{125} = \frac{3}{8}
\]

Fractions to Decimals

We can simply divide the numerator by the denominator and the answer is our decimal. A calculator would make this easy. Our other option is long division.

\[
\frac{3}{8} = 0.375.
\]
In each of the following scenarios, a sale has made the item fly off the shelves. Find the total cost in each situation. You want to be quick, before someone else scoops up the item and it’s gone! Use mental math to speed up the process. (Consider all totals to be prior to tax.)

At Walmart following the Christmas holiday, Maryann sees a tag like the green one above in the candy aisle. If she puts enough candy in her cart to total $25 before the discount, how much can she expect it to be after the discount?

Jeff wants to get a new MP3 player that will hold all of his music. The one he wants is normally $250. However, he gets this coupon in his inbox and prints it off to take with him. How much will the MP3 player be with the coupon?

While shopping for back-to-school clothes online, you remember you have this coupon. If you put what would normally be $110 worth of clothes in your cart, how much will it be with the discount code?

Frank takes his kids shopping for spring clothes at Kohl’s when he sees the ad above in the kid’s department. If he puts $120 worth of clothing in his cart, how much will it be with the discount?
While shopping for new shoes prior to the start of school, Will sees this sign at Foot Locker. If the pair of shoes he wants is normally $70, how much will they be on sale?

Gamestop is having a sale on their games. Kyle wants to buy a game that is normally $40. How much will he pay today?

Bill receives Kmart coupons in the mail as shown above. While at the store, he decides to buy a new TV. If the TV costs $400 without any discounts, what will it cost if he applies one of the coupons above?

Deciding she could use a few more sweaters for winter, Michelle buys $80 worth of new shirts. If she uses her coupon at checkout, what will her new total be?
In each of the following scenarios, a sale has made the item fly off the shelves. Find the total cost in each situation. You want to be quick, before someone else scoops up the item and it’s gone! Use mental math to speed up the process. (Consider all totals to be prior to tax.)

At Walmart following the Christmas holiday, Maryann sees a tag like the green one above in the candy aisle. If she puts enough candy in her cart to total $25 before the discount, how much can she expect it to be after the discount?

$12.50

Jeff wants to get a new MP3 player that will hold all of his music. The one he wants is normally $250. However, he gets this coupon in his inbox and prints it off to take with him. How much will the MP3 player be with the coupon?

$225

While shopping for back-to-school clothes online, you remember you have this coupon. If you put what would normally be $110 worth of clothes in your cart, how much will it be with the discount code?

$77

Frank takes his kids shopping for spring clothes at Kohl’s when he sees the ad above in the kid’s department. If he puts $120 worth of clothing in his cart, how much will it be with the discount?

$96
While shopping for new shoes prior to the start of school, Will sees this sign at Foot Locker. If the pair of shoes he wants is normally $70, how much will they be on sale?

$49

Gamestop is having a sale on their games. Kyle wants to buy a game that is normally $40. How much will he pay today?

$34

Bill receives Kmart coupons in the mail as shown above. While at the store, he decides to buy a new TV. If the TV costs $400 without any discounts, what will it cost if he applies one of the coupons above?

$320

Deciding she could use a few more sweaters for winter, Michelle buys $80 worth of new shirts. If she uses her coupon at checkout, what will her new total be?

$68
CALCULATING COSTS

**Student/Class Goal**
Students want to be able to use estimations in an everyday setting so that they can quickly and easily figure out costs.

**Outcome** (lesson objective)
Students will be able to compare and order values in decimal, fraction, and percent form. They will also be able to estimate solutions to problems using rounding and compatible numbers.

**Time Frame**
2 hours

**Standard** Use Math to Solve Problems and Communicate
(Primary benchmarks in bold.)

<table>
<thead>
<tr>
<th>Number Sense</th>
<th>Benchmarks</th>
<th>Geometry &amp; Measurement</th>
<th>Benchmarks</th>
<th>Algebra &amp; Patterns</th>
<th>Benchmarks</th>
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<tr>
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<td>Solve problems using computations</td>
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<td>Evaluate/solve expressions/equations</td>
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<tr>
<td>Order of operations</td>
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<td>Perimeter/area/volume</td>
<td>Connect relationships to representations</td>
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<tr>
<td>Compare/order numbers</td>
<td>3.3, 4.4</td>
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<tr>
<td>Estimate &amp; compute to solve problems</td>
<td>4.5, 5.4</td>
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<td>Evaluate using roots and exponents</td>
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**Data Analysis & Probability**
Benchmarks

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<th>Reason mathematically</th>
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<td>5.36</td>
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**Materials**
(Optional) SmartPal kit – inserting a blank sheet of paper into the sleeves will give students a reusable sheet of paper that they can quickly try answers out on and erase without using up a pencil eraser. It’s quicker as well.

(Optional) Local grocery store mailers may prove helpful for examples.

**Rounding and Ordering: A Guide—Handout**

**Keeping Costs Down—Handout**

**Learner Prior Knowledge**
Students should have prior knowledge of decimals, fractions, and percentages. (Percentages and Prices lesson plan would be sufficient)

Rounding to the nearest dollar. (To the nearest tenth or hundredth would be better.)

Ordering/comparing of whole numbers.

**Vocabulary**
Compatible Numbers - numbers that go together well to allow for easy, mental computation and estimation of addition, subtraction, multiplication, or division problems. The numbers must also be close to the original values in the problem.

**Instructional Activities**

**Note:** Keep in mind that your class may not need to go through each of the parts below. Please pick and choose which elements to incorporate into your actual lesson based on what you know of your students. In addition, extra sample problems may need to be incorporated based upon your particular class.

Part 1: If you want them to use the SmartPals, make sure they have those to begin. The overall idea behind this lesson is to be able to find out the best deals and the estimated overall price of items while shopping. As such, we should give them some context. Let’s say that we make a quick run to the store after work and all we have on us is a $20 bill – no checks or credit/debit cards – and our shopping list. We’re going to need to be very careful to make sure we do not go over our funds.

- **(I do)** Make a grocery list on the board. You can take suggestions with estimated prices, but we want some ugly numbers in there. For instance, here is a sample list:
  - Milk $2.99
  - Bread $2.19
1. **(Understand the problem)** With our grocery list and $20, we want to be able to get as many items as possible without getting to the register only to find we don’t have enough money. Unfortunately, we cannot pull out a pen and paper because we have none on us, nor can we pull out a calculator for the same reason. This is all mental math. We must find a way to add these values quickly and easily.

2. **(Pick a strategy)** We have two methods here. The first is rounding. Give the students the handout on Rounding and Ordering if you have yet to do so. We may find rounding is not enough, if that is the case, we will use an idea known as **compatible numbers**.

3. **(Implement the strategy)** If we were to round each of these numbers to, say, the nearest dollar, we would get:
   - Milk $3
   - Bread $2
   - Strawberries $2
   - Ice cream $3
   - Lunch meat $2
   - A case of water $4
   - Apples $3
   
   While adding is now easy, $19, there was a lot of shifting. Are we sure we didn’t round so far as to put us under when really our total is over? Maybe we should have rounded to the nearest dime:
   - Milk $3
   - Bread $2.20
   - Strawberries $2.50
   - Ice cream $2.70
   - Lunch meat $3.50
   - A case of water $3.50
   - Apples $3.50
   
   Now the addition is not as easy as it was when all of our amounts were whole dollars. To add quickly, let’s think about something known as **compatible numbers**. These are numbers that work well together (are compatible) when we are adding, subtracting, multiplying, or dividing. Some examples include:
   - $3 + 7$
   - $16 – 6$
   - $48 + 52$
   - $360 ÷ 10$
   - $25 \cdot 4$
   
   The key to keep in mind is that numbers are compatible if they allow us to do a calculation quickly, easily, and using only mental math. Let’s reorder our list of prices to put compatible numbers together.
   - Milk $3
   - Strawberries $2.50
   - Lunch meat $3.50
   - A case of water $3.50
   - Apples $3.50
   - Bread $2.20
   - Ice cream $2.70
   
   Now we know that the prices with fifty cents are easy to add together, and our two difficult numbers are at the bottom. However, we can estimate these numbers a little more to make them compatible:
   - Bread $2.25
   - Ice cream $2.75
   
   Now, if we add everything together, we get $21. As our rounding/estimations were much closer to our original numbers this time, we should probably put something back to be safe.

4. **(Review)** This can be verified by using a calculator or pencil/paper to add the original prices. The actual values add to $20.78, so we are over our funds.

   • **(We do)** Have the students walk you through this process using:
     - Orange Juice $3.39
     - Bread $1.79
     - Bananas $1.29
Bag of carrots $2.69
Chicken $4.47
A case of water $3.50

You should end up with something similar to:
- Orange Juice $3.30
- Bag of carrots $2.70
- Bananas $1.25
- Bread $1.75
- Chicken $4.50
- A case of water $3.50

Which means that we are under our limit at about $17.

- (You do) Have the students work alone using $10 and:
  - Fruit juice $2.25
  - Buns $1.99
  - Hamburger meat $3.49
  - Ketchup $1.99
  - Cheese slices $1.79

You should end up with something similar to:
- Fruit juice $2.25
- Cheese slices $1.75
- Hamburger meat $3.50
- Ketchup $2
- Buns $2

Which means that we are over at about $11.50.

Part 2: Now that we are estimation queens/kings, we want to be able to compare prices. Often times some brands will be on sale for “3 for $5” or “2 for $7”. What does this do to our prices? Which should we get?

- (I do) Let’s refer back to our shopping list from the beginning.
  - Milk $2.99
  - Bread $2.19
  - Strawberries $2.48
  - Ice cream $2.74
  - Lunch meat $3.49
  - A case of water $3.50
  - Apples $3.48

What if our milk price was for a gallon and we see half-gallons on sale for 3 for $5? Which should we get?

1. (Understand the problem) Here, we are trying to decide which item is the better deal. This means we want to know which has the better price when we get the same amount of the different items.
2. (Pick a strategy) We can consider the prices as “$5 per 3 half-gallons” which can be written as a fraction. We can then use either method on the handout to compare the fractions.
3. (Implement the strategy) The milk is being sold as $5 for 3 half-gallons. In other words, we would pay $5 for 1.5 gallons of milk. Now, if we think about this as a fraction, we have \( \frac{5}{1.5} \). If we do something similar for the single gallon for $2.99, we have \( \frac{2.99}{1} \). Now we have two fractions that we need to compare. We can do this using either method on the handout. If we use the cross multiplication method, and some estimation with compatible numbers, we would have: \( 5 \cdot 1 = 5 \) and \( 3 \cdot 1.5 = 4.50 \). This tells us that the $3 for 1 gallon is the better deal. We could have also used common denominators. In that case we would have put both fractions over 3 and gotten: \( \frac{10}{3} \) and \( \frac{9}{3} \).
4. (Review) We can check this with a calculator by finding unit prices. Without getting too technical, this just means we want to find the price per 1 unit for the sale of 3 for $5. This would tell us that one half-gallon would cost $1.67. As it takes two half-gallons to make one gallon, it would cost us $3.34 to buy the same amount of milk. So yes, the gallon is cheaper.

- (We do) If another brand of ice cream is on sale for 2 for $5, should we get it or the one in our original list for $2.74? Have the students walk you through solving this. The one on sale is cheaper as it would be $2.50 for one.
- (You do) Have the students work alone on whether they should buy a 12 ounce can of pop for $0.50 or a 20 ounce bottle for $1.25. (The can is the better deal.)
Part 3: Now we are going to incorporate the two ideas together. Give them the Keeping Costs Down handout.

- (I do) Looking at the first problem, go over how to solve Steve’s dilemma.
  1. (Understand the problem) We no longer have that $20 constraint. Instead, we just want to estimate his total cost. First, however, we must figure out the cost of each item. This means we need to figure out the better deals on bread and orange juice.
  2. (Pick a strategy) We will compare fractions to find the better cost, then use compatible numbers to estimate total cost.
  3. (Implement the strategy) For the bread, we are comparing \(\frac{1.79}{1}\) and \(\frac{3.50}{2}\). This may actually be easiest to just reduce the second fraction so that we have \(\frac{1.79}{1}\) and \(\frac{1.50}{1}\). Thus, the second one is the better deal. For the orange juice, we are comparing \(\frac{3.19}{1}\) and \(\frac{7.50}{2}\). Again, it may be easiest to reduce the second fraction and get \(\frac{3.19}{1}\) and \(\frac{3.50}{1}\), making the first the better deal. Now, we have our list of prices:
    - Eggs $1.49
    - Bread $1.50
    - Orange juice $3.19
    - Watermelon $3.88

Rounding to compatible numbers, we get:
- Eggs $1.50
- Bread $1.50
- Orange juice $3.20
- Watermelon $3.80

And so our final price is about $10.

- (Review) We can check this with a calculator all the way through, or use other forms of estimation. Either way, we should come to the consensus that our answer is in the right frame for our problem.

- (We do) Have the class walk you through solving Maryann’s problem. In the end, you should get that the 3 for $5 cottage cheese is the better deal, and that the final estimated cost is about $10.50.
- (You do) Have the students do the final two problems to turn in to you. For Kathryn, the gallon is the better deal, as are the individual cans of corn. The final cost is about $9.50. And for Frank, the $2.50 bagged salad is cheaper as is the $4.59 fish. His estimated cost is about $17.

**Assessment/Evidence (based on outcome)**

Each of the you do steps will serve as assessment. The instructor should be able to gauge understanding by having different students provide their solutions and explanations of how they arrived at that solution. In addition, during the we do steps, instructors should be encouraging all students to participate in the discussion. The ability to provide input in these discussions will help the teacher gauge each student’s mastery of the concepts.

At the end of the lesson, have them turn in a few problems from the Keeping Costs Down handout to check for outcome mastery.

**Teacher Reflection/Lesson Evaluation**

*Not yet completed*

**Next Steps**

Rounding can be further explored until students are confident in rounding to any place value. Once roots and exponents are covered, ordering numbers with those involved will follow this plan as well. The fraction comparisons are more like unit rates and proportions. Each of those topics would be great follow-ups.

**Technology Integration**

- [http://www.mathsisfun.com/rounding-numbers.html](http://www.mathsisfun.com/rounding-numbers.html)
  Site on rounding.
- [http://www.mathsisfun.com/ordering_decimals.html](http://www.mathsisfun.com/ordering_decimals.html)
  Site on ordering decimals.
- [http://www.mathsisfun.com/comparing-fractions.html](http://www.mathsisfun.com/comparing-fractions.html)
  Site on comparing fractions.
  Unit price calculator. (Mobile friendly.)

**Purposeful/Transparent**

Since students want to be able to estimate total costs and compare prices, the instructor will use examples involving products found in stores to aid in instruction.
Based on our needs as humans, everyone is going to need to buy something at some point in time. Being able to estimate the cost before getting to the register can help save any unnecessary embarrassment that could arise if not enough money is on hand. In addition, it can help save money by estimating which of two similar products is the better cost. Being able to order or compare numbers allows us to determine better cost as well.

Building Expertise
Building upon the ability to convert among fractions, decimals, and percents, students can now compare and order these numbers. This will open up the possibility of doing algebraic problems using decimals and percents. Estimation allows students to reflect on their solutions to problems. By coming up with an initial estimate, students will be better able to gauge whether their actual solution is reasonable or not.
Rounding and Ordering

A Guide

Rounding

Rounding numbers requires some background knowledge of place value. Place value is determined by the position a digit is in with respect to the decimal point. For example, as we move to the left from the decimal point, we have the place values of ones, tens, hundreds, thousands, and so on. To the right of the decimal point we have the tenths, hundredths, thousandths, and so on. This can be seen more clearly in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>.</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,572.041</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>2</td>
<td>.</td>
<td>0</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>0.019</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>51.2</td>
<td></td>
<td></td>
<td>5</td>
<td>1</td>
<td>.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When it comes to rounding, a three step process can be followed:

1. Find the digit that corresponds to the place value you are rounding to.
   For instance, if we take the first number in our chart (3,572.041) and round it to the nearest 10, then we know that the 7 is in the place we are rounding to.

2. Look to the digit to the right of the place we are rounding to. There are two possibilities here.
   a. If the digit is between 0 and 4, we round down. This means that the digit of the place we are rounding to stays the same.
   b. If the digit is between 5 and 9, we round up. This means that the digit of the place we are rounding to increases by one.

3. All digits to the right of the place we are rounding to become 0.

Ordering/Comparing

Decimals

Putting decimals in order from least to greatest can be tricky. If you are given the numbers 1.37, 1.307, and .72, how do you know which comes first? When dealing with whole numbers, the greatest was the one with the most number of digits. If there was a tie, it was the one with the largest leading digit. However, now that we have introduced place value, we can think about this differently. As with rounding, there are a set of steps that can be followed to order these numbers.
1. Place the numbers so that their decimal places are aligned. A place value chart may be helpful.

<table>
<thead>
<tr>
<th>Place Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Ones</th>
<th>.</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.37</td>
<td>1</td>
<td>.</td>
<td>3</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>1.307</td>
<td>1</td>
<td>.</td>
<td>3</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>.72</td>
<td>0</td>
<td>.</td>
<td>7</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

2. With the numbers aligned, look at the largest place value (the one furthest to the left). Since we are ordering from smallest to largest, we want the smallest value in that column as that will be the smallest number.

3. If there is a tie, as with whole numbers, you go to the next place value. So in the chart above, you would then look at the tenths place. *(Note: If you get to a place where a number does not have a digit in a particular place value, you can put a zero there as a place holder.)*

Fractions

One way to compare fractions would be to convert them into decimals and then order them using the method above. However, it is not necessary to change them into decimals first. There are two other methods to compare fractions.

**Common Denominators**

1. The first method involves finding a common denominator for the two fractions. If we are comparing the fractions \( \frac{7}{8} \) and \( \frac{5}{6} \), we can change them into equivalent fractions over a common denominator, namely \( \frac{21}{24} \) and \( \frac{20}{24} \).

2. Now we must only compare the numerators. Since 20 is less than 21, \( \frac{20}{24} \) is less than \( \frac{21}{24} \).

**Cross Multiplication**

1. The other method requires us to cross multiply. The idea here is we are taking \( \frac{7}{8} \) versus \( \frac{5}{6} \). We will multiply the numerator of one fraction by the denominator of the other, and vice versa. This gives us:

\[
\frac{7}{8} \text{ versus } \frac{5}{6} \quad \rightarrow \quad \frac{7}{8} \times \frac{5}{6} = 7 \cdot 6 \text{ versus } 8 \cdot 5 = 42 \text{ versus } 40
\]

2. The resulting products can now be compared as whole numbers. To decide which fraction was larger, the products go with fraction whose numerator was one of its factors. In the above example, 42 goes with \( \frac{7}{8} \) and 40 goes with \( \frac{5}{6} \).
Keeping Costs Down

In each of the following scenarios, a customer is at the store with a shopping list. They have run into two issues. First, they have found a sale item and want to know if it really is more cost efficient. Second, they want to estimate the total of the items in their cart before they get to the register. (Note: Food items do not have tax.)

Steve runs to the store one afternoon for a few items. Which bread and orange juice should he buy? And based on those items, estimate his final cost.

<table>
<thead>
<tr>
<th>Shopping List</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Eggs - $1.49</td>
</tr>
<tr>
<td>2. Bread $1.79 or 2 for $3</td>
</tr>
<tr>
<td>3. Orange juice</td>
</tr>
<tr>
<td>$3.19 or 2 for $7</td>
</tr>
<tr>
<td>5. Watermelon $3.88</td>
</tr>
</tbody>
</table>

Maryann needs to pick up some items for a barbeque this weekend. Help her decide what the best deal is and then estimate her total cost.

<table>
<thead>
<tr>
<th>Shopping List</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Buns $1.89</td>
</tr>
<tr>
<td>2. Cottage Cheese</td>
</tr>
<tr>
<td>3. $2.39 or 3 for $5</td>
</tr>
<tr>
<td>4. Cheese slices $2.29</td>
</tr>
<tr>
<td>5. Hamburger $3.49</td>
</tr>
<tr>
<td>7. Jar of Pickles $1.19</td>
</tr>
</tbody>
</table>

Kathryn needs the items above for dinner tonight. Some items are on sale. Which is the better deal? Then estimate her total cost based on which items she buys.

<table>
<thead>
<tr>
<th>Grocery List</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk: Gallon - $2.49</td>
</tr>
<tr>
<td>1/2 Gallon - 2/$3</td>
</tr>
<tr>
<td>2 cans of corn: 3/$2</td>
</tr>
<tr>
<td>$0.60 each</td>
</tr>
<tr>
<td>Chicken: $4.30</td>
</tr>
<tr>
<td>1 box of Rice: 2/$3</td>
</tr>
</tbody>
</table>

If Frank needs to get the above items, which will he choose to be the most cost effective? What will his estimated final price be?

<table>
<thead>
<tr>
<th>Shopping List</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk - $2.99</td>
</tr>
<tr>
<td>orange juice - $2.39</td>
</tr>
<tr>
<td>Bagged salad - $2.50 or 2/$6?</td>
</tr>
<tr>
<td>Potatoes - $2.99</td>
</tr>
<tr>
<td>Peaches - $1.49</td>
</tr>
<tr>
<td>Fish filets - $4.59 or 2/$11?</td>
</tr>
</tbody>
</table>
**WHAT TO DO FIRST?**

**Student/Class Goal**
Students want to be able to calculate interest on both deposit and loan accounts.

**Time Frame**
4 hours

**Outcome (lesson objective)**
Students will be able to recognize the difference between a loan and a deposit account based on an interest equation. They will also compute account values and interest rates using exponents and roots. Finally, they will utilize “PEMDAS” to recall the correct order of operations to use when solving problems.

**Standard** *Use Math to Solve Problems and Communicate*
(Primary benchmarks in bold.)

<table>
<thead>
<tr>
<th>Number Sense</th>
<th>Benchmarks</th>
<th>Geometry &amp; Measurement</th>
<th>Benchmarks</th>
<th>Algebra &amp; Patterns</th>
<th>Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connect number words</td>
<td>4.1</td>
<td>Identify/apply basic geometric concepts</td>
<td></td>
<td>Patterns/sequences</td>
<td></td>
</tr>
<tr>
<td>Solve problems using computations</td>
<td>5.1</td>
<td>Connect graphical and algebraic representations</td>
<td></td>
<td>Evaluate/solve expressions/equations</td>
<td>5.16</td>
</tr>
<tr>
<td>Order of operations</td>
<td>5.2, 6.2</td>
<td>Perimeter/area/volume</td>
<td></td>
<td>Connect relationships to representations</td>
<td></td>
</tr>
<tr>
<td>Compare/order numbers</td>
<td></td>
<td>Graphical representations</td>
<td></td>
<td>Graphing</td>
<td></td>
</tr>
<tr>
<td>Estimate &amp; compute to solve problems</td>
<td>5.4</td>
<td>Use of correct units</td>
<td></td>
<td>Solving equations using algebra/graphs</td>
<td></td>
</tr>
<tr>
<td>Evaluate using roots and exponents</td>
<td>4.6</td>
<td>Right triangle trigonometry</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Data Analysis & Probability**

<table>
<thead>
<tr>
<th>Benchmarks</th>
<th>Measurement applications</th>
<th></th>
<th>Solve problems</th>
<th>6.26, 4.26, 5.27, 5.28</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpret data</td>
<td>Measurement conversions</td>
<td></td>
<td>Communicate ideas</td>
<td>5.29, 5.30, 5.31</td>
</tr>
<tr>
<td>Create and display data</td>
<td>Rounding</td>
<td></td>
<td>Reason mathematically</td>
<td>5.33</td>
</tr>
<tr>
<td>Central tendency</td>
<td></td>
<td></td>
<td>Connect concepts</td>
<td>6.36</td>
</tr>
<tr>
<td>Probability</td>
<td></td>
<td></td>
<td>Mathematical performance</td>
<td>5.36</td>
</tr>
</tbody>
</table>

**Materials**
(Optional) SmartPal kit – inserting a blank sheet of paper into the sleeves will give students a reusable sheet of paper that they can quickly try answers out on and erase without using up a pencil eraser. It’s quicker as well.

Calculators - While students should be able to write the steps by hand, the practical application of solving these problems would definitely allow for, and encourage, calculator use.

Cubes
Exponents and Roots - Handout
PEMDAS - Handout
Balancing Act - Handout

**Learner Prior Knowledge**
Recognition of equations
Knowledge of inverse operations: subtraction and addition “undo” each other as do division and multiplication.
Ability to perform all four arithmetic operations on whole numbers, fractions, decimals, and percents.

**Vocabulary**
Simple Interest - interest paid only on the principal (initial deposit).
Compound Interest - interest paid on both the principal and any previously earned interest.

**Instructional Activities**

**Note:** Keep in mind that your class may not need to go through each of the parts below. Please pick and choose which elements to incorporate into your actual lesson based on what you know of your students. In addition, extra sample problems may need to be incorporated based upon your particular class.

**Note:** This lesson is expected to span more than one 2-hour class session. If you need to split this lesson into two parts, a good spot to stop would be between parts 4 and 5. (Between 3 and 4 would work as well, but would not leave as much time for the application problems as a class.)

Part 1: If you want them to use the SmartPals, make sure they have those to begin. Calculators may also prove handy for this lesson, but do not hand them out to begin with. We want them to explore the math concepts without any aid before we allow them to simplify the process with the use of calculators. This lesson stems from the financial world, specifically interest. Interest can be gained on deposit accounts (savings and checking accounts) or it can be something that is paid, as in a loan. Let’s
introduce the idea of starting a savings account to help pay for college expenses. Without going too in depth now, let them know that there are two main ways an account can accumulate interest. One way to calculate interest is based solely on what you put in initially. So if we start a savings account with $100 that earns 5% interest, we gain $5 every year. Another way interest can be calculated is based on what is in the account at the start of each year. In this case, our $100 account earning 5% interest gains $5 that first year. The second year, however, we gain 5% interest on our new balance of $105. This means we gain $5.25 in interest that second year, which is slightly better than our original interest calculation. If we try to write out the two calculations, we will see that the first one is always going to be:

$$100 \cdot 5\%.$$ 

In our second calculation, we have the same expression for the first year. The second year changes the equation, as we now have:

$$105 \cdot 5\%.$$ 

We can rewrite that in terms of our previous equation since we know that $105 is the balance after receiving interest that first year. Now we have:

$$(100 + 100 \cdot 5\%) \cdot 5\%.$$ 

This pattern will continue indefinitely. For every year we want to find our interest, we will multiply once more by 5%. What happens after 20 years? Do we need to write out 5% twenty times? There is actually a short hand notation for this, an exponent. Using the distributive property, we can write the above expression as:

$$(100 \cdot 5\%) + (100 \cdot 5\%^2).$$

Since the notation of the superscript 2 on the second 5% may be new, let’s explore what that 2 means.

Part 2: Make sure everyone has a copy of the Exponents and Roots handout as well as some of the cubes. (Depending on class size and amount of cubes, it is okay to work in pairs or small groups.) Exponents allow us to more easily write multiplication as

- **(I do)** Squaring and cubing (raising to the $2^{nd}$ and $3^{rd}$ powers) are the most common uses of exponents. We can show this using our cubes. We first want to find the values of $2^2$ and $2^3$, using our cubes.
  1. **(Understand the problem)** We are going to find the values of $2^2$ and $2^3$. While we could write these out as multiplication and just find the answers using arithmetic, we want the students to understand what is going on.
  2. **(Pick a strategy)** We will base our implementation on the “area model” for multiplication. When multiplying two numbers, we can think of the two factors as side lengths. The answer then is the area of the rectangle those sides encompass.
  3. **(Implement the strategy)** Using your blocks, grab enough so that you can make a 4x4 square. Since we have 4 along one side and 4 along the adjacent side, we have represented 4 times 4 in an area model. Counting the squares, we see we have 16 altogether. For “4-cubed” we will do something similar, only now we need to add a height dimension. This time, make a 4x4x4 cube. Counting up the total number of blocks (64), we have our answer to $4^3$.
  4. **(Review)** This may help students see how squaring/cubing actually work. If we could visualize further dimensions, the pattern would be the same. For $4^4$, we would add 4 blocks in the next “direction” and fill in our area. However, this is something we cannot visualize, so we have to do the multiplication either by pencil/paper or calculator. Either way, if we made a perfect square and cube with our blocks, then our answer should make sense.

- **(We do)** Unless we want to get into using an impractical amount of cubes, we’ll need to keep the numbers small. Have the students walk you through finding $3^2$ and $3^3$. The solutions should be 9 and 27.

- **(You do)** Have the students work on $2^2$ and $2^3$ as well as $1^2$ and $1^3$.

Part 3: You should have just covered much of the handout. They can read over it themselves, including the properties. We won’t spend too much time on the properties here (unless you choose to) as we are more worried about computation as opposed to manipulation. The next thing to cover, then, is roots, or fractional exponents.

- **(I do)** Since we would cover squaring and cubing with the blocks, we can also cover taking the square root and taking the cube root. We want to solve $\sqrt{9}$ and $\sqrt[3]{27}$.

  1. **(Understand the problem)** We want to find the square root of 9. In other words, what number, when multiplied by itself will give me a product of 9? We also want to find the cube root of 27: what number, when multiplied by itself 3 times will give me 27?
  2. **(Pick a strategy)** We will once again use the blocks, this time doing our original activity in reverse.
  3. **(Implement the strategy)** Start with 9 blocks and form a square with them. Since it comes out perfectly, we just need to see the side length is 3 to have our answer. Do a similar thing with 27 blocks, making a cube this time. It should also come out even with a side length of 3. Thus, our answer to both problems is 3.
  4. **(Review)** Based on our handout, we see that taking a root and taking a power can be inverses of one another.
This means that \( 3^2 = 9 \) immediately gives us \( \sqrt{9} = 3 \). In addition, since \( 3^3 = 27 \), we know that \( \sqrt[3]{27} = 3 \). Since we covered squaring and cubing 3 in part 1, we know our answers make sense.

- **(We do)** Have the students walk you through taking the cube root of 64, as well as the square root of 16. (You can also choose other square roots: 25, 36, 49…. You could do other cube roots, but the amount of blocks will get rather large.)
- **(You do)** Have the students find the cube root of 8 (and 1), and the square root of 4 (and 1).

Following the above activities (or during) it would be appropriate to show students how to solve exponents/roots using a calculator. Introduce them to the following keys: \( x^2 \), \( x^y \), \( \sqrt{\phantom{0}} \), \( \sqrt[3]{\phantom{0}} \), and \( x^{\frac{1}{y}} \). This will allow them to solve imperfect squares/cubes and other roots and exponents quickly.

Part 4: Now that we know what exponents are and how to solve them, we can look back at our interest problem. We need to be able to quickly solve:

\[
\$100 \cdot 5\%^2.
\]

This should be simple, correct? You always go left to right in a problem? If you have yet to do so, give everyone the PEMDAS handout.

- **(I do)** We need to figure out in which order to do the operations in our problems. Students may have heard of the expression, “Please excuse my dear Aunt Sally,” to help them remember the order in which to do mathematical operations. Go over the handout with everyone, working out the problem on there so that everyone can see how the order works.

  1. **(Understand the problem)** We need to use the correct order of operations to solve a problem, that way we know we are using the correct steps and not making any errors in our calculation order.
  2. **(Pick a strategy)** We will use PEMDAS (some people use BODMAS – Brackets, orders, division/multiplication, addition/subtractions) to correctly order the operations we will perform.
  3. **(Implement the strategy)** The steps are marked out on the handout. Make sure to explain each one, as well as to note that when you are performing the operations inside a set of parentheses, you cycle through the rest of the steps first. Then you move on to another set of parentheses or to the rest of the problem and recycle through the steps.
  4. **(Review)** As we can see, we get a different answer than either at the top of the sheet. However, if we follow the correct order (and make no calculation errors) we should always arrive at the same answer as our peers.

- **(We do)** Having the class help you, use order of operations to help find the answer to:

\[
30 - (5 \cdot 2^3 - 15).
\]

It should come out to be 5.

- **(You do)** Have the students solve:

\[
(3^3 - 2 \cdot 7) + (5 \cdot 3 - 2^2).
\]

The answer here would be 24. For further practice they can try

\[
(7 - \sqrt{9}) \cdot (4^2 - 3 + 1),
\]

which is 56, and

\[
\frac{2^4 + (16 - 3 \cdot 4)}{(6 + 3^2) \div (7 - 4)},
\]

which is 4.

Part 5: Depending on where this lesson was split, this could be the beginning of a new class session. We can now go back to our interest problems, and go a bit more in depth on them. Let’s start with “simple interest.” If we think back to the beginning of the lesson, simple interest meant that we always received (or paid) the same amount of interest on just our initial investment. In our initial example, that was 5\% on an investment of $100. To find out what we have after that first year, we would add $100 to the amount of interest earned. In other words,

\[
$100 + $100 \cdot 5\%.
\]

If we did this for multiple years, say 3, we would just multiply our interest portion by 3. In other words, we would have:

\[
$100 + $100 \cdot 5\% \cdot 3.
\]

We can generalize this a bit. If we think of $100 as our “Initial value” or “IV”, 5\% as our “interest rate” or “r”, and 3 as our “time in years” or “t”, then we would have:

\[
\text{Initial Value} + \text{Initial Value} \cdot \text{Interest Rate} \cdot \text{Time in Years},
\]

or:

\[
\text{IV} + \text{IV} \cdot r \cdot t.
\]

Now we can plug in values for any number of problems.
• *(I do)* As an example, let’s say that we invest $1,000 at a simple interest rate of 1% for 10 years. How much would be in the account at the end of those 10 years?

  1. *(Understand the problem)* Our initial value is $1,000. Our interest rate is 1%, or .01, or \( \frac{1}{100} \). Our time in years is 10.
  2. *(Pick a strategy)* We want to find the amount of interest added to our initial investment to find our current value. As we have a generalized formula now, we can just plug in the necessary amounts.
  3. *(Implement the strategy)* Our formula above would become:

\[
$1,000 + \frac{1}{100} \cdot 10.
\]

   If we plug that into our calculators, for ease, we would get $1,100 after 10 years.

  4. *(Review)* 1% is a very little amount. It would give us only $10 earned per year on $1,000. Thus, we would earn $100 over 10 years, so $1,100 after 10 years does make sense.

• *(We do)* Have the class walk you through plugging in the values and solving the scenario where Brett buys a car and gets a loan on simple interest. He borrows $20,000 and gets an interest rate of 5% over 5 years. How much would he owe back in total at the end of the loan? In this case, our formula becomes:

\[
$20,000 + $20,000 \cdot .05 \cdot 5,
\]

and he would owe $25,000 at the end of the loan.

• *(You do)* If Shirley invests $10,000 at 0.5% simple interest for 10 years, how much will her investment have grown to at the end of the investment?

\[
$10,000 + $10,000 \cdot .005 \cdot 10,
\]

which gives her $10,500 after 10 years.

Part 6: While simple interest keeps the interest constant over a period of time, it really isn’t all that helpful when trying to build a savings account. People would rather earn interest on the extra money they are receiving each year. This is possible when interest is compounded, which is the more common way interest is now calculated for bank accounts. In this case, if we remember from the beginning of the lesson, we calculate interest on the amount in the account at that time. Our earlier example gave our current value after as

\[
$100 + $100 \cdot 5%.
\]

If we think about the distributive property, and our ability to use it backwards, we could rewrite the above as:

\[
$100 \left(1 + \frac{5}{100}\right).
\]

Since 5% is the same as 0.05, we can get our current value by multiplying our previous balance by 1.05. The above problem would give us $105 after our first year. Our second year, then, would give us,

\[
$105 \cdot 1.05 = $105 \cdot 1.05 = $100 \cdot 1.05 \cdot 1.05 = $100 \cdot 1.05^2,
\]

which gives us $110.25 no matter how we write it. This shows us that power we had when we first introduced this type of interest. Just like we did with simple interest, we can generalize this into a formula. Let’s call what we have before calculating the interest each year our “Present Value” or “PV”, our 0.05 is our “Interest Rate” or “r”, and 2 is our “Time in Years” or “t.” This gives us:

\[
\text{Present Value}(1 + \text{Interest Rate})^\text{Time in Years},
\]

or,

\[
PV(1 + r)^t.
\]

*Note:* This is for interest that is compounded once yearly. There are other ways to compound interest, which would change the interest rate and time a bit in the above formula. It is up to you whether you mention this for clarity or use the simple formula above.

• *(I do)* Let’s take that same example as before, where we invest $1,000 for 10 years at a rate of 1%. This time, however, the interest is compounded yearly. How does this change our value after 10 years?

  1. *(Understand the problem)* We want to find out how much our account is worth if our present value is $1,000, our interest rate is 1% or 0.01, and our time is 10 years.
  2. *(Pick a strategy)* We will use the above formula and then compare it to our answer for the formula using simple interest.
  3. *(Implement the strategy)* Our formula becomes:

\[
$1,000(1 + .01)^{10},
\]

which gives us about $1,104.62 after 10 years if we plug it into our calculators.

  4. *(Review)* We should note that when comparing the two interest calculations that if the initial value, interest rate, and time are all the same, the compounded interest will always be greater as it calculates interest on a new amount each year. Thus, we knew our answer should be bigger than $1,100. However, 1% is still a small amount, so we didn’t expect it to be much bigger. Our answer, then, should make sense.
(We do) Have the class re-do the earlier example done together using compounded interest. So, Brett buys a car and gets a loan on compounded interest. He borrows $20,000 and gets an interest rate of 5% over 5 years. How much would he owe back in total at the end of the loan? In this case, our formula becomes:

\[ 20,000(1 + .05)^5, \]

which gives us about $25,525.63. **Note:** This would only count if he waited until the very end to repay the entire loan. If he were making payments, the interest would be calculated based on how much he has already paid.

(You do) Have the students re-do the example from earlier. If Shirley invests $10,000 at 0.5% compounded interest for 10 years, how much will her investment have grown to at the end of the investment?

\[ 10,000(1 + .005)^{10}, \]

which gives her $10,511.40 after 10 years.

Part 7: In order to make deciding between simple and compound interest accounts a little more challenging, the interest rates between the two often differ. Otherwise, it would be easy to always choose the compounded interest account. Using the balancing act sheet, we will explore this concept.

**Note:** The handout could be done after the optional activity in the Technology Integration section. It will explain how to insert formulas into Microsoft Excel, but the problems can be done without that program. It would just be a supplement to help students see what is happening. If Excel will not be used, you can just skip to the part with the actual problems.

(I do) Walk the class through the first problem on the handout.

1. **(Understand the problem)** We will be calculating both simple and compounded interest. Our initial value will be $500,000, our interest rates will be 0.015 for simple and 0.013 for compounded, and our time is 20 years.
2. **(Pick a strategy)** We will use the formulas from earlier.
3. **(Implement the strategy)** Our simple interest formula becomes,

\[ 500,000 + 500,000 \cdot .015 \cdot 20, \]

giving us $650,000.00, and our compounded interest formula becomes,

\[ 500,000(1 + .013)^{20}, \]

giving us $647,379.45. Thus, the simple interest account is the better option.
4. **(Review)**

(We do) Have the class walk you through doing problem #2. In this case, our formulas and answers are,

\[ 50,000 + 50,000 \cdot .02 \cdot 10, \]

giving us $60,000.00, and our compounded interest formula becomes,

\[ 50,000(1 + .0175)^{10}, \]

giving us $58,472.22. Thus, the simple interest account is the better option.

(You do) Have the class work on the rest of the problems, turning some into you so you can check their understanding. Answers can be found on the teacher’s answer key.

**Assessment/Evidence (based on outcome)**

Each of the you do steps will serve as assessment. The instructor should be able to gauge understanding by having different students provide their solutions and explanations of how they arrived at that solution. In addition, during the we do steps, instructors should be encouraging all students to participate in the discussion. The ability to provide input in these discussions will help the teacher gauge each student’s mastery of the concepts.

For a summative assessment, have students turn in some of the problems from the Balancing Act worksheet.

**Teacher Reflection/Lesson Evaluation**

Not yet completed

**Next Steps**

One possible next step would be to incorporate roots and exponents into equations and solve those using more Algebraic techniques. Another possibility would be to work on graphing the interest relations and seeing how exponential growth/decay compares with linear growth/decay.

**Technology Integration**

- [http://www.mathsisfun.com/exponent.html](http://www.mathsisfun.com/exponent.html)
  
  Site with basic information on exponents

- [http://www.mathsisfun.com/operation-order-pemdas.html](http://www.mathsisfun.com/operation-order-pemdas.html)
  
  Site with information regarding order of operations

- [http://serc.carleton.edu/sp/library/ssac/examples/15683.html](http://serc.carleton.edu/sp/library/ssac/examples/15683.html)
  
  Optional Simple/Compound interest activity
<table>
<thead>
<tr>
<th><strong>Purposeful/Transparent</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Since students want to be able to perform calculations based on interest accounts, instructors will examples involving bank loan/deposit accounts to introduce the topics of exponents, roots, and order of operations.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Contextual</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>One example of context for roots/exponents is in the world of finance with interest. They also appear in problems of population growth/decay which is a biological application and for freefall problems which is a physics/engineering application.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Building Expertise</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>In prior lessons, instead of using order of operations, students were solving multi-step problems in what were seemingly disjoint steps. Now that order of operations has been introduced, they can do these problems in one step and separate their calculations using necessary parentheses or brackets. This will allow them to move toward adding in unknowns (variables) and working with equations in a more algebraic setting.</td>
</tr>
</tbody>
</table>
Exponents and Roots

The exponent of a number tells us how many times to multiply a number by itself. The base is the number that we are multiplying by. For instance, if our base is 2 and our exponent is 3, then we will multiply 2 by itself 3 times:

\[ 2^3 = 2 \cdot 2 \cdot 2 = 8 \]

We use exponents to make writing multiplication simpler. It is far easier to write

\[ 10^8 \]

than it is to write

\[ 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \]

When we use exponents, we read the expression \( 2^3 \) as “two to the third power.” Power is just a way for us to note the use of an exponent. Some common exponents are given other names. For instance, \( 2^2 \) can be read as “two to the second power” or as “two squared,” while \( 2^3 \) can be read as “two to the third power” as well as “two cubed.”

If our exponent is negative, on the other hand, we are dividing. In this case, if we have \( 2^{-3} \), then we are dividing 1 by \( 2^3 \) times. So we could write:

\[ 2^{-3} = 1 \div 2 \div 2 \div 2 \]

Another way to think of this is to think about taking the reciprocal of \( 2^3 \). In this case, we could write:

\[ 2^{-3} = \frac{1}{2 \cdot 2 \cdot 2} \]

In either case, we find that

\[ 2^{-3} = \frac{1}{8} \]

One special exponent is 0. If we raise any base to the power of zero, our answer is 1. So:

\[ 2^0 = 1 \]
\[ 5^0 = 1 \]
\[ -12^0 = 1 \]
There are some properties of exponents that allow us to simplify problems where they appear. There are five properties we want to be aware of.

<table>
<thead>
<tr>
<th>Property</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $x^a \cdot x^b = x^{a+b}$</td>
<td>1. $2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32$</td>
</tr>
<tr>
<td>2. $\frac{x^a}{x^b} = x^{a-b}$</td>
<td>2. $\frac{2^5}{2^3} = 2^{5-3} = 2^2 = 4$</td>
</tr>
<tr>
<td>3. $(x^a)^b = x^{ab}$</td>
<td>3. $(2^2)^3 = 2^{2\cdot3} = 2^6 = 64$</td>
</tr>
<tr>
<td>4. $(xy)^a = x^a y^a$</td>
<td>4. $(5 \cdot 3)^2 = 5^2 \cdot 3^2 = 25 \cdot 9 = 225$</td>
</tr>
<tr>
<td>5. $(\frac{x}{y})^a = \frac{x^a}{y^a}$</td>
<td>5. $(\frac{3}{5})^2 = \frac{3^2}{5^2} = \frac{9}{25}$</td>
</tr>
</tbody>
</table>

A root can also be known as a radical or a radical exponent. This means that our exponent is a fraction.

Taking the root means that we want to find a number that when multiplied by itself a certain number of times, gives us the number we are taking a root of. For instance, in the example at the right, we want to find a number that when we multiply it by itself a total of three times, we get 8. Based on the exponent example from the beginning, we know that if we take 2 multiplied together 3 times, we get 8. Thus:

$$\sqrt[3]{8} = 2$$

In the example above, we took the “cubed root,” or the “third root.” If we take the “square root,” or the “second root,” of 4, we want to find the number that when multiplied by itself gives us 4. Thus, since 2 multiplied by itself gives us 4, we have:

$$\sqrt{4} = 2$$

We can simplify square roots, as they are the most common, by dropping that 2 and just writing:

$$\sqrt{4} = 2$$

Taking a root is the inverse operation of taking a power. In other words, roots “undo” powers, much like subtraction “undoes” additions and division “undoes” multiplication. For example, if we take the fourth root of a number that was previously raised to the 4th power, we arrive back at the original number:

$$\sqrt[4]{5^4} = 5$$
This will allow us to see why we can represent roots as fractional exponents. Let’s think back to our third property of exponents:

\[(x^a)^b = x^{ab}\]

If we give ourselves a concrete example, let’s say we want to find what root would “undo” taking the square of 3. In other words, what could I do to \(3^2\) to just get back to the number 3? In mathematical terms, we want:

\[\sqrt[2]{3^2} = 3\]

Based on our property, we know that,

\[(3^2)^b = 3^{2b}\]

And, based on the definition of exponents, we know that,

\[3 = 3^1\]

This tells us that we want to find the number that when multiplied by 2 gives us 1. This number is \(\frac{1}{2}\), so,

\[\sqrt[2]{3^2} = (3^2)^{\frac{1}{2}} = 3^{2 \cdot \frac{1}{2}} = 3^1 = 3\]

This tells us that taking the square root is the same as raising to the \(\frac{1}{2}\) power. Similarly, taking the cube root is the same as raising to the \(\frac{1}{3}\) power, taking the fourth root is the same as raising to the \(\frac{1}{4}\) power, and so on.

What if we raise a number to a power that is neither an integer nor a unit fraction? For example, what if we wanted to find \(9^{\frac{3}{2}}\)? In this case, we want to again use that third property of exponents and split that exponent. Then we have,

\[9^{\frac{3}{2}} = 9^{\frac{1}{2} \cdot 3} = \left(9^{\frac{1}{2}}\right)^3 = (3)^3 = 27\]

In other words, we can always consider the bottom part of the fraction to be our root and the top part of the fraction to be our power. So we could have written the initial problem as,

\[9^{\frac{3}{2}} = \sqrt{9^3}\]
In the problem above, there are many options for how to solve it. We could go strictly left-to-right, in which case the answer would be:

\[
\frac{(6)^2 - 4 \times 2 + 10}{2} \rightarrow \frac{36 - 4 \times 2 + 10}{2} \rightarrow \frac{32 \times 2 + 10}{2} \rightarrow \frac{64 + 10}{2} \rightarrow \frac{74}{2} \rightarrow 37
\]

Or, we could do all the multiplication and division first, leaving us with:

\[
\frac{6^2 - \frac{8}{2} + \frac{10}{2}}{2} \rightarrow \frac{3^2 - 4 + 5}{2} \rightarrow \frac{9 - 4 + 5}{2} \rightarrow 10
\]

As you can see, the order in which we do things matters. Just like with definitions of words and grammar rules, a worldwide rule had to be made so that everyone solved their problems similarly. The order that was decided upon is commonly known by the acronym “PEMDAS.” The letters stand for Parentheses (do whatever is inside parentheses, brackets, or braces first), Exponents (evaluate any exponents or roots next), Multiply/Divide (do any multiplication and/or division from left to right next), and Add/Subtract (finish by doing any addition and/or subtraction from left to right).

For our original problem, we first need to notice that the entire top is being divided by 2, so we can think of a set of brackets enclosing the entire numerator. Then, we would solve like this:

\[
\frac{[(18 \div 3)^2 - 4 \times 2 + 10]}{2} \rightarrow \frac{[(6)^2 - 4 \times 2 + 10]}{2} \rightarrow \frac{[6^2 - 4 \times 2 + 10]}{2} \rightarrow \frac{[36 - 4 \times 2 + 10]}{2} \rightarrow \frac{[36 - 8 + 10]}{2} \rightarrow \frac{38}{2} \rightarrow 19
\]

As you can see, you rotate through the order, always doing what’s inside parentheses in the correct order before moving on, left to right.
Using Formulas in Excel

Microsoft Excel allows us to set up formulas that will solve math problems for us. This allows us to enter different values into the cells and see outcomes without having to do the separate calculations each time.

If we open up a new Excel file, we can input formulas for simple and compound interest and then compare the two. Let’s say we want to compare what happens with a $10,000 investment over 25 years when we have either a simple interest of 5% or a compounded interest of 3%. To start, I’m going to set up places to input all my information.

Here, we can see that cell B2 (column B, row 2) has “Initial Value=” in it. In the cell immediately to the right, I will input my initial value. Similarly, I will input my simple interest rate as a % into cell C3, and my compounded interest rate % into C4. Column E will be my years, and F and G will be the calculated values.

To set up my years, I could just go down the column and enter 1 through 25. That is tedious, however, so I’m going to do it with an equation. In E2, I will enter a 1. For E3, however, I will set up an equation, as can be seen in the formula bar. Since I want my years to be one more as I go down the column, I am going to have Excel add one to the cell above it.

Then, when I hit enter, a 2 will be in that cell. I can then highlight that cell and copy it (right click and go to copy or press Ctrl+C). Next, I will highlight all the cells I want to have a year in them, in my case I want 25 years, so I will highlight all the way down to E26. I will then paste that formula to those cells (right click the highlighted cells and go to paste or press Ctrl+V). I now have my years set up.
At the same time, let’s enter our initial value of $10,000, and our interest rates of 5% and 3% (be sure to include the $ and % signs!).

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>Initial Value</td>
<td>$10,000</td>
<td>Years</td>
<td>Simple Account Value</td>
<td>Compounded Account Value</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>Simple Interest Rate</td>
<td>5%</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>Compounded Interest Rate</td>
<td>3%</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Before we do the formulas for the account values, let’s set up our cells to show us money values. Highlight all the cells that will show account values:
Then, right click and choose “Format Cells…”

In the new window, you want to make sure you have selected Currency as the category. Then, since we are dealing with money, I have set my Decimal places to 2 to represent cents. Negative numbers will
not be something we will deal with in this activity, but you can also choose how those appear here as well. After selecting your options, click the OK button to go back to your spreadsheet.

Now, we will enter our interest formulas. Before, when we did years, we entered E2 to represent the cell above ours. (Note: You can also click on the desired cell instead of typing in its name.) If you highlight any of the cells with a year in it, you will notice that value changes. For instance, in cell E10, the formula has E9 in it. Excel adapts our formulas to work with the cell it currently occupies. It took our idea of “add 1 to the cell above this one” and adapted it to all the cells we pasted that into. For our account value calculations, we will want to modify this a bit.

For simple interest, our formula is:

\[
\text{Initial Value} + \text{Initial Value} \cdot \text{Interest Rate} \cdot \text{Time in Years},
\]

so we will take the values in cell C2, C3, and column E. First, we will highlight cell F2. We will then type in the formula:
Notice that this differs from before as we have put $ signs around C2 and C3. This is because when I copy the formula to the rest of column F, I do not want Excel to shift C2 and C3 to the cells below them. I want the formula to always have those specific amounts in it. Also, note that for multiplication we use the asterisk symbol (*). Now, I can copy and paste that formula into cells F3 through F26 to get the calculated amounts for the Simple Interest Account. (This can be done the same way as we did for the years formula.)
Now, I will do the Compounded Account formula:

\[
\text{Present Value}(1 + \text{Interest Rate})^{\text{Time in Years}},
\]

and in Excel:

\[=\text{CS2}*(1+\text{CS4})^\text{E2}\]

<table>
<thead>
<tr>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Value=</td>
<td>$10,000.00</td>
<td>1</td>
<td>$10,500.00</td>
<td>=CS2*(1+CS4)^E2</td>
<td></td>
</tr>
<tr>
<td>Simple Interest Rate=</td>
<td>5%</td>
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<tr>
<td>Compounded Interest Rate=</td>
<td>3%</td>
<td>3</td>
<td>$11,500.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Again we use the $ symbol to tell Excel to always use those two cells, and we use the ^ symbol to denote raising to a power. Also note that we cannot assume multiplication between the initial value and the parenthesis, we must tell Excel exactly when we mean to multiply. After copying and pasting this to the rest of column G, I have:

<table>
<thead>
<tr>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Value=</td>
<td>$10,000.00</td>
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<td></td>
<td>$20,937.78</td>
</tr>
</tbody>
</table>

Now, we can manipulate the values in our initial value and interest rate boxes and the account values will reflect this.
Problems

1. Jen has won the lottery and decides to take a lump sum payment that amounts to $500,000. She has the option of putting this into either an account with a simple interest rate of 1.5%, or an account with a yearly compounded rate of 1.3%. Which account should she choose, and how much will she have after 20 years?

2. Frank receives an inheritance after his parents pass away. After expenses and taxes, he is left with $50,000. He can put this into a simple interest account with a rate of 2%, or into a compounded interest account with a rate of 1.75%. Which should he choose, and how much will he have after 10 years?

3. Daphne decided to consolidate her money into one checking account. She was offered either a simple interest rate of 0.5% or a compounded interest rate of 0.2%. If she plans on leaving the money in there for at least 25 years, how much will she have after those 25 years for each type of account?

4. Bradley sells his car for $7,500. He plans to put the money into an interest earning account to let it grow for 5 years. He can choose from a simple interest account with a rate of 1.5%, or a compounded rate of 1.15%. Which is the better option for him and how much will he have after 5 years?

Challenge Problem

Sara’s parents start a savings account for her when she is born. They deposit $500 into the account to start it, and plan to deposit an additional $500 into the account each year on her birthday. In addition, the bank allows them to choose between a savings account that earns 0.3% simple interest or one that earns 0.2% compounded interest yearly. Figure out how much would be in each account on Sara’s 18th birthday for college use.

(Hints: Year 1 would be the year Sara is born. Her 1st birthday, then, would be year 2, and her 18th birthday would be Year 19. Also, her parents put in an additional $500 into the account every year. Would this change the way simple interest is calculated? Would it change the formula for how much is in an account at the end of each year? Would it change the way compounded interest is calculated? Would it change the formula for how much is in that account at the end of each year?)
1. Jen has won the lottery and decides to take a lump sum payment that amounts to $500,000. She has the option of putting this into either an account with a simple interest rate of 1.5%, or an account with a yearly compounded rate of 1.3%. Which account should she choose, and how much will she have after 20 years?

Simple Interest: $650,000.00
Compounded Interest: $647,379.45
She should choose the account with simple interest.

2. Frank receives an inheritance after his parents pass away. After expenses and taxes, he is left with $50,000. He can put this into a simple interest account with a rate of 2%, or into a compounded interest account with a rate of 1.75%. Which should he choose, and how much will he have after 10 years?

Simple Interest: $60,000.00
Compounded Interest: $59,472.22
She should choose the account with simple interest.

3. Daphne decided to consolidate her money into one checking account. She has $35,000 and was offered either a simple interest rate of 0.5% or a compounded interest rate of 0.48%. If she plans on leaving the money in there for at least 25 years, how much will she have after those 25 years for each type of account?

Simple Interest: $39,375.00
Compounded Interest: $39,451.06
She should choose the account with compounded interest.

4. Bradley sells his car for $7,500. He plans to put the money into an interest earning account to let it grow for 5 years. He can choose from a simple interest account with a rate of 1.5%, or a compounded rate of 1.35%. Which is the better option for him and how much will he have after 5 years?

Simple Interest: $8,062.50
Compounded Interest: $8,020.10
She should choose the account with simple interest.
### Challenge Problem

Sara’s parents start a savings account for her when she is born. They deposit $500 into the account to start it, and plan to deposit an additional $500 into the account each year on her birthday. In addition, the bank agrees to give them an account that earns 0.25% interest compounded yearly. Figure out how much would be in the account on Sara’s 18th birthday for college use.

Each year, the interest is compounded on the new amount. So the first year, it is calculated on $500. The amount after that first year, then, is $501.25. Then, the parents add $500, so the second year’s interest is calculated on $1,001.25. The amounts following each year are:

<table>
<thead>
<tr>
<th>Years</th>
<th>Account Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$501.25</td>
</tr>
<tr>
<td>2</td>
<td>$1,003.75</td>
</tr>
<tr>
<td>3</td>
<td>$1,507.51</td>
</tr>
<tr>
<td>4</td>
<td>$2,012.53</td>
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<tr>
<td>5</td>
<td>$2,518.81</td>
</tr>
<tr>
<td>6</td>
<td>$3,026.36</td>
</tr>
<tr>
<td>7</td>
<td>$3,535.18</td>
</tr>
<tr>
<td>8</td>
<td>$4,045.26</td>
</tr>
<tr>
<td>9</td>
<td>$4,556.63</td>
</tr>
<tr>
<td>10</td>
<td>$5,069.27</td>
</tr>
<tr>
<td>11</td>
<td>$5,583.19</td>
</tr>
<tr>
<td>12</td>
<td>$6,098.40</td>
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<tr>
<td>13</td>
<td>$6,614.90</td>
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<tr>
<td>14</td>
<td>$7,132.68</td>
</tr>
<tr>
<td>15</td>
<td>$7,651.76</td>
</tr>
<tr>
<td>16</td>
<td>$8,172.14</td>
</tr>
<tr>
<td>17</td>
<td>$8,693.82</td>
</tr>
<tr>
<td>18</td>
<td>$9,216.81</td>
</tr>
<tr>
<td>19</td>
<td>$9,741.10</td>
</tr>
</tbody>
</table>

If using Excel, the formula to enter in year one is the same as the normal compounded interest formula. For year two (and on), you will want to put:

\[
=(F2+C2)*(1+C3)
\]

F2 (which will adapt for each subsequent cell) tells Excel to use the value at the end of the previous year, while the C2, C3 values will always tell Excel to use the specified yearly deposit and interest rate.
MATHIN’ AROUND THE HOUSE

Student/Class Goal
Home remodels, or “flips” are hot lately. When remodeling, designers need to be sure shapes are similar/congruent and need to make sure certain pieces are parallel/perpendicular to one another.

Outcome (lesson objective)
Students will be able to apply formulas and theorems about shapes, angles, parallel lines, and perpendicular lines.

Standard Use Math to Solve Problems and Communicate
(Primary benchmarks in bold.)

<table>
<thead>
<tr>
<th>Number Sense</th>
<th>Benchmarks</th>
<th>Geometry &amp; Measurement</th>
<th>Benchmarks</th>
<th>Algebra &amp; Patterns</th>
<th>Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connect number words</td>
<td>Identify/apply basic geometric concepts</td>
<td>3.6, 4.7, 5.6</td>
<td>Patterns/sequences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solve problems using computations</td>
<td>Connect graphical and algebraic representations</td>
<td>Evaluate/solve expressions/equations</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Order of operations</td>
<td>Perimeter/area/volume</td>
<td>Connect relationships to representations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compare/order numbers</td>
<td>Graphical representations</td>
<td>Graphing</td>
<td></td>
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<td></td>
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<tr>
<td>Estimate &amp; compute to solve problems</td>
<td>Use of correct units</td>
<td>3.11, 4.12</td>
<td>Solving equations using algebra/graphs</td>
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<td></td>
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<tr>
<td>Evaluate using roots and exponents</td>
<td>Right triangle trigonometry</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Data Analysis & Probability Benchmarks
| Measurement applications |
| Solve problems |
| 4.25, 5.25, 4.26, 4.27 |
| Interpret data |
| Measurement conversions |
| Communicate ideas |
| 4.29, 5.30, 5.31 |
| Create and display data |
| Rounding |
| Reason mathematically |
| 4.32 |
| Central tendency |
| Connect concepts |
| 5.35 |
| Probability |
| Mathematical performance |
| 5.36 |

Materials
Problem Solving Steps—Handout
SmartPal kit (SmartPal sleeves, wipe off cloths, dry erase markers) – inserting a blank sheet of paper into the sleeves will give students a reusable sheet of paper that they can quickly try answers out on and erase without using up a pencil eraser. It’s quicker as well.
Slide N’ Measure compass
Tessellating Tiles Exploration—Handout
Parallel Flooring — Handout

Learner Prior Knowledge
Ability to identify and classify the attributes of 2-dimensional and 3-dimensional figures.

- 2-D figures: at least Square, Circle, Diamond, Rectangle, and Triangle
- 3-D figures: at least Rectangular solid, Cube, Cylinder, Sphere, and Cone

Attributes: at least length, width, height, diameter, radius, simple, complex, regular, irregular.

Ability to identify and classify angles by degrees.

Vocabulary
Complex figures – This is a figure that crosses over, or intersects, itself. For example:

The two polygons above both have 5 edges (sides). The one on the left is simple since the edges intersect only at the vertices (endpoints of the segments). The one on the right is complex as the edges intersect at a place other than their endpoints.
Congruent – Two objects with the same size and shape, though their positioning may differ. For line segments, this would mean they have the same length. For angles, the two angles would have the same degree measure. For shapes, this would mean that their corresponding sides and angles are congruent. The end idea is that we can take one shape and overlay it onto another shape. If we can get them to match up perfectly, they are congruent.

Similar – Two objects with the same shape, but their sizes are scaled. For shapes, this means that all corresponding angles are congruent, but their corresponding sides are in proportion.

Regular figures – A shape where all sides are congruent and all angles are congruent.

Irregular figures – A shape that does not meet the definition for a regular figure due to at least one side/angle not being congruent to the rest.

Exterior angle – Angle formed between any side of a shape and the line extended from that side.

Interior angle – Any angle in the interior of a shape. Note: An interior angle and its exterior angle form a straight angle, and thus their sum is 180°.

Complementary angles - Two angles whose sum is 90°.

Supplementary angles - Two angles whose sum is 180°.

Reflection symmetry – A shape can be folded on any line (reflected over any line) and the two halves will be congruent.

Rotation symmetry - A shape can be rotated between 0° and 360° and be congruent to the original shape.

Point symmetry - A shape is congruent in two opposite directions of a point. (Note: This is a little hard to explain without a graphical representation. One can be found here: http://www.regentsprep.org/Regents/math/geometry/GT1a/Psymmet.htm and there is another in the technology integration section.)

Transversal – A line that intersects two or more lines.

Alternate Interior Angles – Congruent angles that are located between two parallel lines and on opposite sides of a transversal.

Alternate Exterior Angles – Congruent angles that are located outside two parallel lines and on opposite sides of a transversal.

Corresponding Angles – Congruent angles located in the same position when two parallel lines are cut by a transversal.

Same-side Interior Angles – Supplementary angles that are located on the same side of the transversal and inside two parallel lines.

Instructional Activities

Part 1: Give them the handout on Polya’s 4 step problem solving process.
We want our students to be able to follow a sequence of steps when solving problems. Whether they know it or not, they probably already sort of do this. We want them to follow Polya’s four step process:
1. Understand the problem (What is the unknown? The data? The conditions?)
2. Pick a strategy to solve the problem (Have you seen a similar problem? One with a similar unknown?)
3. Implement that strategy to come to a solution
4. Review the work and the solution to make sure the solution makes sense in the given context.

After step 4, if there seems to be an error with the solution, students should go back to step 1 and repeat the process until they come to a solution that makes sense.

(For the first few lessons, these steps should be discussed and written down so that students can refer to them as a guide when solving problems. During the I do steps, your thinking aloud should show you going through all four steps in the process. Though they are not explicitly stated in the lesson plan, you should be going through them as you solve the problem. While you begin the problem, make sure to state what you need to pull out of the problem to show you are understanding what is being asked. Then be sure to describe the strategy you are going to use to solve the problem. This could be as simple as stating that there is a given procedure for adding fractions. Then go through the implementation. Finally, make sure to go back and review the answer so students see that the solution makes sense given the context.)

Note: Keep in mind that your class may not need to go through each of the parts below. Please pick and choose which elements to incorporate into your actual lesson based on what you know of your students. In addition, extra sample problems may need to be incorporated based upon your particular class.

Part 2: To set up the lesson, let’s consider a family, the Johnson family, that wants to remodel/redesign their home. They choose to focus on redoing the living room, bathroom, and kitchen. In the kitchen and bathroom, they decide to go with tile patterns that are more than just the normal checkerboard patterns.

For example:

• (I do) Let’s consider the above pattern. It is also on the Tessellating Tiles Exploration handout, so make sure each student has one of those. It may also be a good time to hand out the compass/protractors/rulers at this time as they will be using them for this part of the lesson. The Johnsons want the above pattern to be in their shower. Go over the sample questions, as well as any others you can come up with, that are listed on the handout. Also, fill out the sample chart for the problem as well.

   1. (Understand the problem) Taking the shape on the paper, we want to explore the interior shapes that make up the entire pattern. We want to name them, find out about them, find out about their side lengths and angle measures, and decide if there are any similar or congruent shapes.
   2. (Pick a strategy) We will use the ruler/compass/protractor (Slide N’ Measure) to find out more about the sides and angles. Then, we will use the definitions of similar and congruent to figure out if we have any of those types of shapes. Finally, we will use some properties of angles to help us out as well.
   3. (Implement the strategy) First, let’s notice that we have three different colors in the pattern. It appears that each color is made up of the same shape, so we are working with no more than 3 shapes. Let’s look at the largest one first.

I randomly picked to start with the very center piece, but any of those pieces will work. Based on our chart, there are a few things we can answer right now. We see the shape has 6 sides. This makes it a Hexagon. We also can see that none of the edges crosses over another edge, thus we have a simple shape. Using our slide n’
measure on the figure on the handout, we find that the shape has a side length of about an inch (or 2.5 cm, depending on which scale you would like to use). It should be noted that each side has this same length. Now we can take our slide n' measure and check each angle. We should be getting 120° for each angle inside of our hexagon. While we are on degrees, this would be a good time to figure out the amount of degrees in an exterior angle.

In the image above, ∠𝐵𝐴𝐶 would be an interior angle while ∠𝐵𝐴𝐷 would be its exterior angle. This is because ∠𝐵𝐴𝐷 is formed by extending 𝐴𝐶 outward. A special property of exterior angles is that the sum of the exterior angles of a shape add to 360° (the reasoning as well as a graphic for this can be found here: http://www.mathsisfun.com/geometry/exterior-angles-polygons.html). Since we have 6 sides to our shape we will have 6 exterior angles. In addition, we know that since each interior angle is 120°, each exterior angle must be 60° since they will all be supplementary to the interior angles. Thus, we have 6 ∙ 60° = 360°. The very last part of our chart asks us if the shape is symmetric. There are three types of symmetries. The different types, their definitions, and examples can be found here: http://www.mathsisfun.com/geometry/symmetry.html. Based on that, we can see our hexagon has 6 lines of reflection symmetry, rotational symmetry of order 6 (or 60° since that is how much we would spin the shape to have it be the same as our original), and point symmetry. If we take measurements for any of the other hexagons, we will find that the angles are all the same. Based on our definitions for Similar shapes and Congruent shapes, we have a candidate here. Now we need to check the sides. Doing so will tell us that the side lengths are the same for all the hexagons. In this case, we have similar hexagons, where the ratio of proportion for the sides is 1:1. That also means we have congruent hexagons. As such, it will not matter how George keeps track of his hexagon tiles. He does not need to indicate which one goes where in the pattern as any hexagon he cuts that is congruent with the first will work.

Let’s do the blue shape next. Following the same initial steps, we have a 3-sided figure or triangle. We can measure the angles to find each is 60° and measure the side length to see that each is 1/2 of an inch (which makes sense as two blue triangles make up the side of the hexagon which was 1 inch). Since all the sides/angles are the same, we have a regular triangle, and nothing overlaps so it is simple. Based on our symmetry definitions, we do not have point symmetry this time. (Note, if a figure does not have 180° rotational symmetry, it should not have point symmetry.) And, once again, all of our white triangles are congruent to one another.

Finally, we are left with the white triangle. Like the previous shape, we know it’s a triangle by its three sides. Since not all triangles are necessarily equilateral/equiangular, we need to measure this one to check. We can use our protractor, or we can look at our pattern.

Here, we can see that the white triangle and the blue triangles have angles that lie on the same line. Since these three angles form a line, they add to 180°. This means the indicated angle of the white triangle must be 60° since the two blue angles will sum to 120°. By this same argument, we could find that the other 1 angles of our triangle are 60°. Then, by measuring, we can find our side lengths are all 1/2 inch. This tells us that the white triangles are congruent to the blue triangles. So our information in those two rows should be identical.
Just using our judgment, it looked like the pattern had congruent triangles and that all the shapes of the same color were congruent to one another. Using our measuring tool we were able to find that was indeed the case.

- **(We do)** Have the students work with you to do the same thing with the bathroom floor tiles.

- **(You do)** Have the students do the final two examples. They can do this in class, or save one to turn in. The hope is that by the last one, they can see that the exterior angles of a regular polygon can be found by:

  \[
  \frac{360^\circ}{n},
  \]

  and the interior angles of a regular polygon can be found by:

  \[
  \frac{180^\circ(n - 2)}{n},
  \]

  where \(n\) is the number of sides of the polygon.
Before moving on, there are some other points we should discuss. It should be noted that the triangles have 180°. In fact, all triangles, even if they aren't equilateral/equiangular, will have 180° in their interior angles. From there, we can find the interior angle sum of all other polygons. We do this by finding how many triangles will fit into a particular shape. Take a hexagon for example:

Since we can make four triangles inside the hexagon (they all need to go from the same vertex or corner), then we have $4 \cdot 180° = 720°$ in the interior of that shape. This is how we derive the formula from above:

$$\frac{180°(n - 2)}{n},$$

since we can always make 2 less triangles than our total number of sides.

Part 3: Beyond shapes, another area in which angles are important is with parallel lines. If you take two parallel lines and intersect them with another line, very interesting properties arise with the angles formed. But, let’s approach this from the opposite end. Let’s say Maureen and George have taken a break after finishing their kitchen and bathroom. Pleased with the results, they decide to put hardwood flooring into their living room. Once again, wanting to save money, George needs to cut some of the boards himself so that they will fit once he gets to the end of the room. For appearances, they want to make sure the boards stay parallel to both walls. In order to ensure this, George wants to check to make sure his walls are parallel. To do this, he wants to utilize some angles and parallel lines properties.

When it comes to parallel lines, the following diagram is very common:

In the diagram above, segment AB is part of a line that is parallel to segment CD. In addition, segment EF is a random transversal that intersects our two parallel lines. So we have two parallel lines cut by a transversal. In this situation, we know that alternate interior angles are congruent ($\angle 3 \cong \angle 6$, $\angle 4 \cong \angle 5$), alternate exterior angles are congruent ($\angle 1 \cong \angle 8$, $\angle 2 \cong \angle 7$), corresponding
angles are congruent \((\angle 1 \cong \angle 5, \angle 2 \cong \angle 6, \angle 3 \cong \angle 7, \angle 4 \cong \angle 8)\), and same-side interior angles are supplementary \((\angle 3 + \angle 5 = 180^\circ, \angle 4 + \angle 6 = 180^\circ)\). Fortunately, we can work backwards here as well. If I have two lines cut by a transversal and I want to know if segment AB is parallel to segment CD, then I just need to check to see if I have congruent alternate interior angles, or congruent alternate exterior angles, etc.

- **(I do)** Make sure everyone has the Parallel Flooring handout. To start out, George needs to check to see if his walls are parallel. He makes a dotted chalk line between the two walls so that he can measure an angle between the walls and this newly formed transversal. He takes some measurements that he writes down as in the diagram on the handout. Are his walls parallel?

  1. **(Understand the problem)** We see that George took two angle measurements. The red box tells us that we have a right angle \((90^\circ)\) in the bottom left corner of the room. In addition, we see that his measurements denote a \(36^\circ\) angle and a \(54^\circ\) angle.

  2. **(Pick a strategy)** Since we want to find out if we have parallel lines, we need to see if we have either supplementary same-side interior angles, or if we have congruent angles of one of our other three types (alternate interior, alternate exterior, or corresponding). We don’t have the set-up for corresponding or alternate exterior angles since George can only measure interior angles, but either of the other two ways will work. To get used to the congruent angles, let’s use alternate interior.

  3. **(Implement the strategy)** We do not currently have a pair of alternate interior angles labeled. However, since we know we have a \(90^\circ\) angle in the bottom left, we know that the missing portion of the angle must be complementary to the \(36^\circ\) angle. Thus, that missing portion is \(54^\circ\). Now, we do have a pair of alternate interior angles, and since they are the same degree measure, they are congruent. This tells us those two walls (top and bottom) are parallel.

  4. **(Review)** We can check this by using same-side interior angles. It may look like we currently have a pair, with the two given angles. However, in order for our angles to be useful, they must be measured from the parallel line. Our \(36^\circ\) angle does not have the parallel line as one of its sides, so it doesn’t count. But, we can extend our bottom wall out to get the following diagram:

   ![Diagram](image)

   Since we knew that the left-side wall created a \(90^\circ\) angle inside the room, and we know if we extend out that bottom line we should have a straight angle, then the new angle formed must also be \(90^\circ\). (We also know that if two lines intersect and form one right angle, they are perpendicular and actually form 4 right angles.) Now we have an angle that extends from the bottom wall all the way to the dotted line made by George. This angle will be the \(36^\circ\) angle he measured, plus the right angle we just formed: \(36^\circ + 90^\circ = 126^\circ\). Now we can check same-side interior angles. We have \(54^\circ + 126^\circ = 180^\circ\), so our same-side interior angles are supplementary, making our lines parallel.

- **(We do)** Have the class do the first flooring cut problem with you. George needs to see if his first cut is parallel.

  (Note: The \(90^\circ\) angle is probably the precut angle, though that won’t really affect our calculations.) We once again only have interior angles, though we can use the trick of extending lines around the right angle to give ourselves more angles. With the given picture, it would make sense to use same-side interior angles and see that we do not have supplementary angles, thus do not have a parallel cut.
• *(You do)* Have the students do the final problem on the handout.

In this scenario, George was able to measure the angle between the angled wall and the wall he first dealt with. That large angle is 150°. However, that will obviously not be the angle of a board that goes there. Instead, we must think about the angle between the wall and the already installed flooring. In that case, we have:

The previous boards were placed parallel to the top and bottom walls. We were first told that the bottom left hand corner of the room is a right angle. That’s going to tell us that if we box off the room, as above, we will have 4 right angles. (This can be shown using same-side interior angles, and alternate interior angles.) This tells us that drawing in the edge of the already installed flooring will create the right angle above. Then, we have that 150° - 90° = 60°, as shown above. Since we can consider laying the board alongside the dotted line in the left picture, we basically want to check to see if we have parallel lines. We want the unknown angle to be congruent to that 60° angle, so the marked angle on the board needs to be 60° when George cuts it for it to lay flush against the wall.

As a final part of the lesson, especially if there is time in class, have the students check different lines in the room to see if they are parallel.

**Assessment/Evidence (based on outcome)**

Each of the you do steps will serve as assessment. The instructor should be able to gauge understanding by having different students provide their solutions and explanations of how they arrived at that solution. In addition, during the we do steps, instructors should be encouraging all students to participate in the discussion. The ability to provide input in these discussions will help the teacher gauge each student’s mastery of the concepts.
Students can turn in one part of the Tessellating Tiles handout. In addition, for the second part of the lesson, they can pick different lines in the room that seem parallel and test to see if they actually are. A chalk tray to the top of the board. Lines on the floor. Lines on paper. Table edges. They can then draw the scenario as well as their measurements to show whether or not the lines they picked were parallel and why.

**Teacher Reflection/Lesson Evaluation**

*Not yet completed*

**Next Steps**

The Pythagorean Theorem would be a great follow-up to this lesson. It would allow students to further explore right triangles, which are used heavily in geometry courses. Another option would be to start working with the coordinate plane and graphing lines with their equations. This would allow them to further explore properties of parallel and perpendicular lines as well as plot shapes.

**Technology Integration**

- Reflection Symmetry
- http://www.mathsisfun.com/geometry/symmetry-rotational.html
- Rotation Symmetry
- http://www.mathsisfun.com/geometry/symmetry-point.html
- Point Symmetry
- Geometry Theorems/Properties

**Purposeful/Transparent**

Students want to see how geometric properties can be incorporated in an everyday setting. Instructors will use examples from around the home to help accomplish this task.

**Contextual**

Geometric properties are inherently real-world. The properties of angles and shapes are very hands on and can be seen in real-world representations of those objects. Angle theorems and properties are used by those in architecture and construction to make sure their plans/blueprints are feasible and to make precise cuts when building. Right angles and triangles, especially the Pythagorean Theorem, are useful here.

**Building Expertise**

Much of geometry is built upon definitions and theorems. Having a solid foundation here will allow students to build upon that knowledge using formulas for area, perimeter, and volume, as well as further explore properties and theorems with shapes and angles.
George and Maureen Johnson decide to do some remodeling of their home. They found some tile patterns on the internet that they liked. To save money, George decides that he will buy tiling of the colors they want in bulk and cut out the shapes they need. He will then try to recreate the pattern.

You may find it useful to recreate the following table to aid you in answering each question.

<table>
<thead>
<tr>
<th>Shape</th>
<th># of sides</th>
<th>Regular or Irregular?</th>
<th>Simple or Complex?</th>
<th>Symmetric?</th>
<th>Degrees in an interior angle</th>
<th>Degrees in an exterior angle</th>
<th>Length of a side</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shower Floor</td>
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<tr>
<td>Bathroom Floor</td>
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<tr>
<td>Kitchen Sink Backdrop</td>
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<td>Kitchen Floor</td>
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</tbody>
</table>
The first room they want to work on is the bathroom. They want to upgrade the floor of their shower to have the following pattern:

![Hexagonal Tile Pattern](image.png)

What shortcuts can George take when cutting out the shapes? Can he interchange their positions? Can they be rotated? Are the same? Similar? What can be said about the shapes, angles, and sides in the pattern?
The next area that George and his wife want to retile is the bathroom itself. They decide to go with the following pattern:

![Tile Pattern Image]

Again, what sort of shortcuts can George take? Does he need to keep the cut pieces facing a certain direction? Does it matter if he just throws them all onto a pile? What does he know about the shapes? How can they be compared?
After finishing up in the bathroom, Maureen decides they should work on the kitchen. They choose the following pattern to put as a backdrop behind their sink:

As Maureen cuts the tiles for this pattern, what do you notice about the shapes? The angles?
The final area they need tile for is the kitchen floor. Maureen brings George back in to help her recreate this pattern:

In this final pattern, can you find the interior angles of each shape without measuring? How? Do you notice a pattern to the degrees in an interior angle of certain shapes? Regular or irregular? Complex or simple?
George and Maureen Johnson decide to do some remodeling of their home. They found some tile patterns on the internet that they liked. To save money, George decides that he will buy tiling of the colors they want in bulk and cut out the shapes they need. He will then try to recreate the pattern.

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<tr>
<td>Bathroom Floor</td>
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<tr>
<td>Kitchen Sink Backdrop</td>
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<tr>
<td>Kitchen Floor</td>
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</table>
The first room they want to work on is the bathroom. They want to upgrade the floor of their shower to have the following pattern:

What shortcuts can George take when cutting out the shapes? Can he interchange their positions? Can they be rotated? Are the same? Similar? What can be said about the shapes, angles, and sides in the pattern?
The next area that George and his wife want to retile is the bathroom itself. They decide to go with the following pattern:

![Tiling Pattern](image)

Again, what sort of shortcuts can George take? Does he need to keep the cut pieces facing a certain direction? Does it matter if he just throws them all onto a pile? What does he know about the shapes? How can they be compared?
After finishing up in the bathroom, Maureen decides they should work on the kitchen. They choose the following pattern to put as a backdrop behind their sink:

As Maureen cuts the tiles for this pattern, what do you notice about the shapes? The angles?
The final area they need tile for is the kitchen floor. Maureen brings George back in to help her recreate this pattern:

In this final pattern, can you find the interior angles of each shape without measuring? How? Do you notice a pattern to the degrees in an interior angle of certain shapes? Regular or irregular? Complex or simple?
**Parallel Flooring**

George and Maureen Johnson are ready to put hardwood flooring into their living room and want to make sure that the flooring is put down so that all pieces are parallel, not only to each other, but to the walls as well.

First, George must make sure his walls are parallel, so that if he is making the pieces parallel to one wall, they will also be parallel to the other wall. He takes the following measurements of his living room:

Are the two walls at the top and bottom of the picture parallel?

As George gets to the end of his first row of flooring, he needs to cut his first board. He feels a little shaky with the saw and isn’t sure he cut it perfectly parallel. He decides that if he can check to see if his walls are parallel, he can check to see if the board is as well. He takes the following angle measurements:

Is his cut parallel?
The Johnson’s living room has an angled wall. Knowing it will give him issues, George decides to do this part last. When he gets to it, he cannot line a board up to mark the angle at which to cut. How can he use what he knows about parallel lines to help him? He already knows the following:

What angle will he need to cut at in order to make sure the board fits flush against the wall?
### WALKING THE GRID

**Student/Class Goal**
Students want to be able to calculate distances on a map without using a ruler and the scale provided. They instead want to use the gridlines over top of the map.

**Outcome** *(lesson objective)*
Students will be able to identify coordinate systems and plot points on a graph. In addition, they will use the Pythagorean Theorem to solve contextual problems.

**Time Frame**
2 hours

**Standard** *(Use Math to Solve Problems and Communicate)*
(Primary benchmarks in bold.)

### NRS EFL
Levels 3-6

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<th>Number Sense</th>
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<th>Geometry &amp; Measurement</th>
<th>Benchmarks</th>
<th>Algebra &amp; Patterns</th>
<th>Benchmarks</th>
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<tr>
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<td>Identify/apply basic geometric concepts</td>
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<tr>
<td>Solve problems using computations</td>
<td>Connect graphical and algebraic representations</td>
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<tr>
<td>Order of operations</td>
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<td>Evaluate using roots and exponents</td>
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**Data Analysis & Probability** *(Benchmarks)*

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<tr>
<td>Measurement conversions</td>
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<td>4.29, 4.30, 4.31</td>
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<tr>
<td>Probability</td>
<td>Mathematical performance</td>
<td>5.36</td>
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</table>

**Materials**
SmartPal kit – inserting a blank sheet of paper into the sleeves will give students a reusable sheet of paper that they can quickly try answers out on and erase without using up a pencil eraser. It’s quicker as well. For this lesson, we will want the grid handout provided inserted.
Calculators (Optional)
Gridiron City – Handout
Mapping it Out – Handout

**Learner Prior Knowledge**
Ability to define and identify spatial relationships: vertical, horizontal, adjacent.
Ability to identify and define points, lines, rays, segments, and planes in mathematical and everyday settings.

**Vocabulary**

- **Coordinate grid** - A grid (in our case, a 2-dimensional grid) that allows us to plot points and describe their coordinates. It allows us to efficiently pinpoint locations.
- **Origin** - The point (0,0) lying at the intersection of the x- and y-axis.
- **Axes** - The (often) darker shaded lines on a graph which serve as reference lines from which distances are measured.
- **X-axis** - This line runs from left to right. The horizontal distance from 0 is found using this axis. As with a number line, to the left of 0 would be negative numbers and to the right would be positives.
- **Y-axis** - This line runs from top to bottom. The vertical distance from 0 is found using this axis. As with a thermometer, above 0 the numbers are positive and below 0 the numbers are negative.
- **Quadrants** - The four parts of the coordinate plane, divided based on the x- and y-axis. They are labeled I, II, III, IV in a counter-clockwise fashion beginning with the top right.

![Coordinate Grid Diagram]
Coordinates - An ordered pair that looks like: (x,y). Our first value is always given as the amount travelled in the x-direction, and the second value is always the amount travelled in the y-direction. A point’s coordinates are given by the intersection of these amounts. (A point with the ordered pair (3,2) as its coordinates, for example, would lie at the intersection of positive 3 in the x-direction, and positive 2 in the y-direction.)

Pythagorean Theorem - When working with right triangles, given the lengths of two sides, we can find the third using this formula: leg² + leg² = hypotenuse²

Distance formula - (Derived from the Pythagorean Theorem) Given the coordinates for two points on a grid, we can find the distance between them using the formula: distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. Note: \((x_2 - x_1)^2\) means that we take the x-value from the second point’s coordinates and subtract from that the x-value from the first point’s coordinates. We then take that difference and square it. The sub-notation just tells us which point we are working with.

Instructional Activities

Note: Keep in mind that your class may not need to go through each of the parts below. Please pick and choose which elements to incorporate into your actual lesson based on what you know of your students. In addition, extra sample problems may need to be incorporated based upon your particular class. See the technology integration portion for a website that gives many example problems.

Part 1:

Maps like the one above are quite common. Many now come with grid overlays as well as an index to easily find cities. For instance, maybe you are from out of state and want to find Cincinnati. You aren’t sure exactly where to look on the map, but the index will list the cities alphabetically along with a coordinate to help you find each space. We will come back to this map once we know a bit more about graphs and coordinates.

Part 2: Let’s begin with what may be a simpler map and scenario. Make sure everyone has the Gridiron City handout. The SmartPal kits may prove useful as well so that students can write “on” the handout and make corrections as needed. Go through the scenario on the first and second page with the class, including the questions. On the second page, the movie theater is a half mile from the grocery store (2 quarter-mile gridlines) and the stadium is 1 and 3/4 of a mile from the department store (7 quarter-mile gridlines). In order to find an answer to the final question, we need to put the circus on our sketch. This happens on the third page. Now that we have a coordinate grid, go over some vocabulary. Necessary terms are: coordinate grid (coordinate plane), origin, axes (x-axis and y-axis), quadrants, and coordinates. Once the terms are out there, go over how to find coordinates.
• *(I do)* (This will be a fairly quick explicit instruction model.) Starting with point A, we will eventually find the coordinates of each of the six points on the graph.

1. *(Understand the problem)* Using what we know about a coordinate plane and axes, we want to come up with the coordinates of the movie theater (A).

2. *(Pick a strategy)* We will use our knowledge that the distance between grid lines is \(\frac{1}{4}\) (of a mile). Otherwise, there are not really multiple strategies to use here.

3. *(Implement the strategy)* If we go to the point on the grid, we want to find the x-coordinate and y-coordinate. If we travel straight down along the vertical grid line that A sits on, we will arrive at the x-axis. We then want to count back to the origin and we see that we were two steps away from the origin. So our x-coordinate is \(2 \cdot \frac{1}{4} = \frac{1}{2}\). We want to do the same thing for our y-coordinate. This time, from A, we will travel left along that grid line until we reach the y-axis. We then see we are 4 steps from the origin. So our y-coordinate is \(4 \cdot \frac{1}{4} = 1\). Thus, our ordered pair for point A is \(\left(\frac{1}{2}, 1\right)\).

4. *(Review)* We can double check this by finding the intersection of \(\frac{1}{2}\) on the x-axis and 1 on the y-axis. We can do this by placing a finger on each value, and traveling into the first quadrant until they intersect. We know to go to the first quadrant since both values are positive. (In the second quadrant the x-value is negative and the y-value is positive, in the third quadrant the x-value is negative and the y-value is negative, and in the fourth quadrant the x-value is positive and the y-value is negative.)

• *(We do)* As a class, have the students help you find the coordinates of the grocery store (B): \(\left(\frac{1}{2}, \frac{1}{2}\right)\). You may also do point C, or save that for the students to do alone \((-2 \frac{1}{4}, -2)\).

• *(You do)* Have the students do any remaining points on the graph. For additional practice, pair them up and have them use the graphs on the 4th page of the handout to plot points and have their partner find the coordinates, or state coordinates and have their partner plot the point. [Point D: \((-\frac{1}{2}, -2)\), Point E: \((1 \frac{3}{4}, -1 \frac{1}{2})\), Circus: \((0,0)\).]

Part 3: Now we are going to work on distance and lines. One of the previous questions was about the distance from the circus to the amusement park (the blue dot at the origin to point E).

• *(I do)*

1. *(Understand the problem)* We can have the students come up with different routes to get from one place to the other. If we are driving, we will have to follow the gridlines as those are the roads. If we start at the origin (the circus) we can travel east until we are directly north of the park. We can then go south until we reach the park. This would have us traveling \(13/4\) or 3 and \(1/4\) miles (\(7/4\) east, \(6/4\) south). Is there a shorter route? What if we were walking and were able to cut across the gridlines?

2. *(Pick a strategy)* There is a distance formula when we know coordinates that will tell us the distance between two points. That formula tells us that the distance = \(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\).

3. *(Implement the strategy)* In our case, let’s call our first point \((x_1, y_1)\) the circus position, which is at the origin, or \((0,0)\). Our second point \((x_2, y_2)\), then, is the park located at \((1 \frac{3}{4}, -1 \frac{1}{2})\). Using the formula above, we get:

\[
\sqrt{\left(1 \frac{3}{4} - 0\right)^2 + \left(-1 \frac{1}{2} - 0\right)^2} = \sqrt{\left(1 \frac{3}{4}\right)^2 + \left(-1 \frac{1}{2}\right)^2}.
\]

Simplifying this will give us:

\[
\sqrt{\left(\frac{7}{4}\right)^2 + \left(-\frac{3}{2}\right)^2} = \sqrt{\frac{49}{16} + \frac{9}{4}} = \sqrt{\frac{49}{16} + \frac{36}{16}} = \sqrt{\frac{85}{16}}.
\]

Using a calculator (or, if you have covered simplifying fractions and square roots, doing it by hand) we will find that the distance here is about 2.3 miles, which is significantly less than if we had followed the gridlines.

4. *(Review)* If we draw the possible routes in – first the one that goes east and then south, and then the one that cuts across diagonally – we will see that a triangle is formed. Since a property of triangles is that the sum of two sides must be greater than the third side, we knew that the distance we found in step 3 would be less than the first way of travelling.

• *(We do)* If the circus troupe wants to see a movie that doesn’t begin until 8pm and they want to kill time until then at the department store, how far will they have to travel? Call the department store point 1 \((-\frac{1}{2}, -2)\), and the movie theater point 2 \((\frac{1}{2}, 1)\). Then we have:

\[
\sqrt{\left(\frac{1}{2} - -\frac{1}{2}\right)^2 + (1 - -2)^2} = \sqrt{(1)^2 + (3)^2} = \sqrt{1 + 9} = \sqrt{10} \approx 3.16.
\]
- **(You do)** Have the students check other points. They can even use the formula to recheck the easier distances (A to B, C to D).

**Part 4:** Now let’s go back to our original map (give students the Mapping it Out handout so they can use it in their SmartPals to do the activity):

We know now that we can describe points based on ordered pairs. Let’s re-label along the top so that we have numbers that we can put into our distance formula. A=1, B=2, C=3, D=4, E=5, F=6, and G=7.

- **(I do)** Let’s say we want to find the distance from Lancaster to Xenia.
  1. **(Understand the problem)** We see that Lancaster is basically at the intersection given by (D3) or (4,3). Xenia is also basically at an intersection (B3) or (2,3). Since we have ordered pairs we can find the distance.
  2. **(Pick a strategy)** We will use the ordered pairs above as well as the distance formula. (As these are simply a horizontal distance, we could just count, but let’s reinforce the formula.) Let Xenia be point 2 and Lancaster be point 1.
  3. **(Implement the strategy)**

\[
\sqrt{(2 - 4)^2 + (3 - 3)^2} = \sqrt{(-2)^2 + (0)^2} = \sqrt{4 + 0} = \sqrt{4} = 2.
\]

We are not done however, as the answer just tells us the distance if we consider the distance between grid lines to be “1”. The distance between gridlines is 40 miles, though. So if we travel a distance of 40 miles twice (since we cross 2 grid lines), we get that the distance from Lancaster to Xenia is about 80 miles.

- **(Review)** We can check this two ways. First, since we had chosen points that we did not have to cut diagonally across grid lines to get the shortest distance, we could have just counted how many grid lines we crossed. Secondly, we could plug this into an online map and find the distance. (Google maps gives me a distance of 79.4 miles from Lancaster to Xenia.)

- **(We do)** As a class, find the distance from Columbus to Norwood. Columbus is pretty much in the center of some grid lines. We will need to estimate its coordinates. Since is seems midway between the lines in both the x- and y-directions, let’s call its coordinates (3.5,3.5). Norwood is quite close to an intersection, so let’s call its coordinates (1,2). Then our distance would be:

\[
\sqrt{(1 - 3.5)^2 + (2 - 3.5)^2} = \sqrt{(-2.5)^2 + (-1.5)^2} = \sqrt{6.25 + 2.25} = \sqrt{8.5} \approx 2.92.
\]

2.92 would be the distance if our grid lines were 1 mile apart, instead they are 40 miles apart. So our distance is 2.92 \cdot 40 \text{ miles} = 116.8 \text{ miles}.
• (You do) Have the students answer the remaining questions or pick cities to find the distance between.
  - Zanesville and Kent:
    \[
    \sqrt{(6 - 5)^2 + (6 - 3.5)^2} = \sqrt{(1)^2 + (2.5)^2} = \sqrt{1 + 6.25} = \sqrt{7.25} \approx 2.69.
    \]
    Multiplying this by 40, we get that the distance is about 107.7 miles.
  - Youngstown and Toledo:
    \[
    \sqrt{(2.5 - 7)^2 + (7 - 6)^2} = \sqrt{(-4.5)^2 + (1)^2} = \sqrt{20.25 + 1} = \sqrt{21.25} \approx 4.61.
    \]
    Multiplying this by 40, we get that the distance is about 184.39 miles.
  - Springfield and Portsmouth:
    \[
    \sqrt{(3.5 - 2)^2 + (1 - 3.5)^2} = \sqrt{(1.5)^2 + (-2.5)^2} = \sqrt{2.25 + 6.25} = \sqrt{8.5} \approx 2.92.
    \]
    Multiplying this by 40, we get that the distance is about 116.8 miles.

Part 5: Finally, if time permits, let’s look back to our Gridiron City layout. In part 3, we found the distance from the circus to the amusement park by going east then south. If we draw in that route, as well as the shortcut that we found using the distance formula, we should have a right triangle on our hands. What we just derived was the Pythagorean Theorem. The side we found using the distance formula is across from the right angle, making it the hypotenuse of the triangle. The other two sides, then, are legs. Since distance and length are synonymous here, then we found that squaring the length of one leg and adding it to the square of the other leg gives us the square of the hypotenuse. In other words: \( \text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2 \). Or, more commonly: \( a^2 + b^2 = c^2 \), though don’t let the letters throw anyone off in case a leg is labeled “\( c \)” and the hypotenuse “\( a \)” for instance. It is always \( \text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2 \). Let’s pull some examples off the map.

• (I do) If Athens is located 53 miles directly south of Zanesville, and Zanesville is located 56 miles directly east of Columbus, how far is Athens from Columbus?
  1. (Understand the problem) The question is asking us to find the distance. We want to use the distance formula, but we don’t have coordinates. In this situation, we want to draw a picture to see what we have going on. Once we do that, we should have a right triangle. One leg will be the distance from Athens to Zanesville (53 miles) and the other leg will be the distance from Zanesville to Columbus (56 miles). Our hypotenuse then is the missing distance.
  2. (Pick a strategy) Since we have made a right triangle, we want to use Pythagorean Theorem.
  3. (Implement the strategy) \( \text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2 \) will give us \( 53^2 + 56^2 = \text{hypotenuse}^2 \). We then have \( 2809 + 3136 = \text{hypotenuse}^2 = 5809 \). Thus, the hypotenuse, or the distance from Athens to Columbus is roughly 76 miles.
  4. (Review) We can check our work with a calculator, or an online map. Google maps tells me that Athens to Columbus is 75.7 miles.

• (We do) Have the class walk you through a similar problem. This time, let’s use a non-map example as the Pythagorean Theorem is useful in multiple contexts, especially involving lengths and distances. Softball infields, while known as diamonds, are actually squares. If the distance between successive bases is 60 feet, how far must the catcher throw the ball if they are trying to throw somebody out at second base? (In other words, if we make a square tipped on its side to resemble a diamond, what is the distance from the bottom point to the top point?) Here we have the Pythagorean Theorem once more. Drawing in that vertical line gives us two right triangles. The hypotenuse is the line we drew in, and all the legs are 60 feet. Thus, \( \text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2 \) will give us \( 60^2 + 60^2 = \text{hypotenuse}^2 \). Since \( 60^2 + 60^2 = 7200 \), our distance is about 85 feet.

• (You do) You can use Google maps to find the distance between cities that appear to form right angles, or you can make up problems such as the previous one for more practice. Some contextual examples can be found in the technology integration section.) When building structures, right angles are very important. While putting up the wall frame for a new building, Brad wants to make sure the wall is at a 90 degree angle from the floor. After putting up the wall they put a support beam up to hold it in place. Brad realizes they didn’t grab anything to measure an angle with, so he must figure out a way to check they have a right angle using only his tape measure.
If he measures the angled support to be 4.5 feet, the vertical distance from the floor up the wall to the support as 4 feet, and the horizontal distance from the wall along the floor to the support as 3 feet, is the wall creating a right angle with the floor? Since leg² + leg² = hypotenuse², we want to check if 3² + 4² is the same as (4.5)². Since 3² + 4² = 9 + 16 = 25 and (4.5)² = 20.25, then no, the wall is not square to the floor.

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<tr>
<th>Assessment/Evidence (based on outcome)</th>
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<tr>
<td>Each of the You do steps will serve as assessment. The instructor should be able to gauge understanding by having different students provide their solutions and explanations of how they arrived at that solution. In addition, during the We do steps, instructors should be encouraging all students to participate in the discussion. The ability to provide input in these discussions will help the teacher gauge each student’s mastery of the concepts.</td>
</tr>
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</table>

Have students turn in the final sheet of the Gridiron City handout if they used it with a partner to find coordinates and plot points. In addition, have them turn in any calculations of distances between cities on the map of Ohio from Part 4. And to check for Pythagorean Theorem understanding, have them work out one or two contextual examples to check from the site below.

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<td>Not yet completed</td>
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<th>Next Steps</th>
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<tr>
<td>As we have given ourselves the coordinate grid now, we can easily plot shapes on here. If we make shapes that follow the gridlines, finding perimeter and area of basic/irregular shapes would be a good follow-up. In addition, now that the coordinate grid has been introduced, graphing equations or connecting algebraic and graphical representations of lines/curves would be a good follow-up.</td>
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<tr>
<th>Technology Integration</th>
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<tr>
<td>Sample contextualized Pythagorean Theorem problems.</td>
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<tr>
<td><a href="http://www.mathsisfun.com/pythagoras.html">http://www.mathsisfun.com/pythagoras.html</a></td>
</tr>
<tr>
<td>Information on the Pythagorean Theorem. A link at the bottom will take you to sample problems.</td>
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<th>Purposeful/Transparent</th>
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<tr>
<td>Since students want to be able to read a map and find distances, instructors will go over what goes into reading a map and plotting coordinates. And, while it will be a rough estimate depending on the way the map is laid out, instructors will go over how to find distance using just coordinates and not a ruler and map scale. Finally, instructors will go over the basics of what lines and curves look like on a graph and why.</td>
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<thead>
<tr>
<th>Contextual</th>
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<tr>
<td>Maps often have a grid overlaid on them now. While they do not have a set of axes as this lesson did, you can still consider one corner the origin as many maps will label boxes by a notation similar to “A4”, where letters denote how far across to go, and numbers denote how far up or down to go. In addition, finding distances between points is a good introduction to the Pythagorean Theorem which is used in carpentry and other architectural/construction jobs.</td>
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<tr>
<th>Building Expertise</th>
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<tr>
<td>Much of geometry, and algebra, utilizes the coordinate grid. Being able to plot points and interpret what is already on a graph will give students a head start into more advanced topics such as graphing lines and curves.</td>
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</tbody>
</table>
Nancy is the ringleader of a traveling circus. Due to her job, she often finds herself in new cities on a regular basis. When she tells her crew that they’ll be heading to Gridiron City next, one of the clowns excitedly tells her that’s where he is originally from. She asks if he would give them a few attractions to see, which he agrees to.

He gives them a quick sketch of five places they may be interested in going during their stay in the city. He said he based his sketch on the assumption that the circus will be roughly in the middle of the drawing. His points represent the following places:

- A: Movie Theater
- B: Grocery Store
- C: Sports Stadium
- D: Department Store
- E: Amusement Park

Nancy thanks the clown for his help, but as the crew looks at the sketch they have a bunch of questions. How far away are things? Is the distance between the stadium and the department store really that much further than the distance between the grocery store and movie theater?
After thinking about the questions for a bit, the clown believes that his sketch does have distances between the places preserved. He edits the sketch to include the grid overlay seen below.

Before they can ask him about distances or travel routes, he tells them that each gridline represents a regularly travelled road. When they designed the city, they wanted travel to be easy. In addition, the clown tells them that the distance between each gridline is a quarter of a mile. How far does this place the Movie Theater from the Grocery Store? How about the Stadium from the Department Store? What would the distance be from the circus to the Amusement Park? How would we go about that?
In order to find the distance from the circus to the Amusement Park, one of the trapeze artists suggests placing a blue dot in the center since the clown said he based his sketch on the circus being near the middle. In addition, the trapeze artist adds in the two darkened lines shown below, saying that it reminds him of the coordinate grid they learned in school.

Now, they can give each other directions and find distances using the grid.
While there is no scale on the map, you are told that the distance between gridlines is 40 miles.

Can you find the distance between Lancaster and Xenia?

Columbus and Norwood?

Zanesville and Kent?

Youngstown and Toledo?

Springfield and Portsmouth?
# BUYING FOR IRREGULARITIES

## Student/Class Goal
Many building materials are sold by the square foot or square yard. Students want to know how to find the amount they will need to purchase for different home projects.

## Time Frame
2 hours

## Standard Use Math to Solve Problems and Communicate
(Primary benchmarks in bold.)

## NRS EFL Levels 3-6

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## Data Analysis & Probability

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## Materials
- SmartPal kit – inserting a blank sheet of paper into the sleeves will give students a reusable sheet of paper that they can quickly try answers out on and erase without using up a pencil eraser. It’s quicker as well. For this lesson, we will want the grid handout provided inserted.
- Calculators (Optional)
- House Designs - Handout
- Yard Dimensions – Handout
- Volume of Composite Shapes – Handout

## Learner Prior Knowledge
- Vocabulary about properties and attributes of 2-D and 3-D figures
- Ability to work with arithmetic of whole numbers and decimals (for pi), as well as knowledge of raising to the powers of 2 and 3.

## Vocabulary
- **Perimeter** - Distance around a two-dimensional shape, for circles this is called the circumference.
- **Area** - The amount of space contained within the edges (or boundary for a circle) of a two-dimensional shape.
- **Volume** - The amount of space a three-dimensional object takes up. (Think about how much an object can hold.)
- **Surface area** - The total amount of space contained on all surfaces of an object. On a three-dimensional object, this is simply the sum of the areas of each face.

## Instructional Activities

### Note: Keep in mind that your class may not need to go through each of the parts below. Please pick and choose which elements to incorporate into your actual lesson based on what you know of your students. In addition, extra sample problems may need to be incorporated based upon your particular class. See the technology integration portion for a website that gives many example problems.

- **Part 1:** Measurements for perimeters, areas, and volumes are important when it comes to working around the home. When buying supplies for home projects, these measurements are often needed so that there is enough material and at the same time, to be cost efficient, not too much material. As such, let’s go over examples, and contexts, for these measurements.
  - **(I do)** For the beginning, students will need the SmartPals, a calculator (optional), and the House Designs handout.

Starting with the perimeter section, we will go through and do the first problem from each section for the I do portion,
and the second problem for the We do portion. Any remaining problems will be for student completion. First, students will need to know what perimeter is. Give them a basic definition. This could be discussion based. Students may already have an idea about perimeter being the distance around an object. While formulas exist for shapes, they are rather unnecessary for this particular concept and are probably more difficult than just remembering to sum all the side lengths. This would be a personal preference, though, and can be done as instructors choose.

1. **(Understand the problem)** Our first problem in the perimeter section tells us we will be building a fence around a dog house. The house is 6 feet by 8 feet and the fence will extend out an additional 8 feet from each side of the dog house.

2. **(Pick a strategy)** In order to add the sides, we must first find them. Let’s draw a picture to help visualize our scenario.

3. **(Implement the strategy)**

![Diagram of a dog house with fencing]

This tells us that the width of the enclosed space is 6 ft. plus 8 ft. on each side, so 6 + 8 + 8 = 22 ft. The length becomes 8 ft. plus 8 ft. on each side, or 8 + 8 + 8 = 24 ft. Then, since we have 2 widths and 2 lengths we have 22 + 22 + 24 + 24 = 92 feet of fencing needed.

4. **(Review)** If we wanted to use the “formula” for the perimeter of a rectangle, we would take:

\[
2 \cdot l + 2 \cdot w = 2 \cdot 22 + 2 \cdot 24 = 44 + 48 = 92,
\]

which checks our work. 92 feet of fencing is necessary.

- **(We do)** Have the class walk you through solving the second problem. We have a circle, so we do have a formula this time around as circles are the oddballs when it comes to formulas. We have to do something to get pi in there. The formula is \( C = 2\pi r \). For this example, that becomes: \( C = 2\pi (7.5) = 15\pi \approx 47.1 \) feet. **(Note:** While multiplying by 3.14 does give a concrete number that is probably easier for students to grasp and visualize, they should be familiar with leaving answers in terms of pi. Many tests will often ask for answers to be exact, or in terms of pi.)

- **(You do)** Have the students work on at least the 3rd problem alone. They could turn in the 4th problem as an assessment piece, or you can use it as further in-class practice.

Part 2

- **(I do)** We are now moving on to the area section. Here, formulas will become more important. But there are still just a few that we need to use. For instance, all triangles use the same formula, \( A = \frac{1}{2}bh \). In addition, since squares are a special case of rectangles, we can use \( A = l \cdot w \) for those as well. And finally, for circles, we will use \( A = \pi r^2 \).

1. **(Understand the problem)** Our first problem deals with finding the area of a rectangular space. We know the length and width are 8 ft. and 6 ft.

2. **(Pick a strategy)** Since we are working with a rectangle, we will be using the formula \( A = l \cdot w \).

3. **(Implement the strategy)** For our problem, \( A = l \cdot w \) will become \( A = 8 \cdot 6 = 48 \). Since we have a contextual problem, we need to go back and add in units. When we have area, we are multiplying two dimensions, so our answer should be in square units. Thus, Buster would have 48 ft\(^2\) of space.

4. **(Review)** We could check our answer using a grid model. In this case, if we take grid paper, we would mark off a width of 6 blocks and a length of 8 blocks. After boxing in that space, we would count the number of blocks in our area. We should come up with 48 blocks, which gives us an area of 48, which we found using the formula.

- **(We do)** The second problem asks us about the area of the pool. While we do have a 3-d object and it does have a volume, we are looking for area as we want to know how many people can be contained within the circular barrier of the pool. We don’t want to see just how many people we can stuff above/below the water line in the actual pool, but just who can fit if everyone was standing up. Have the class walk you through setting up the correct formula and finding an answer: \( A = \pi r^2 = \pi (7.5)^2 = \pi (56.25) = 56.25\pi \approx 176.6 \) ft\(^2\).

- **(You do)** Have students do the final problems on their own, with the option of using the final one as a homework assessment. **(Note:** Make sure students know that the height of the triangle must make a right angle with the base!)
Part 3: We are now moving on to irregular measurements. Here, this just means that our shapes are no longer the simple, basic figures we are used to. Instead, they are combinations of those basic shapes. Our goal here, then, is to break the shapes up into pieces that we know. For perimeter, this just means breaking it up so that we can find any unknown sides and then adding up all the outside edges.

- **(I do)**
  1. **(Understand the problem)** We want to find the perimeter of the desk shown. We are given many dimensions already, but not quite all of them. We do, know, however, that since we are working with perimeter, we just need to add up all the exterior lengths of the desktop.
  2. **(Pick a strategy)** Since we have an irregular shape, there really is no formula to use as a shortcut. Instead, we must first find any missing lengths and then just use addition to find the sum.
  3. **(Implement the strategy)** We need to find that missing length. Since the back portion of that piece is 72 inches, and the left portion of the L cuts across 24 inches, then the remaining length is 72 – 24 = 48 inches. Now, we just add up all of our lengths: 84 + 72 + 36 + 48 + 48 + 24 = 312 inches.
  4. **(Review)** Our problem was to find the distance around the outside of the desk. We don’t want to take the perimeter of the individual parts, because there are some sides of the rectangles that are no longer the “perimeter” of the entire shape. We want to take the lengths that we would walk if we were to walk around the outside of the desk. Those are the lengths we added, so our solution should make sense.

- **(We do)** Here, have students walk you through how to find the solution. We need to find the perimeter denoted by solid lines in this image:

![Diagram of desk with measurements](image)

The missing portion is that half circle that is shown by a solid line. We can use the circumference formula and then take half, but we found the circumference of the pool in the first problem. Thus, we just need to take half of that amount (47.1) to get 23.55. We then want to add that to all the sides, remembering to include 2.5 and 15 twice each. We then have 23.55 + 2.5 + 2.5 + 15 + 15 + 20 = 78.55 feet.

- **(You do)** Have the students do the final problem on the handout. It should be noted that the dotted, vertical line cuts the bottom edge in half. Also, drawing in the dotted green line below should help students see that there are two right triangles created at the top. We can then use the Pythagorean Theorem to find that the slanted sides of the top triangle are both 25 inches.

Part 4: When finding the area of irregular shapes, the need to break the overall shape down into more basic ones is even more important. For area, we use formulas to find the area of rectangles, triangles, and circles. Thus, we will need to break up more difficult figures into these basic shapes. Then, we will need to add or subtract to find the desired area.
1. **(Understand the problem)** For this first problem, we are asked to find the area between the fencing and the doghouse. In other words, the shaded area:

   ![Diagram of the first problem](image1)

2. **(Pick a strategy)** To do this, we will take the area of the entire fenced in space, and subtract the area of the doghouse.

3. **(Implement the strategy)** Since we found the width and length of the fenced in space to be 22 and 24 earlier, we know the area of that space is just their product, or 528 ft\(^2\). If we take out the area of the doghouse which had been found earlier to be 48 ft\(^2\) we end up with an area of 480 ft\(^2\).

4. **(Review)** There are other ways to go about this. We could divide the shaded area into smaller rectangles and find the area of each before taking their sums. No matter how we do it, we should arrive at the same answer. It makes sense as we knew the area had to be less than the entire enclosed space, but only by that minor space taken up by the doghouse.

- **(We do)** Have the class walk you through doing the second problem. In this case we are back to working with the pool and circles. We want to find the shaded area:

   ![Diagram of the second problem](image2)

   To do this, we will take the entire rectangle with dimensions 15x20 and take out the half circle that the pool occupies. Since we already know the area of the entire pool to be 176.6 ft\(^2\), then we know the area of half of that would be 88.3 ft\(^2\). We subtract that from the 300 ft\(^2\) area of the rectangle to get 211.7 ft\(^2\).

- **(You do)** Have the students do the final problem on the handout. Once again, they'll need to use the triangle at the top of the image to help them find the overall area. This time, it should be easier to add the rectangle at the bottom to the triangle at the top instead of subtracting anything.

Give them the Yard Dimensions handout to turn in for you to check their understanding.

**Note:** Due to time constraints, volume has been left out. However, if time permits, you can go over contextual examples of volume and irregular volume. After doing perimeter/area for basic and irregular shapes, volume follows the same idea. You want to break up the objects into known 3-dimensional figures. It should also be noted that volume formulas for most encountered shapes can be simplified into two formulas:

- **Prisms** (including cubes) and **Cylinders**: \( V = B \cdot h \), where \( B \) = the area of the base
- **Pyramids** and **Cones**: \( V = \left( \frac{1}{3} \right) B \cdot h \)

This may help students who have a hard time remembering multiple formulas. Sample problems can be found in the Volume of Composite shapes handout.
Assessment/Evidence (based on outcome)
Each of the You do steps will serve as assessment. The instructor should be able to gauge understanding by having different students provide their solutions and explanations of how they arrived at that solution. In addition, during the we do steps, instructors should be encouraging all students to participate in the discussion. The ability to provide input in these discussions will help the teacher gauge each student’s mastery of the concepts.

You can have students turn in any 4th problems on the House Designs handout and/or the Yard Dimensions handout.

Teacher Reflection/Lesson Evaluation
Not yet completed

Next Steps
After this lesson, a follow-up may be to plot shapes or partial shapes onto coordinate grids to reinforce their properties or to see if students can complete the pictures. Another possibility is to go further into formulas of more complicated shapes, especially that of surface area.

Technology Integration
http://www.geogebra.org
    Online graphing tool. Tutorials can be found here: http://wiki.geogebra.org/en/Tutorial:Main_Page, or you can search Google for “GeoGebra tutorials”. This tool can be utilized to create figures and find their areas and perimeters.

Purposeful/Transparent
Since students want to be able to apply perimeter, area, and volume to contextual situations, instructors will cover these topics theoretically, but also with applied examples from around the home.

Contextual
Many topics in geometry give way to application problems with ease and this topic is no different. All shapes found in the real world have perimeter, area, and/or volume depending on their dimensions. These topics are useful to everyone as well. In a house, do you have enough room for your furniture? Is there enough space in your bedroom for your bed, desk, and dresser? Is there enough volume space in your car for all the things you need to haul? How much fencing is needed to enclose the backyard?

Building Expertise
Perimeter, area, and volume can be introduced with or without equations/formulas. If they are introduced before students are completely comfortable with equations, they may help reinforce how to solve equations and how to find answers using formulas. This topic gives a concrete and visual representation to some often used formulas.
House Designs

**Perimeter**

1. There are plans to build a doghouse with the dimensions above. If we wanted to fence in a space that is an additional 8 feet from each side of the doghouse, what amount of fencing would we need to do so?

2. The family decides to put a pool up in the backyard. Since they have children, they want to check the price of some safety fencing that could go up around the outside of the pool as seen above. If the pool is 15 feet across and the fencing is sold by length, how much fencing should be purchased?

3. This new house they are designing has a half bathroom on the first floor that is rather small. To save space, they decide to put the counter on the sink in the corner. For a modern design, they go with the triangular shape shown above. Find the length of the front of the sink as well as the total perimeter of the counter.

4. The owners want to put the above painting by Escher in a frame. It measures 26 inches in width by 22 inches in height. If wood is bought by total length, how much should be purchased?
Area

1. If the above doghouse is built for Buster, how much area will he have to move about inside the house?

2. Once again, the family wants to put in the above ground pool seen above. They want it to be 15 feet across. What will be the area that people can swim in?

3. After installing the new counter in the half bathroom, how much area is being used up based on the dimensions above?

4. After buying materials to frame above picture from our previous problem, the frame extended the picture by 2 inches on all sides. How much space on the wall will the framed picture take up?
1. In the office, the owners want to put the desk shown above. Given those dimensions, find the distance around the entire outside of the desk.

2. Deciding they’d rather have a deck surrounding part of the pool than the safety fence going all the way around, the above design is decided upon. The deck will start at the widest portion of the pool. Without the cutout portion where the pool is, the dimensions of the deck are 20 feet by 15 feet. The fencing along the bottom, below the deck portion, surrounds the entire deck (including the portion between the pool wall and the actual deck). What length of fencing would be necessary to complete that bottom portion?

3. The TV stand above is designed to go in the corner of a room. The shape on top is similar to that of a home plate in baseball. If you know the given dimensions, what is the perimeter of the TV stand top?
Irregular Area

1. Going back to the fenced in area we created in the first part of this activity, how much area will Buster have to play in between the fence and his house?

2. We are once again working with the deck pictured above. The deck will start at the widest portion of the pool. Without the cutout portion where the pool is, the dimensions of the deck are 20 feet by 15 feet. What is the area of the portion of the deck that can be walked on?

3. Once again, we are working with the TV stand and the dimensions above. This time, what is the area of the TV stand top?
House Designs

Perimeter

1. There are plans to build a doghouse with the dimensions above. If we wanted to fence in a space that is an additional 8 feet from each side of the doghouse, what amount of fencing would we need to do so?

92 feet of fencing is needed

2. The family decides to put a pool up in the backyard. Since they have children, they want to check the price of some safety fencing that could go up around the outside of the pool as seen above. If the pool is 15 feet across and the fencing is sold by length, how much fencing should be purchased?

About 47.1 feet of fencing is needed

3. This new house they are designing has a half bathroom on the first floor that is rather small. To save space, they decide to put the counter on the sink in the corner. For a modern design, they go with the triangular shape shown above. Find the length of the front of the sink as well as the total perimeter of the counter.

The perimeter of the piece is 68.28 inches

4. The owners want to put the above painting by Escher in a frame. It measures 26 inches in width by 22 inches in height. If wood is bought by total length, how much should be purchased?

96 inches of frame is needed
Area

1. If the above doghouse is built for Buster, how much area will he have to move about inside the house?
   \[ 48 \text{ ft}^2 \]

2. Once again, the family wants to put in the above ground pool seen above. They want it to be 15 feet across. What will be the area that people can swim in?
   \[ 176.6 \text{ ft}^2 \]

3. After installing the new counter in the half bathroom, how much area is being used up based on the dimensions above?
   \[ 200 \text{ in}^2 \]

4. After buying materials to frame above picture from our previous problem, the frame extended the picture by 2 inches on all sides. How much space on the wall will the framed picture take up?
   \[ 624 \text{ in}^2 \]
Irregular Perimeter

1. In the office, the owners want to put the desk shown above. Given those dimensions, find the distance around the entire outside of the desk.

312 inches

2. Deciding they’d rather have a deck surrounding part of the pool than the safety fence going all the way around, the above design is decided upon. The deck will start at the widest portion of the pool, with 2.5 feet of extra space to either side. Without the cutout portion where the pool is, the dimensions of the deck are 20 feet by 15 feet. The fencing along the bottom, below the deck portion, surrounds the entire deck (including the portion between the pool wall and the actual deck). What length of fencing would be necessary to complete that bottom portion?

78.55 feet

3. The TV stand above is designed to go in the corner of a room. The shape on top is similar to that of a home plate in baseball. If you know the given dimensions, what is the perimeter of the TV stand top?

120 inches
1. Going back to the fenced in area we created in the first part of this activity, how much area will Buster have to play in between the fence and his house?

\[480 \text{ ft}^2\]

2. We are once again working with the deck pictured above. The deck will start at the widest portion of the pool. Without the cutout portion where the pool is, the dimensions of the deck are 20 feet by 15 feet. What is the area of the portion of the deck that can be walked on?

\[211.7 \text{ ft}^2\]

3. Once again, we are working with the TV stand and the dimensions above. This time, what is the area of the TV stand top?

\[900 \text{ in}^2\]
Yard Dimensions

The Matthews family was looking into buying a new house. With two children, they knew they would want a yard for them to play in. After looking at the house below, they realized they would need to build a fence in order to block the kids from the creek and the crop of trees. If they built the fence outlined below, how many feet of material would be needed and what be the area of the enclosed yard that the kids could play in? (Hint: The area the house itself takes up would not be included as yard since you cannot play on that patch of land.)
Yard Dimensions

The Matthews family was looking into buying a new house. With two children, they knew they would want a yard for them to play in. After looking at the house below, they realized they would need to build a fence in order to block the kids from the creek and the crop of trees. If they built the fence outlined below, how many feet of material would be needed and what be the area of the enclosed yard that the kids could play in? (Hint: The area the house itself takes up would not be included as yard since you cannot play on that patch of land.)

Overall Area of yard = 3866 m²
Volume of Composite Shapes

1. If some homeowners decide to build a garage with the dimensions below, how much space will they have inside the structure?

```
Volume = (1/2 * 2 * 1) + (2 * 2 * 3) = 1 + 12 = 13 m³
```

2. If a new factory is built with the dimensions as given in the image below, what would be the overall volume of the new building?

```
Volume = (1/2 * 7.5 * 20) + (30 * 20 * 3) = 75 + 1800 = 1875 m³
```
1. If some homeowners decide to build a garage with the dimensions below, how much space will they have inside the structure?

\[ 18.75 \text{ m}^3 \]

2. If a new factory is built with the dimensions as given in the image below, what would be the overall volume of the new building?

\[ 3,300 \text{ m}^3 \]
## HOUSE TRANSFORMATIONS

### Outcome (lesson objective)
Students will be able to complete partial 2-d figures on the coordinate plane as well as represent and analyze figures using the coordinate plane. Finally, students will be able to graph the results of translations, reflections, and rotations.

### Time Frame
4 hours

### Standard
Use Math to Solve Problems and Communicate

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### Materials
SmartPal kit – inserting a blank sheet of paper into the sleeves will give students a reusable sheet of paper that they can quickly try answers out on and erase without using up a pencil eraser. It’s quicker as well. For this lesson, we will want the grid handout provided inserted.
Patty Paper
Slide N’ Measure (optional)
Symmetries and Transformation - Handout
Room Layouts - Handout

### Learner Prior Knowledge
Mathin’ Around the House lesson
Students should be familiar with plotting points on a graph.
Coordinate plane vocabulary.
Ability to distinguish and measure angles.

### Vocabulary

**Review**
- **Reflection symmetry** – A shape can be folded on any line (reflected over any line) and the two halves will be congruent.
- **Rotation symmetry** - A shape can be rotated between 0° and 360° and be congruent to the original shape.
- **Point symmetry** - A shape is congruent in two opposite directions of a point. (Note: This is a little hard to explain without a graphical representation. One can be found here: [http://www.regentsprep.org/Regents/math/geometry/GT1a/Psymmet.htm](http://www.regentsprep.org/Regents/math/geometry/GT1a/Psymmet.htm) and there is another in the technology integration section.)

**New**
- **Transformation**: A function that changes an object in form, shape, or appearance.
- **Translation**: A transformation consisting of a constant shift with no rotation or stretching.
- **Reflection**: A transformation that exchanges all points of an object with their mirror image.
- **Rotation**: A transformation that turns an object around a fixed point.
**Instructional Activities**

**Note:** Keep in mind that your class may not need to go through each of the parts below. Please pick and choose which elements to incorporate into your actual lesson based on what you know of your students. In addition, extra sample problems may need to be incorporated based upon your particular class. See the technology integration portion for a website that gives many example problems.

Part 1: Since our overall goal here is to be able to take a room and (mathematically) transform the items in it so that the layout is completely new, we want to know what the different transformations do as well as what the different symmetries mean for each object. Make sure everyone has a handout on Symmetries and Transformations. This will help with the vocabulary for the rest of the lesson. **(Note: A possible point for discussion would be what sort of shapes have point symmetry? Since we’re looking for objects that are the same on both sides of the line, we need objects that already have 180° (or 2nd order) rotational symmetry.)**

We are going to start with checking symmetries of different shapes as this will help us with our transformations. It will help us know whether a rectangular coffee table, for example, will “look” the same once we rotate it. It will also tell us if we need to worry about certain pieces of furniture having a “front” side. If we reflect that, will the front stay in the front?

1. **(I do)** Using a blank coordinate grid (2 can be found at the end of the Symmetries and Transformations handout), let’s explore shapes and symmetries. If we think back to the shapes we know, we can work with squares, rectangles, and triangles rather easily. For our furniture, we will most likely have squares and rectangles. If needed, go back and review any properties of these shapes such as congruent sides, parallel sides, and perpendicular sides. Now, let’s say we have the following graph:

![Graph of a rectangle]

We have two segments, one that goes between (4,3) and (4,7) and one that goes between (4,3) and (7,3). We are told that this is two sides of a rectangle and are asked to complete the rectangle.

1. **(Understand the problem)** Given the graph, we must find the other 2 sides of our shape. We know that rectangles have opposite sides parallel and congruent and that there are four right angles. We also know that rectangles have two lines of reflection symmetry along the midpoints of the sides.
2. **(Pick a strategy)** We have a few options here. One is to find the length of one given side so that we know the length of the opposite side. Since we know we have only horizontal and vertical lines, it will be easy to make perpendicular/parallel lines.
3. **(Implement the strategy)** If we take the vertical side, we see its length is 4. Thus, I need a parallel side with length 4. This side will start from the other endpoint of the horizontal, bottom side. Once we draw that in, we have three of the 4 sides of our rectangle. Our final side just needs to attach the two free endpoints.
4. **(Review)** Another option in part 2 would have been to use our patty paper to use reflection symmetry. Let’s use that method to check our answer. If we take a piece of patty paper and trace the “L” shape we started with onto it, we can now work with it. Picking it up, if we fold along the midpoint of one of the sides (so that both pieces of the segment match up on either side of our fold), we have found a line of reflection symmetry in our shape. Now, we must just copy the complete side from one side of our fold to the other. We can then do this once more folding along the midpoint of the other sides.
Answer:

- *(We do)* Have the students walk you through what to do for the following image if you are only told that this is the bottom-most side of a square:

Since the segment goes from (-8,-6) to (-3,-6), we know the side length for our square is 5. Since we have a horizontal line segment, perpendicular lines will be vertical. We need vertical segments from each endpoint of the given segment that are each 5 units in length. Once we draw those in, we will only need to attach the two free endpoints at the top of these vertical segments.
(You do) Have the students work on what to do to complete this image. You are told that the shape has both rotational and reflection symmetry. (The segment extends from (-2,2) to (2,2).)
Part 3: Our next transformation is a reflection. If a translation = a slide, a reflection = a flip.

- **(I do)** Looking at the Room Layout handout, show students how to do a reflection using the TV stand from the bedroom.

  1. **(Understand the problem)** Due to another mistake on the part of the movers, the Harrisons need to move their entertainment center up 6 spaces. On a coordinate plane, the vertical direction corresponds to the y-value, or the second number in our ordered pair. We did not change anything in the horizontal, or x-direction. So, it seems like our points should change by adding 6 (moving upwards - the positive direction - along the y-axis) to the y-values. If we look at how our corner coordinates change, we see that we added 6 to the y-values for all four corners. Thus, our answer makes sense.

  - **(We do)** Have the students walk you through doing another piece of furniture. For example, if choosing the coffee table, they will first need to find the corner points (if they haven’t already). For the coffee table, those are (-3,-1.5), (3,-1.5), (3,-3.5), and (-3,-3.5). After copying the coffee table to the patty paper and shifting it up 6, we should have new coordinates at (-3,4.5), (3,4.5), (3,2.5), and (-3,2.5).

  - **(You do)** Have students complete the other furniture pieces (at least one) on their own.

Part 3: Our next transformation is a reflection. If a translation = a slide, a reflection = a flip.

- **(I do)** Looking at the Room Layout handout, show students how to do a reflection using the TV stand from the bedroom.

  1. **(Understand the problem)** Due to another mistake on the part of the movers, the Harrisons need to move their furniture around in their bedroom as well. Here, the Harrisons want their bed on the other side of the room. However, if they just translate it, as they did with the furniture in the living room, the bed will be facing the wall. They want the headboard against the wall, instead, so a translation will not work. We want to change the coordinates of every piece of furniture, starting with the TV stand, so that the objects in the room are directly across the room from where they are now. Since we want things to be “across” from each other, we want to flip, or reflect, the furniture in this room. The coordinates of the TV stand are currently (-3,10), (3,10), (3,7), and (-3,7). **(Note):** While a reflection will flip the room as we are describing, we need to be a little lenient of the fact that an actual “flip” would place all the objects facedown. If students bring this up, and ask why we didn’t just rotate the furniture, that could lead into Part 4.)
2. (Pick a strategy) Since we have the patty paper as a tool, we will use it to copy the object we are moving and then physically manipulate its position.

3. (Implement the strategy) If we overlay the piece of patty paper over the “Before” image, we can trace the TV stand. Since we need a line of reflection this time, let’s also trace the x-axis since that is what we are reflecting everything over. Next, pick up the patty paper and fold it along this line of reflection. Lay the now folded piece of patty paper back where it was. Our x-axis should be lined up with the folded axis on our patty paper, and the patty paper (both halves as it is folded) should be over top of the bottom half of the graph. This will show us where the dresser will go after reflecting it. Note its position and place it on the blank graph. Our new coordinates are: (-3,-10), (3,-10), (3,-7), and (-3,-7).

4. (Review) Let’s once again think about this from a graphing perspective. Once more, we changed the furniture’s vertical position but not its horizontal position. We also changed the vertical orientation, or whether it was facing top or bottom, but not its horizontal orientation. As such, our x-values should not change. And between our starting coordinates and our ending coordinates, we see this to be the case. The y-values, however, should change. We flipped over the x-axis. As reflections preserve distance (as stated on the handout Symmetries and Transformations), then whatever distance we had between the x-axis and a point before reflecting it should still be the distance between the x-axis and that point after reflecting it. So a point with a y-value of 10 prior to a reflection over the x-axis should have a y-value of -10 afterwards. Comparing our points, we see that all of our y-values have this property intact, so our new coordinates do make sense.

- (We do) Have the students walk you through doing another piece of furniture. For example, if choosing the bed, they will first need to find the corner points (if they haven’t already). For the bed, those are (-5,3), (4,3), (4,-9), and (-5,-9). After copying the bed to the patty paper and reflecting it, we should have new coordinates at (-5,-3), (4,-3), (4,9), and (-5,9).
- (You do) Have students complete the other furniture pieces (at least one) on their own.

Part 4: Our final transformation is a rotation. The word to help remember rotations would be “turn.” For this activity, students can use the Slide N’ Measures if they need to measure what a 90° angle looks like, however, this is a 1/4 turn, so it may not be necessary.

- (I do) Looking at the Room Layout handout, show students how to do a rotation using the desk from the office.
  1. (Understand the problem) The movers made a final mistake and the Harrisons need to move their furniture around one more time. In the office, Mr. Harrison wants to shift everything around one wall to the left (counterclockwise). If he shifts everything using translations, nothing will face the right way. If he uses a reflection, some things will move two walls, while others will not change walls at all. Instead, he will use a rotation. If he wants to shift everything one wall to the left in a square room, that will be a 90° rotation.

- (Pick a strategy) Since we have the patty paper as a tool, we will use it to copy the object we are moving and then physically manipulate its position.

- (Implement the strategy) If we overlay the piece of patty paper over the “Before” image, we can trace the desk. Next, we are told that everything must rotate 90° counterclockwise about the center of the room. This makes the center of the room our point of rotation. If we place our finger on the origin, or “center of the room”, we can then rotate the patty paper about this point while leaving the original graph in place. Once we move the paper a quarter turn (90°) counterclockwise, note the new position of the desk. Our new coordinates are: (-10,-4), (-10,4), (-6,4), and (-6,-4).

- (Review) This time, we clearly see we changed both the x- and y-values, which makes sense as we changed the orientation and positioning of our object. We can use our Slide N’ Measure’s to check our rotation, though. If we make a line from any point on the desk before the rotation to the origin, and then make a line from that point’s position in our rotated image to the origin, we will have formed an angle. That angle should be 90° when we measure it. If it is, we have done our rotation correctly.

- (We do) Have the students walk you through doing another piece of furniture. For example, if choosing the bookcase, they will first need to find the corner points (if they haven’t already). For the bookcase, those are (-10,7), (-6,7), (-6,-7), and (-10,-7). After copying the coffee table to the patty paper and shifting it up 6, we should have new coordinates at (-7,-10), (-7,-6), (7,-6), and (7,-10).
- (You do) Have students complete the other furniture pieces (at least one) on their own.

**Note:** A question after these activities might be whether or not the students were able to find the coordinates using symmetry (as in the first activity) or if they just looked at the graph to find all four points. In addition, if time permits, it may be a good idea to see if students can find an object’s translation or reflection using just coordinates and no graph. Or, if they can figure out the rules for how the coordinates change for rotations of different angles around the origin.
### Assessment/Evidence (based on outcome)
Each of the you do steps will serve as assessment. The instructor should be able to gauge understanding by having different students provide their solutions and explanations of how they arrived at that solution. In addition, during the we do steps, instructors should be encouraging all students to participate in the discussion. The ability to provide input in these discussions will help the teacher gauge each student’s mastery of the concepts.

To do a final check on each student’s grasp of the concepts, have them work individually on the following to turn into you (if there is not sufficient time at the end of the lesson, have them bring it to turn in at the beginning of the next class): Have each student create their own furniture layout for a room in their house. This can be any room in the house, not necessarily the living room as in the lesson. Their room must have at least three pieces of movable furniture. The student must then move one piece of furniture using a translation, another piece using a reflection, and a final piece using a rotation. Have them turn in the following:

1. On graph paper, they should have the initial layout of their room. On that same graph, they should notate the new positions of the three moved pieces of furniture (the markers would come in handy here).
2. A description of each transformation. For example: a translation up 2 units and to the left 3 units, a reflection over the x-axis, or a rotation 270° about the origin.
3. The coordinates of the corners of the pieces of furniture they’ll be moving before and after the transformation.

### Teacher Reflection/Lesson Evaluation
Not yet completed

### Next Steps
Lines of reflection other than the two axes and points of rotation other than the origin. In addition, transformations can be done solely algebraically using functions. After working with the coordinate plane, it would be a good time to introduce graphing equations and functions.

### Technology Integration
  - Transformation activities on GeoGebra (below)
- [http://www.geogebra.org/webstart/geogebra.html](http://www.geogebra.org/webstart/geogebra.html)
  - Online graphing tool
  - Online room designer

### Purposeful/Transparent
Students want to be able to apply the concepts of translations, rotations, and reflections into a real-world problem. Teachers will model and then guide them in using these concepts with respect to moving objects around the home.

### Contextual
Transformations are normally difficult to put into a real-world context for many students. Many do not see the application outside of the coordinate plane. However, moving objects, like furniture, is one way to put this in context. Movers and contractors use the idea of rotations and translations and the fact that length is preserved when moving objects around inside a house to make sure they have enough room. Artists often use all three transformations in works of art. Basketball, pool, and mini golf all use the concept of a reflection and its ability to preserve angles when the ball bounces off the backboard or the railing.

### Building Expertise
In geometry and algebra, the coordinate plane is extremely important. This lesson should further each student’s familiarity and comfort with the coordinate plane. In addition, recognizing that one shape is just a transformation of another shape will come in handy when working with similar and congruent shapes.
**Symmetries and Transformations**

**Symmetry in the Coordinate Plane**

Symmetry is a geometric property of an object. Two shapes are said to be symmetric if one is obtained from performing transformations on the other. However, shapes can have internal symmetry as well. In this respect, there are three types of symmetry.

**Reflection Symmetry**

In this type of symmetry, a line can be drawn across a shape so that if we fold our shape along that line, the two sides will match up perfectly. This line that we fold on is called the line of reflection. It is possible for a shape to have more than one line of reflection, one line of reflection, or no lines of reflection. In the images below, lines of reflection, if they exist, are shown for different objects.

**Rotation Symmetry**

For rotational symmetry, the idea is that a shape will be exactly the same as its original image if we rotate it around a point. For shapes, often we will choose the center point for this. Then, if we hold the center point, and rotate the shape 360°, how many times will the shape be congruent to the original? For example, in the image below, if we rotate the equilateral triangle around its center, it will match up to the original three times and have rotational symmetry of order 3. The pentagon will do this five times, so will have order 5. Any shape will do it once, namely when we hit 360°, so any figure has
rotational symmetry of order 1. However, we are more interested in shapes that are the same before we get all the way around to 360°.

Point Symmetry

Point symmetry is slightly more difficult to grasp than the first two. For point symmetry, we plot a point either within our shape or outside of it. If we draw a line that intersects that point, then everything that intersects the line on one side of the point must intersect that line on the other side of the point.

Each object above has point symmetry. If we were to draw in the center point for each object, any line we draw through that point will intersect something at the same distance on both sides of the line. For example:

The line intersects both the inside and outside of the circle at the same distance from the point on either side of the line. On the other hand, the following shape does not have point symmetry:
as the figure does not intersect at the same distance on both sides of the line.

**Transformations in the Coordinate Plane**

In general, a transformation takes an object and moves it to a new position. If we move a point P, we typically label the point as P’ after the transformation.

**Translations**

- A translation “slides” an object to a new position in the coordinate plane.
- Translations preserve the following properties:
  - Distance: Lengths in the original are preserved in the translation. (For example: A side length in equilateral triangle ABC is 5. When we translate this, we will now have equilateral triangle A’B’C’ with side length 5.)
  - Angle measures
  - Parallel lines will still be parallel if translated.
  - Midpoints
  - Co-linearity
  - Orientation: The order of labeling is preserved.

**Reflections**

- A reflection is a “flip” or a mirror image of an object over a line in the coordinate plane.
- The distance from the object being reflected to the line of reflection is the same as the distance from the reflected object to the line of reflection.
- Reflections preserve the following properties:
  - Distance
  - Angle measures
  - Parallel lines will still be parallel if translated.
  - Midpoints
  - Co-linearity
- It is important to note that while the two objects will be congruent, the labeling order is *not* preserved in reflections. As this is a mirror image, the labeling order is reversed.
• The two most common lines of reflection in the coordinate plane are the $x$-axis and the $y$-axis. It is possible to reflect over any line, however.

**Rotations**

• A rotation is a “turning” of an object about a point of rotation in the coordinate plane.
• The angle formed between the original point and its rotated image is the angle of rotation. (Note: if the object rotated is a shape, the angle of rotation will be the same between all corresponding points—the original point and its rotated image.)
• Angles measured in the counter-clockwise direction are positive while angles measured in the clockwise direction are negative.
• Rotations preserve the following properties:
  o Distance
  o Angle measures
  o Parallel lines will still be parallel if translated.
  o Midpoints
  o Co-linearity
  o Labeling order is preserved.
• The most common point of rotation is the origin and the three most common angles of rotation in the coordinate plane are $90^\circ$, $180^\circ$, and $270^\circ$. With a protractor, however, we could do a rotation of any angle.
Room Layouts

After moving into a new house, the Harrisons realize the moving company did not put their furniture where they wanted it. Help them figure out what their rooms will look like with the following transformations. (**The distance between gridlines is 1/2 foot.**)

Living Room (Before)

Find the four corner points of each piece of furniture in the diagram below:

<table>
<thead>
<tr>
<th>Piece of Furniture</th>
<th>Corner 1 Coordinates</th>
<th>Corner 2 Coordinates</th>
<th>Corner 3 Coordinates</th>
<th>Corner 4 Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entertainment Center</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coffee Table</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>End Table</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sofa</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>End Table</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Diagram of the living room layout](image.png)

1. **Entertainment Center**
2. **Coffee Table**
3. **End Table**
4. **Sofa**
Living Room (Translation)
In order to create a space to walk into the home from the door, the Harrisons decide to move all the furniture back 3 feet until the Entertainment center is against the wall. Find the four corner points of each piece of furniture in the new setup:

<table>
<thead>
<tr>
<th>Piece of Furniture</th>
<th>Corner 1 Coordinates</th>
<th>Corner 2 Coordinates</th>
<th>Corner 3 Coordinates</th>
<th>Corner 4 Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entertainment Center</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coffee Table</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>End Table</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Sofa</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>End Table</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Bedroom (Before)
Find the four corner points of each piece of furniture in the diagram below:

<table>
<thead>
<tr>
<th>Piece of Furniture</th>
<th>Corner 1 Coordinates</th>
<th>Corner 2 Coordinates</th>
<th>Corner 3 Coordinates</th>
<th>Corner 4 Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV Stand</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bed</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Night Stand</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dresser</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Upon entering the bedroom, the Harrisons once again see room for change. This time they know they don’t want the bed to be on the same side of the room as the door. They want the TV stand where the bed is and the bed where the TV stand is. Mrs. Harrison says this seems like a reflection. If they reflect all the furniture over the horizontal “center line” of the room, find the four corner points of each piece of furniture in the new layout:

<table>
<thead>
<tr>
<th>Piece of Furniture</th>
<th>Corner 1 Coordinates</th>
<th>Corner 2 Coordinates</th>
<th>Corner 3 Coordinates</th>
<th>Corner 4 Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV Stand</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bed</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Night Stand</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dresser</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Office (Before)

Find the four corner points of each piece of furniture in the diagram below:

<table>
<thead>
<tr>
<th>Piece of Furniture</th>
<th>Corner 1 Coordinates</th>
<th>Corner 2 Coordinates</th>
<th>Corner 3 Coordinates</th>
<th>Corner 4 Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desk</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bookcase</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lamp</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chair</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Diagram with labeled coordinates for each piece of furniture:
Office (Rotation)

Just like the previous two rooms, the Harrisons want to make some updates in the office as well. Mr. Harrison does not like the bookcase in front of the window taking up all the light. He wants to put his desk there, instead. In fact, he seems to just want to shift everything around the room by one wall. Find the four corner points of each piece of furniture in the new layout if they rotate everything around the center of the room counterclockwise by 90 degrees:

<table>
<thead>
<tr>
<th>Piece of Furniture</th>
<th>Corner 1 Coordinates</th>
<th>Corner 2 Coordinates</th>
<th>Corner 3 Coordinates</th>
<th>Corner 4 Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desk</td>
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</tr>
<tr>
<td>Chair</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Room Layouts

After moving into a new house, the Harrisons realize the moving company did not put their furniture where they wanted it. Help them figure out what their rooms will look like with the following transformations. (**The distance between gridlines is 1/2 foot.**)

Living Room (Before)

Find the four corner points of each piece of furniture in the diagram below:

<table>
<thead>
<tr>
<th>Piece of Furniture</th>
<th>Corner 1 Coordinates</th>
<th>Corner 2 Coordinates</th>
<th>Corner 3 Coordinates</th>
<th>Corner 4 Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entertainment Center</td>
<td>(-10,4)</td>
<td>(10,4)</td>
<td>(10,0)</td>
<td>(-10,0)</td>
</tr>
<tr>
<td>Coffee Table</td>
<td>(-3,-1.5)</td>
<td>(3,-1.5)</td>
<td>(3,-3.5)</td>
<td>(-3,-3.5)</td>
</tr>
<tr>
<td>End Table</td>
<td>(-9,-6)</td>
<td>(-6,-6)</td>
<td>(-6,-9)</td>
<td>(-9,-9)</td>
</tr>
<tr>
<td>Sofa</td>
<td>(-6,-6)</td>
<td>(6,-6)</td>
<td>(6,-10)</td>
<td>(-6,-10)</td>
</tr>
<tr>
<td>End Table</td>
<td>(6,-6)</td>
<td>(9,-6)</td>
<td>(9,-9)</td>
<td>(6,-9)</td>
</tr>
</tbody>
</table>
Living Room (Translation)
In order to create a space to walk into the home from the door, the Harrisons decide to move all the furniture back 3 feet until the Entertainment center is against the wall. Find the four corner points of each piece of furniture in the new setup:

<table>
<thead>
<tr>
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<th>Corner 3 Coordinates</th>
<th>Corner 4 Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entertainment Center</td>
<td>(-10,10)</td>
<td>(10,10)</td>
<td>(10,6)</td>
<td>(-10,6)</td>
</tr>
<tr>
<td>Coffee Table</td>
<td>(-3,4.5)</td>
<td>(3,4.5)</td>
<td>(3,2.5)</td>
<td>(-3,2.5)</td>
</tr>
<tr>
<td>End Table</td>
<td>(-9,0)</td>
<td>(-6,0)</td>
<td>(-6,-3)</td>
<td>(-9,-3)</td>
</tr>
<tr>
<td>Sofa</td>
<td>(-6,0)</td>
<td>(6,0)</td>
<td>(6,-4)</td>
<td>(-6,-4)</td>
</tr>
<tr>
<td>End Table</td>
<td>(6,0)</td>
<td>(9,0)</td>
<td>(9,-3)</td>
<td>(6,-3)</td>
</tr>
</tbody>
</table>
Bedroom (Before)
Find the four corner points of each piece of furniture in the diagram below:

<table>
<thead>
<tr>
<th>Piece of Furniture</th>
<th>Corner 1 Coordinates</th>
<th>Corner 2 Coordinates</th>
<th>Corner 3 Coordinates</th>
<th>Corner 4 Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV Stand</td>
<td>(-3,10)</td>
<td>(3,10)</td>
<td>(3,7)</td>
<td>(-3,7)</td>
</tr>
<tr>
<td>Bed</td>
<td>(-5,3)</td>
<td>(4,3)</td>
<td>(4,-9)</td>
<td>(-5,-9)</td>
</tr>
<tr>
<td>Night Stand</td>
<td>(-8,-5)</td>
<td>(-5,-5)</td>
<td>(-5,-8)</td>
<td>(-8,-8)</td>
</tr>
<tr>
<td>Dresser</td>
<td>(6,-5)</td>
<td>(10,-5)</td>
<td>(10,-10)</td>
<td>(6,-10)</td>
</tr>
</tbody>
</table>
Upon entering the bedroom, the Harrisons once again see room for change. This time they know they don’t want the bed to be on the same side of the room as the door. They want the TV stand where the bed is and the bed where the TV stand is. Mrs. Harrison says this seems like a reflection. If they reflect all the furniture over the horizontal “center line” of the room, find the four corner points of each piece of furniture in the new layout:

<table>
<thead>
<tr>
<th>Piece of Furniture</th>
<th>Corner 1 Coordinates</th>
<th>Corner 2 Coordinates</th>
<th>Corner 3 Coordinates</th>
<th>Corner 4 Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV Stand</td>
<td>(-3,-10)</td>
<td>(3,-10)</td>
<td>(3,-7)</td>
<td>(-3,-7)</td>
</tr>
<tr>
<td>Bed</td>
<td>(-5,-3)</td>
<td>(4,-3)</td>
<td>(4,9)</td>
<td>(-5,9)</td>
</tr>
<tr>
<td>Night Stand</td>
<td>(-8,5)</td>
<td>(-5,5)</td>
<td>(-5,8)</td>
<td>(-8,8)</td>
</tr>
<tr>
<td>Dresser</td>
<td>(6,5)</td>
<td>(10,5)</td>
<td>(10,10)</td>
<td>(6,10)</td>
</tr>
</tbody>
</table>
Find the four corner points of each piece of furniture in the diagram below:

<table>
<thead>
<tr>
<th>Piece of Furniture</th>
<th>Corner 1 Coordinates</th>
<th>Corner 2 Coordinates</th>
<th>Corner 3 Coordinates</th>
<th>Corner 4 Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desk</td>
<td>(-4,10)</td>
<td>(4,10)</td>
<td>(4,6)</td>
<td>(-4,6)</td>
</tr>
<tr>
<td>Bookcase</td>
<td>(-10,7)</td>
<td>(-6,7)</td>
<td>(-6,-7)</td>
<td>(-10,-7)</td>
</tr>
<tr>
<td>Lamp</td>
<td>(-6,-8)</td>
<td>(-4,-8)</td>
<td>(-4,-10)</td>
<td>(-6,-10)</td>
</tr>
<tr>
<td>Chair</td>
<td>(-3,-6)</td>
<td>(2,-6)</td>
<td>(2,-10)</td>
<td>(-3,-10)</td>
</tr>
</tbody>
</table>
Office (Rotation)
Just like the previous two rooms, the Harrisons want to make some updates in the office as well. Mr. Harrison does not like the bookcase in front of the window taking up all the light. He wants to put his desk there, instead. In fact, he seems to just want to shift everything around the room by one wall. Find the four corner points of each piece of furniture in the new layout if they rotate everything around the center of the room counterclockwise by 90 degrees:

<table>
<thead>
<tr>
<th>Piece of Furniture</th>
<th>Corner 1 Coordinates</th>
<th>Corner 2 Coordinates</th>
<th>Corner 3 Coordinates</th>
<th>Corner 4 Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desk</td>
<td>(-10,-4)</td>
<td>(-10,4)</td>
<td>(-6,4)</td>
<td>(-6,-4)</td>
</tr>
<tr>
<td>Bookcase</td>
<td>(-7,-10)</td>
<td>(-7,-6)</td>
<td>(7,-6)</td>
<td>(7,-10)</td>
</tr>
<tr>
<td>Lamp</td>
<td>(8,-6)</td>
<td>(8,-4)</td>
<td>(10,-4)</td>
<td>(10,-6)</td>
</tr>
<tr>
<td>Chair</td>
<td>(6,-3)</td>
<td>(6,2)</td>
<td>(10,2)</td>
<td>(10,-3)</td>
</tr>
</tbody>
</table>
EQUATIONS TRANSLATIONS

Student/Class Goal
Based on bills and other financial situations, students want to be able to take their own scenarios and turn them into solvable math problems.

Outcome (lesson objective)
Students will connect words and mathematical symbols.
Students will compute answers to multi-step word problems.

Time Frame
2 hours

Standard
Use Math to Solve Problems and Communicate
(Primary benchmarks in bold.)

<table>
<thead>
<tr>
<th>Number Sense</th>
<th>Benchmarks</th>
<th>Geometry &amp; Measurement</th>
<th>Benchmarks</th>
<th>Algebra &amp; Patterns</th>
<th>Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connect number words</td>
<td>4.1</td>
<td>Identify/apply basic geometric concepts</td>
<td></td>
<td>Patterns/sequences</td>
<td></td>
</tr>
<tr>
<td>Solve problems using computations</td>
<td></td>
<td>Connect graphical and algebraic representations</td>
<td></td>
<td>Evaluate/solve expressions/equations 4.16, 5.16</td>
<td></td>
</tr>
<tr>
<td>Order of operations</td>
<td></td>
<td>Perimeter/area/volume</td>
<td></td>
<td>Connect relationships to representations 4.17, 5.17</td>
<td></td>
</tr>
<tr>
<td>Compare/order numbers</td>
<td></td>
<td>Graphical representations</td>
<td></td>
<td>Graphing</td>
<td></td>
</tr>
<tr>
<td>Estimate &amp; compute to solve problems</td>
<td></td>
<td>Use of correct units</td>
<td></td>
<td>Solving equations using algebra/graphs 6.19</td>
<td></td>
</tr>
<tr>
<td>Evaluate using roots and exponents</td>
<td></td>
<td>Right triangle trigonometry</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Data Analysis & Probability
Benchmarks
Measurement applications
Solve problems 4.25, 5.25, 6.25, 5.26, 6.28

Interpret data
Measurement conversions
Communicate ideas 4.29, 6.31, 6.32, 6.33

Create and display data
Rounding
Reason mathematically

Central tendency
Connect concepts 3.27, 6.36

Probability
Mathematical performance 4.35

Materials
SmartPal kit (SmartPal sleeves, wipe off cloths, dry erase markers) – inserting a blank sheet of paper into the sleeves will give students a reusable sheet of paper that they can quickly try answers out on and erase without using up a pencil eraser. It’s quicker as well.
Calculator
Algebra Tiles
Equations Translations worksheet

Learner Prior Knowledge
Students should be familiar with the order of operations, be able to solve simple one- and two-step linear algebraic equations, be familiar with multiple methods of solving linear equations, and use formal (sum, difference, product, quotient) and informal (plus, minus, times, split, etc.) terms associated with mathematical operations. Students should also be familiar with Polya’s problem solving steps; if not, provide them with the Polya’s 4-step handout.

Instructional Activities

Note: Throughout the lesson, students may find a calculator helpful. As the problems are contextual and there was an attempt to keep values realistic, there will be decimals. Also, the SmartPals may be used so that students can try problems multiple times. With the dry erase, there is less worry of an error. And, finally, instead of using a different manipulative for each problem, the Algebra Tiles can be used to represent our unknowns in each problem. As they are double-sided, we can use them to express the positive and negative values. For the systems, you can use the x² block as another variable or include another manipulative (such as the centimeter cubes).

Part 1: (I do) To get students prepared for the worksheet, present them with a simple real life example that they will be familiar with. One such example could be setting up an equation that represents the cost of a cell phone bill where one pays $50 a month for 500 minutes and $0.05 for each minute over 500 minutes. If their bill was $54.35 last month, how many minutes did they go over?

1. **Understand the problem**: Begin by breaking down the question into what you know and what you don’t know (see Teacher Answer Sheet).
2. **Devise a plan:** Present how all the variables are related and create an equation. Since there is a constant $50 charge and we need to pay $0.05 for every minute over 500 minutes to get our bill total of $54.35, we get the equation $50 + .05x = 54.35$.

3. **Carry out your plan:** Solve the equation, making sure to label your answer. Subtracting 50 from each side of the equation, we get $0.05x = 4.35$. Now dividing by 0.05, the equation is solved for $x$ and looks like $x = 87$.

4. **Look back:** Discuss with the class about how you know your answer makes sense by calculating a balance statement (plugging your solution back into your original equation and simplifying each side of the equation, see Teacher Answer Sheet) and that your answer is a possibility (phone companies always round up to a whole number of minutes).

Part 2: (I do) To prepare students for systems of equations, present the previous problem with the addition of $0.05 for each text message. During a particular month, you go over your 500 minute allotment. In addition, you send twice as many text messages as minutes that you went over by. How many minutes over did you go and how many text messages did you send? Tell students that you are going to solve this problem three different ways: (a) using one variable, (b) substitution, and (c) elimination. Be sure to mention that the substitution and elimination methods are used when there is more than one unknown (as in this case). To begin, make a list of your knowns and unknowns. For method (a), you know if you keep your minutes over 500 as $x$, you can write the number of text messages as $2x$ as there are twice as many of them. Setup your equation and solve for $x$ (see Teacher Answer Sheet). After you solve for $x$, you can use the fact that there are twice as many text messages as average minutes to solve for the number of text messages. For methods (b) and (c), if you keep your minutes over 500 as $x$, you can label the number of text messages as $y$. Hence, you know $y = 2x$. Using substitution, you end up with the same equation as you did in method (a). For elimination, we are attempting to eliminate one of the variables from the equations. Label the top equation (on the teacher answer sheet) as Equation 1 and the bottom equation as Equation 2. It would help to put each of the equations in standard form ($Ax + By = C$). You have the choice to eliminate the $x$'s or the $y$'s. One way to eliminate each variable is shown on the teacher answer sheet. The key to the elimination method is to set up your equations so that when you put the equations together, one of the variables will end up with a coefficient of zero after adding the equations. This is where the elimination name comes in as we have eliminated one of the variables. Be sure to point out that both methods will yield the same answer. Ask students which method they prefer and why. After allowing students to voice their opinions, be sure to point out that there will be times where substitution will be the easiest method (when one variable is already solved for or easily solved for), and the same is true with elimination (when the same variable in different equations match or can be easily canceled out).

Part 3: (We do) Pass out the Equations Translations worksheet and read problem one out loud. Have the students take turns defining the knowns and unknowns of the problem (take notes on board). Ask for volunteers to suggest a possible equation to represent the problem. Be sure to give students enough time, but if no students are able/willing to speak up, you may want to start by writing “$x = 26$” on the board to represent the 26 pieces of candy remaining. Then talk about what happened in the story. For example, ask the students, “What is meant by the phrase ‘gave out’?” Typically, the hardest part of the equation for students to understand is the “6x” that represents the total amount of candy that Sarah began with. To help students get to the solution, it may be helpful to give examples such as “if you buy three packs of gum with 12 sticks of gum in each pack, how many sticks of gum did you buy?” After the equation has been written on the board, reread the problem while pointing to each term in the equation that represents that part of the story. Have the students walk through the steps involved with solving the equation (combining like terms, moving the constant term to the other side of the equation, and dividing by the coefficient). Have students decide if they believe the answer is reasonable, and then justify their solution. Probe students with questions like, “Is it possible to have 13 pieces of candy in a bag?” or, “What are some possible answers that you know would be wrong?” (Negative answers, fractional/decimal answers.)

Part 4: (You do) Allow students to work in groups or individually to set up and solve problems 2 through 5. For each problem, have students take turns presenting their solutions and justifying their work. Make sure students list their knowns and unknowns, provide reasoning for their equations, and review their solutions. If you have students that finish early, ask them if there are any other ways to solve the problems (especially for problems 3, 4, and 5 using one variable, substitution, and elimination).

Part 5: (I do) Write the equation $5(x - 2) + 3 = 2x + 2$ on the board. Give different ways of saying the equation using words and come up with a real life situation that represents the equation. For example, “Three added to the product of five and the difference of $x$ and two is equal to two more than twice $x$,” or “five times the quantity $x$ minus 2, plus 3 equals two times $x$ plus two” and “buying five items at two dollars off with a three dollar surcharge is the same as buying two regular priced items with a two dollar surcharge. What is the regular price of the item?”

Part 6: (We do) Ask for any volunteers to attempt to read problem 6 of the handout. If the first student reads the equation correctly, ask if there are any other ways to read the equation. Be sure that you discuss how and why any incorrect answer is, in fact, incorrect. A common incorrect answer is “three times $x$ minus four plus two equals 20” which would look like $3x - 4 + 2 = 20$ where the student forgot to distinguish the placement of the parenthesis. Then ask students to come up with possible story problems that would be represented by the equation. Answers can and should vary. If one student gives an answer involving
money (most common response), ask if there is any situation involving time that can be represented by the equation. One such story could be “Tom and Phillip repair bikes. It takes Tom three days of work, each for four hours shorter than Phillip’s normal business day, and two hours of work on a fourth day to finish a bike. If it takes Tom a total of 20 hours, how long is Phillip’s normal business day?”

Part 7: (You do) Have your students work individually on problems 7 through 10. After students have completed the worksheet, allow time to discuss different solutions and check the accuracy of their own answers. One problem at a time, have students share their solution(s), and allow the other students to agree or disagree (only interjecting when students are unable to diagnose incorrect solutions).

Assessment/Evidence (based on outcome)
Parts 4 and 7 provide students with the opportunity to present their mastery; students should actively listen to other students’ solutions and justifications for signs of understanding and possible misconceptions. Having students present their solutions not only requires them to communicate mathematically, but allows other students to assess the accuracy of their peers’ answers.

Exit Slip:
1. Janice runs three times a week to stay in shape. She runs two miles further on Mondays than she does on Wednesdays. On Fridays, Janice runs one mile shorter than twice what she runs on Wednesday. If she runs a total of 13 miles a week, how far does she run each day? (5 miles on Monday, 3 miles on Wednesday, and 5 miles on Friday)
2. Write in words and come up with a story that would represent the equation 4(\(x + 2\)) = 72. (Answers may vary, 4 times the sum of \(x\) and 2 equals 72. James worked four days and got paid for two extra hours each day. If James got paid for 72 hours, how many hours did he work each day?)

Teacher Reflection/Lesson Evaluation
Not yet completed

Next Steps
After students have mastered the translation between story problems and their mathematical representation, present students with equations and problems that consist of decimal/fractional components or answers, solutions that do not make sense, and inequalities (can also be used to fill extra time if needed).

Another possible next step would be to have students attempt to solve the equations graphically, thus connecting the verbal, mathematical, and graphical representations of equations.

Technology Integration
As this lesson is designed to help develop students’ ability to translate and communicate mathematically, technology is not integrated into this lesson. If you so desire, it may be helpful to students to use graphing calculators, or GeoGebra (http://www.geogebra.org), to solve the linear equations graphically.

Purposeful/Transparent
Students want to be able to translate everyday problems into mathematical equations that allow them to be solved algebraically. The teacher will model how to define knowns and unknowns of a word problem, how to translate word problems into mathematical equations, solve the equations, and interpret/review one’s solution(s) and then guide students through similar exercises. This is followed by reading equations and creating everyday situations that would be represented by each equation.

Contextual
There are many everyday situations that can be modeled by mathematical equations and solved algebraically. Setting budgets, calculating costs, determining the length of fencing needed to fence in one’s yard, determining the stopping points on a road trip, and setting a schedule that meets all participants’ needs can all be modeled mathematically and solved algebraically.

Building Expertise
Students will gain knowledge and experience in translating and solving real world situations that they can use to model problems they face in their own lives. This lesson will also prepare students to solve systems of equations involving more than two variables.
Vocabulary Sheet

**Constant** — a term that doesn’t change value. In the expression $3x + 5$, 5 is a constant as it is a term that never changes value.

**Variable** — letters that are used to represent unknown numbers that may differ depending on circumstances. In the expression $3x + 5$, $x$ is a variable and $3x$ is the variable term as it is a term that contains a variable.

**Coefficient** — numbers multiplied by variables. In the expression $3x + 5$, 3 is the coefficient.

**Knowns** — the collection of information that is known about a situation.

**Unknowns** — the information that is unknown about a situation. His information will usually help us decide what we want our variables to be.

**Term** — a number, a variable, or the product of numbers and variables. In the expression $3x + 5$, $3x$ and 5 are different terms.
Equations Translations

Define the unknown(s), write an equation, and solve each of the following problems.

1. Sarah bought six identical bags of candy to hand out during Halloween. She gave out 20 pieces of candy in the first hour, 14 pieces of candy in the second hour, and 5 pieces of candy in the third hour. She also gave a bag of candy to her neighbor Mr. Rodgers who ran out of candy after the first round of trick-or-treaters. She has 26 pieces of candy remaining. How many pieces of candy were in each bag?

2. Sean’s mom gave him $5 to add to his piggy bank. Sean convinced his grandpa to then triple the amount of money he had in his wallet after he mowed his lawn. After purchasing a $45 video game, he had $4 less than he started with. How much money did Sean originally have in his wallet?

3. John’s saving account has $35 more than twice the amount of his checking account. If John has $433 in his savings account, how much does he have in his checking account?

4. Dianne needs to build a fence for a rectangular garden where the length is twice as long as the width and she has total of 54 ft. of fence. If she wants to use all of her fencing, what will be the dimensions of her garden?

5. Sally has a bag of red, blue, and yellow marbles. She has three more red marbles than blue marbles and two more yellow marbles than twice the number of red marbles. If she has a total of 59 marbles, how many of each color does she have?

Write out each equation in words, solve it, and then create a story problem to match the equation.

6. $3(x - 4) + 2 = 20$

7. $10x - 20 = 100 + 4x$

8. $2x + 2(x + 4) = 36$

9. $\begin{cases} y = 2x + 3 \\ 57 = x + y \end{cases}$

10. $8x - 5 - 2x + 4 = 35$
Equations Translations: Teacher Answer Sheet

From the Lesson Plan:

Part 1

<table>
<thead>
<tr>
<th>Knowns</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50 for up to 500 minutes</td>
<td>Amount of minutes over 500 minutes (x).</td>
</tr>
<tr>
<td>$0.05 for each minute over 500 minutes</td>
<td></td>
</tr>
<tr>
<td>Total charge of $54.35</td>
<td></td>
</tr>
</tbody>
</table>

Equation: \( 50 + 0.05x = 54.35 \)

Solution: 87 minutes

Balance Statement:

\[
50 + 0.05(87) = 54.35 \\
50 + 4.35 = 54.35 \\
54.35 = 54.35
\]

Part 2

<table>
<thead>
<tr>
<th>Knowns</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50 for up to 500 minutes</td>
<td>Amount of minutes over 500 minutes (x).</td>
</tr>
<tr>
<td>$0.05 for each minute over 500 minutes</td>
<td></td>
</tr>
<tr>
<td>$0.05 for each text message</td>
<td>Amount of text messages (y or 2x).</td>
</tr>
<tr>
<td>Twice as many text messages as overage minutes</td>
<td></td>
</tr>
<tr>
<td>Total charge of $54.35</td>
<td></td>
</tr>
</tbody>
</table>

Equations: for (a) \( 50 + 0.05x + 0.05(2x) = 54.35 \) or for (b) and (c) \[
\begin{align*}
y &= 2x \\
50 + 0.05x + 0.05y &= 54.35
\end{align*}
\]

One variable:

<table>
<thead>
<tr>
<th>Original Equation</th>
<th>50 + 0.05x + 0.05(2x) = 54.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1: Multiply 0.05 and 2</td>
<td>50 + 0.05x + 0.1x = 54.35</td>
</tr>
<tr>
<td>Step 2: Simplify equation by combining like-terms</td>
<td>0.15x = 4.35</td>
</tr>
<tr>
<td>Step 3: Subtract 50 to each side of the equation</td>
<td>( x = 29 )</td>
</tr>
<tr>
<td>Step 4: Divide by 0.15</td>
<td>So the number of text messages is ( 2 \times 29 = 58 )</td>
</tr>
<tr>
<td>Step 5: Plug back in to find number of text messages</td>
<td></td>
</tr>
</tbody>
</table>

Elimination:

<table>
<thead>
<tr>
<th>Standard Form</th>
</tr>
</thead>
</table>
| \[
\begin{align*}
-2x + y &= 0 \\
0.05x + 0.05y &= 4.35
\end{align*}
\]

Eliminate x’s

|     | \[
\begin{align*}
-2x + y &= 0 \\
2x + 2y &= 174
\end{align*}
\] |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1: Multiply Equation 2 by 40</td>
<td>Step 2: Add equations together</td>
</tr>
<tr>
<td>Step 3: Solve for y</td>
<td>0x + 3y = 174</td>
</tr>
<tr>
<td>Step 4: Plug back into original equation and solve for x</td>
<td>y = 58</td>
</tr>
<tr>
<td>58 = 2x</td>
<td>29 = x</td>
</tr>
</tbody>
</table>

Eliminate y’s

|     | \[
\begin{align*}
-2x + y &= 0 \\
x + y &= 87
\end{align*}
\] |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1: Multiply Equation 2 by 20</td>
<td>Step 2: Subtract equation 1 from equation 2</td>
</tr>
<tr>
<td>Step 3: Solve for x</td>
<td>(-3x + 0y = -87)</td>
</tr>
<tr>
<td>Step 4: Plug back into original equation and solve for y</td>
<td>( x = 29 )</td>
</tr>
<tr>
<td>y = 2(29)</td>
<td>y = 58</td>
</tr>
</tbody>
</table>

Solution: 29 minutes over and 58 text messages.

Check Statement:

\[
50 + .05(29) + .05(58) = 54.35 \\
50 + 1.45 + 2.9 = 54.35 \\
54.35 = 54.35
\]
Equations Translations Worksheet:

1.

<table>
<thead>
<tr>
<th>Knowns</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 bags of candy</td>
<td>Number of pieces of candy in each bag (x).</td>
</tr>
<tr>
<td>Gave away 20 pieces</td>
<td></td>
</tr>
<tr>
<td>Gave away 14 pieces</td>
<td></td>
</tr>
<tr>
<td>Gave away 5 pieces</td>
<td></td>
</tr>
<tr>
<td>Gave away 1 bag</td>
<td></td>
</tr>
<tr>
<td>26 pieces left</td>
<td></td>
</tr>
</tbody>
</table>

Equation: $6x - 20 - 14 - 5 - x = 26$

Solution: 13 pieces of candy in each bag

Balance Statement:

\[
6(13) - 20 - 14 - 5 - 13 = 26 \\
78 - 20 - 14 - 5 - 13 = 26 \\
26 = 26
\]

2.

<table>
<thead>
<tr>
<th>Knowns</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gained $5</td>
<td>Original amount of money in Sean’s wallet (x).</td>
</tr>
<tr>
<td>Then tripled money</td>
<td></td>
</tr>
<tr>
<td>Spent $45</td>
<td></td>
</tr>
<tr>
<td>Now has $4 less than he started with</td>
<td></td>
</tr>
</tbody>
</table>

Equation: $3(x + 5) - 45 = x - 4$

Solution: $13$

Balance Statement:

\[
3(13 + 5) - 45 = 13 - 4 \\
3(18) - 45 = 9 \\
54 - 45 = 9 \\
9 = 9
\]

For problems 3 through 5, there are two popular ways to set up the problem. For each problem, a) will be set up using one variable, and b) will be set up using multiple variables.

3a)

<table>
<thead>
<tr>
<th>Knowns</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings is $35 more than twice checking</td>
<td>Amount of money in checking account (x).</td>
</tr>
<tr>
<td>$433 in savings</td>
<td></td>
</tr>
</tbody>
</table>

Equation: $2x + 35 = 433$

Solution: $199$

Balance Statement:

\[
2(199) + 35 = 433 \\
398 + 35 = 433 \\
433 = 433
\]

3b)

<table>
<thead>
<tr>
<th>Knowns</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings is $35 more than twice checking</td>
<td>Amount of money in checking account (x).</td>
</tr>
<tr>
<td>$433 in savings</td>
<td>Amount of money in savings account (y).</td>
</tr>
</tbody>
</table>

Equations:

\[
\begin{align*}
2x + 35 &= y \\
433 &= y
\end{align*}
\]

Solution: $199$

Balance Statement:

\[
2(199) + 35 = 433 \\
398 + 35 = 433 \\
433 = 433
\]
4a)  
<table>
<thead>
<tr>
<th>Knowns</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length is twice the width</td>
<td>Width of fence (x).</td>
</tr>
<tr>
<td>Perimeter is 54 ft.</td>
<td>Length of fence (2x).</td>
</tr>
</tbody>
</table>

Equations: \(2x + 2(2x) = 54\)  
Solution: width = 9 ft. and length = 18 ft.  
Balance Statement:  
\[
2(9) + 2(2(9)) = 54  
18 + 36 = 54  
54 = 54
\]

4b)  
<table>
<thead>
<tr>
<th>Knowns</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length is twice the width</td>
<td>Width of fence (x).</td>
</tr>
<tr>
<td>Perimeter is 54 ft.</td>
<td>Length of fence (y).</td>
</tr>
</tbody>
</table>

Equations: \(\begin{align*}
y &= 2x  
2x + 2(y) &= 54
\end{align*}\)  
Solution: width = 9 ft. and length = 18 ft.  
Balance Statement:  
\[
18 = 2(9)  
18 = 18
\]

5a)  
<table>
<thead>
<tr>
<th>Knowns</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 more red marbles than blue</td>
<td>Number of blue marbles (x).</td>
</tr>
<tr>
<td>2 more yellow than twice the number of red</td>
<td>Number of red marbles (x+3).</td>
</tr>
<tr>
<td>Total of 59 marbles</td>
<td>Number of yellow marbles (2(x+3)+2)</td>
</tr>
</tbody>
</table>

Equation: \(x + (x + 3) + 2(x + 3) + 2 = 59\)  
Solution: 12 blue marbles, 15 red marbles, and 32 yellow marbles  
Balance Statement:  
\[
12 + (12 + 3) + 2(12 + 3) + 2 = 59  
12 + 15 + 30 + 2 = 59  
59 = 59
\]

5b)  
<table>
<thead>
<tr>
<th>Knowns</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 more red marbles than blue</td>
<td>Number of blue marbles (x).</td>
</tr>
<tr>
<td>2 more yellow than twice the number of red</td>
<td>Number of red marbles (y).</td>
</tr>
<tr>
<td>Total of 59 marbles</td>
<td>Number of yellow marbles (z)</td>
</tr>
</tbody>
</table>

Equations: \(\begin{align*}
y &= x + 3  
z &= 2y + 2  
x + y + z &= 59
\end{align*}\)  
Solution: 12 blue marbles, 15 red marbles, and 32 yellow marbles  
Balance Statement:  
\[
12 + (12 + 3) + 2(12 + 3) + 2 = 59  
12 + 15 + 30 + 2 = 59  
59 = 59
\]
6. Words: Two more than three times the difference of $x$ and four equals twenty.
Solution: $x = 10$
Situation: Juan has three boxes of pizza with four pieces missing and two extra slices. He has a total of 20 slices. How many slices are in a box of pizza?

7. Words: Twenty less than the product of ten and $x$ is equal to 100 plus the product of 4 and $x$.
Solution: $x = 20$
Situation: Tonya’s swimming club is selling t-shirts as fundraiser. She plans to sell the t-shirts for $10 each and has to pay $20 for a setup fee. If she wants to make $100 and cover the $4 cost of each t-shirt, how many t-shirts does she have to sell?

8. Words: Twice $x$ plus two times the sum of $x$ and 4 equals 36.
Solution: $x = 7$
Situation: The perimeter of a throw rug is 34 ft. If the width is 4 feet longer than the length, what is the length of the rug?

9. Words: $y$ equals three more than twice $x$ and 57 is equal to the sum of $x$ and $y$.
Solution: $x = 18, y = 39$
Situation: Veronica and Chelsea both work at the grocery store. Veronica works three hours more than twice Chelsea’s hours. If together they work a total of 57 hours, how many hours do each of them work?

10. Words: The product of 8 and $x$ minus 5 minus twice $x$ plus 4 equals 35.
Solution: $x = 6$
Situation: Mrs. Robinson bought eight boxes of pencils to keep in her desk. During a test, she handed out 5 pencils and lent Mr. Wilson two boxes. If she got 4 pencils back at the end of the test and has 35 pencils remaining, how many pencils were in each box?
## Sequence Sense

### Student/Class Goal
Students see data in a list and want to be able to find future values based on the trend.

### Time Frame
2 hours

### Outcome (lesson objective)
Students will analyze mathematical sequences.
Students will use formulas to find missing terms in patterns and sequences.

### Standard  Use Math to Solve Problems and Communicate
(Primary benchmarks in bold.)

<table>
<thead>
<tr>
<th>NRS EFL</th>
<th>Levels 4-6</th>
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</table>

### Number Sense

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Number Sense</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connect number words</td>
<td>Identify/apply basic geometric concepts</td>
</tr>
<tr>
<td>Solve problems using computations</td>
<td>Connect graphical and algebraic representations</td>
</tr>
<tr>
<td>Order of operations</td>
<td>Perimeter/area/volume</td>
</tr>
<tr>
<td>Compare/order numbers</td>
<td>Graphical representations</td>
</tr>
<tr>
<td>Estimate &amp; compute to solve problems</td>
<td>Use of correct units</td>
</tr>
<tr>
<td>Evaluate using roots and exponents</td>
<td>Right triangle trigonometry</td>
</tr>
</tbody>
</table>

### Geometry & Measurement

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Geometry &amp; Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify/apply basic geometric concepts</td>
<td>Solve problems</td>
</tr>
<tr>
<td>Connect graphical and algebraic representations</td>
<td>Communicate ideas</td>
</tr>
<tr>
<td>Graphical representations</td>
<td>Reason mathematically</td>
</tr>
<tr>
<td>Use of correct units</td>
<td>Connect concepts</td>
</tr>
<tr>
<td>Right triangle trigonometry</td>
<td>Mathematical performance</td>
</tr>
</tbody>
</table>

### Algebra & Patterns

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Algebra &amp; Patterns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connect concepts</td>
<td>Connect number words</td>
</tr>
<tr>
<td>Connect relationships to representations</td>
<td>Connect concepts</td>
</tr>
<tr>
<td>Connect concepts</td>
<td>Connect number words</td>
</tr>
<tr>
<td>Evaluate/solve expressions/equations</td>
<td>Connect relationships to representations</td>
</tr>
<tr>
<td>Connect relationships to representations</td>
<td>Connect number words</td>
</tr>
</tbody>
</table>

### Processes

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.15, 5.15, 6.15</td>
<td>Connect concepts</td>
</tr>
<tr>
<td>5.16</td>
<td>Connect concepts</td>
</tr>
<tr>
<td>4.17, 5.17</td>
<td>Connect concepts</td>
</tr>
<tr>
<td>4.25, 5.25, 6.25, 5.26, 6.28, 4.28, 5.28</td>
<td>Connect concepts</td>
</tr>
<tr>
<td>4.29, 6.31, 6.32, 6.33</td>
<td>Connect concepts</td>
</tr>
<tr>
<td>4.35</td>
<td>Connect concepts</td>
</tr>
</tbody>
</table>

### Materials

- SmartPal kit (SmartPal sleeves, wipe off cloths, dry erase markers) – inserting a blank sheet of paper into the sleeves will give students a reusable sheet of paper that they can quickly try answers out on and erase without using up a pencil eraser. It’s quicker as well.
- Centimeter Cubes
- Calculator
- Arithmetic and Geometric Sequences Excel sheet
- Arithmetic and Geometric Sequences worksheet

### Learner Prior Knowledge
Students should be able to read and construct tables and charts to organize data and be familiar with the order of operations. Students should also be familiar with Polya’s problem solving steps, if not provide them with Polya’s 4-step handout.

### Instructional Activities

**Note**: Students may find the calculator useful when working with the explicit formulas as there will be exponents involved. In order to find the formulas and see the patterns and trends, students may find the Centimeter Cubes helpful. With them, they can actually set out a manipulative to represent each term in the sequence.

**Part 1**: Start the lesson by writing the notation $a_1, a_2, a_3, a_4, ... a_n, ...$ to describe a sequence of numbers where the subscript denotes where a term is in a sequence of numbers: $a_{10}$ denotes the $10^{th}$ number of a sequence. Then make a chart on the board with the following headings: sequence, definition, example, recursive formula, and explicit formula (see Teacher Answer Form). If students are not familiar with the terms “recursive” and “explicit,” be sure to define them before continuing.

- A recursive formula calculates the current number of a sequence based on the previous number.
- An explicit formula allows the calculation of any term of a sequence directly without needing to know the previous number.

For arithmetic sequences, be sure to emphasize the need for a common difference between the numbers in the sequence (either addition or subtraction). For geometric sequences, be sure to emphasize the need for a common ratio between the numbers of a sequence (multiplication or division). Following, you should use both the recursive and explicit formulas to calculate the next term of each of the examples you used in your chart.
Part 2: (I do) Hand out the Arithmetic and Geometric Sequences Worksheet to the students as well as some Centimeter Cubes. Depending on class size, you may need to have students work in pairs or small groups so that everyone has access to plenty of cubes. Read problem one out loud. Reread what the problem is asking for (understand). Taking the fact the question asks for both a recursive and explicit formula of a sequence, we know we must first understand the sequence (devise a plan). It may help to make a table or list of the year and the number of members of the Hall of Fame for the first five years (enact the plan). Since there were 20 members originally inducted, we know the first term of the sequence is 20, and since 5 new members are added each year we know that it is an arithmetic sequence with a common difference of 5 and the second term is 25, the third term is 30, and so on. Straight from the table made previously, we know the recursive formula is \(a_n = a_{n-1} + 5\) (another way of thinking of it is the current number = the previous number +5) and the explicit formula is \(a_n = 20 + 5(n - 1)\) (another way of thinking of it is the amount of members in the hall of fame in the nth year is equal to the original number inducted plus five times one less than the nth year. Be sure to point out that the ninth term represents the number of members in 2009; the 28th term represents the number of members in 2028. Use both formulas to check against the table or list you made earlier (check). For the second part of the problem, the long way would be to continue your list or table until you reach 200. The quicker way would be to plug 200 in for \(a_n\) of the explicit formula and solve for \(n\). You may want to do both ways to show that it can be done either way, but using the explicit formula is more efficient.

Part 3: (We do) Ask for a volunteer to read problem two out loud. Take a quick poll of whether students think the situation is an arithmetic or geometric sequence. If you have any volunteers to provide their reasoning for their conjecture, let them share their thoughts. If not, start by determining that the sequence is not arithmetic by pointing out the fact that the difference between consecutive terms is not constant. To determine if it is geometric or not, take the second term and divide it by the first term (you will get 0.8). Then take the third term and divide by the second term (again you get 0.8). Repeat with each pair of consecutive numbers to show that there exists a common ratio of 0.8. You can also have each row of students calculate a different ratio (row 1 do term 2 divided by term 1, row 2 do term 3 divided by term 2, and so on). Ask if there are any volunteers to come up with the recursive formula. Ask a different student to explain why they think the formula is correct or not (be sure to have a correct answer before moving on). Then ask if any students have a guess at an explicit formula. Again ask another student to check their answer. Once the formulas have been completed, have half of the students calculate the value of the car in year 10 using the recursive formula and the other half calculate the value using the explicit formula. Allow students to compare answers and discuss which way was easier.

Part 4: (You do) Have students work individually on the remainder of the problems. As students finish the worksheet, have them pair up and check their answers against their partner’s answers. Once all students have finished and have checked their answers with their partner, go over each problem one at a time having a student present their answer to the problem. Ask students to provide reasoning for their actions.

Assessment/Evidence (based on outcome)
Part 4 will serve as evidence of student mastery. During part 4, the teacher should actively listen to partner discussions for signs of understanding or of misconceptions. When students are working alone, the teacher should ask students speak out loud as they solve the problem.

Exit Slip:
1. Determine the next two terms, the recursive formula, the explicit formula, and the 15th term for the sequence 2, 5, 8, 11, ...
   (Answers: 14, 17, Recursive: \(a_n = a_{n-1} + 3\), Explicit: \(a_n = 2 + 3(n - 1)\), \(a_{15} = 2 + 3(15 - 1) = 2 + 3(14) = 44\))
2. Determine the next two terms, the recursive formula, the explicit formula, and the 15th term for the sequence 2, 1, \(\frac{1}{2}\), \(\frac{1}{4}\), ...
   (Answers: 1/8, 1/16, Recursive: \(a_n = a_{n-1} + 2\), Explicit: \(a_n = 2 \left(\frac{1}{2}\right)^{n-1}\), \(a_{15} = 2 \left(\frac{1}{2}\right)^{15-1} = 2 \left(\frac{1}{2}\right)^{14} = \frac{1}{8192}\))

Teacher Reflection/Lesson Evaluation
Not yet completed

Next Steps
Have students graph the sequences on an x-y plane with the term number as the x-variable and the value as the y-value to visually represent the linear relationship of arithmetic sequences and exponential relationship of geometric sequences. You can also present students with ways of representing patterns that are neither linear nor exponential.

Technology Integration
Excel spreadsheets can be used to help build and organize data (see Excel spreadsheet for problem one in teacher resource).

Purposeful/Transparent
This lesson starts with defining terms and notation related to arithmetic and geometric sequences to give students a vocabulary set to describe and analyze different patterns. It then moves to contextual situations that involve sequences to give students an idea of where sequences are found in everyday life. The lesson wraps up with determining if patterns are arithmetic, geometric, or neither to provide students the opportunity to focus on the ideas of a common difference or common ratio and to analyze sequences that may not be arithmetic or geometric.
**Contextual**
This lesson focused on representing everyday situations involving salaries, asset depreciation, population growth, and exponential decay. This is important as understanding sequences are vital in social, financial, and medical environments.

**Building Expertise**
Students will use the definitions related to sequences to analyze patterns and use formal mathematics to represent and extend those patterns.
Sequence □ An ordered set of numbers that are defined by the position they hold. Represented by $a_1, a_2, a_3, a_4, \ldots a_n, \ldots$ where the subscript denotes where a term is in a sequence of numbers. $a_{10}$ denotes the 10th number of a sequence.

Recursive Formula □ terms of a sequence are calculated based upon the value of the previous term(s).

Explicit Formula □ terms of a sequence are calculated based upon its place in the order of the sequence.

Arithmetic Sequence □ A sequence of numbers where a common difference, $d$, exists between consecutive numbers.

Geometric Sequence □ A sequence of numbers where a common ratio, $r$, exists between consecutive numbers.
Arithmetic and Geometric Sequences

1. The Professional Lawn Bowling Association Hall of Fame opened in 2001. It originally inducted 20 members to start the hall of fame and, beginning with 2002, inducts 5 new members every year. Write a recursive formula and explicit formula that expresses the total number of members in the hall of fame for each year after the doors opened. In what year will there be 200 members of the Professional Lawn Bowling Association Hall of Fame?

2. After graduation, Josh bought a brand new car for $20,000. Below is a table displaying the value of the car for the first 5 years. Find a recursive formula and an explicit formula to represent the value of the car. Then figure out how much the car will be worth in year 10 (rounded to the nearest dollar).

<table>
<thead>
<tr>
<th>Year</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$20,000</td>
</tr>
<tr>
<td>2</td>
<td>$16,000</td>
</tr>
<tr>
<td>3</td>
<td>$12,800</td>
</tr>
<tr>
<td>4</td>
<td>$10,240</td>
</tr>
<tr>
<td>5</td>
<td>$8,192</td>
</tr>
</tbody>
</table>

3. Sally’s year-long job offers to pay her salary in one of two ways. The first option is to give her $10 the first week, $20 the second week, $30 the third week and so on. While option two offers to give her a penny end the end of the first week, two pennies at the end of the second week, four pennies the third week, and continuing to double it every week. What option should Sally choose? Create a recursive and explicit formula to express each situation.

4. The local radio station is running a promotion where they ask a question and if the 10th caller answers the question correctly they win $250. If the caller doesn’t get the question right, the money increases by $250 each day until the caller gets the question right. If the promotion starts on Monday, make a table for a week (Monday through Friday) for the prize amount if nobody gets the question right all week. Write a recursive formula and an explicit formula to express the prize amount on the nth day if nobody answers the question correctly. If nobody answers the question correctly for three weeks, how much will the prize amount be on Monday of the fourth week?

5. The population of the Smallville is 5,000 people and grows 2% every year. What will the population be in three years? What will the population be in 50 years?

6. John makes widgets to sell at the market. He worked all day long between the 13th and 18th day to get ready for the biggest market sale at the end of the month. He knows he had 145 widgets at the start of the 13th day and 205 widgets at the start of 18th day. How many widgets can he make in one day? If he continues to work all day until the end of the month, how many widgets will he have made by the end of the 30th day?

7. For chemotherapy to work properly there needs to be exactly 50mg of medicine in the bloodstream on the 6th day of treatment. If half of the medicine is removed from the body each day, how much should doctors administer on the first day of treatment?

For 8-13 Determine whether each of the following sequences is arithmetic, geometric, or neither. Find a formula that represents the sequence, and then use it to find the next two terms and the 10th term.

8. \(
\frac{2}{3}, 2, 6, 18, \ldots
\)

9. \(
1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots
\)

10. \(5, -5, 5, -5, \ldots\)

11. \(3, 3, 3, 3, \ldots\)

12. \(7, 4, 1, -2, \ldots\)

13. \(2, 4, 2, 4, \ldots\)
### Sequence Type

<table>
<thead>
<tr>
<th>Definition</th>
<th>Example</th>
<th>Recursive Formula</th>
<th>Explicit Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic where a common difference, ( d ), exists between consecutive numbers.</td>
<td>5, 8, 11, 14, 17, ...</td>
<td>( a_n = a_{n-1} + d ) For example ( a_n = a_{n-1} + 3 )</td>
<td>( a_n = a_1 + d(n - 1) ) For example ( a_n = 5 + 3(n - 1) )</td>
</tr>
<tr>
<td>Geometric where a common ratio, ( r ), exists between consecutive numbers.</td>
<td>5, 10, 20, 40, 80, ...</td>
<td>( a_n = a_{n-1}(r) ) For example ( a_n = a_{n-1}(2) )</td>
<td>( a_n = a_1(r)^{n-1} ) For example ( a_n = 5(2)^{n-1} )</td>
</tr>
</tbody>
</table>

1. **Recursive:** \( a_n = a_{n-1} + 5 \)  
   **Explicit:** \( a_n = 20 + 5(n - 1) \)  
   200 members in 2037  
   \[
   200 = 20 + 5(n - 1) \\
   200 = 5n + 15 \\
   185 = 5n \\
   37 = n
   \]

2. **Recursive:** \( a_n = a_{n-1} \times 0.8 \)  
   **Explicit:** \( a_n = 20,000(0.8)^{(n-1)} \)  
   The car is worth $2,684.35 in year 10.

3. **Option 1**  
   **Recursive:** \( a_n = a_{n-1} + 10 \)  
   **Explicit:** \( a_n = 10 + 10(n - 1) \) or \( a_n = 10(n) \)  
   Sally should choose option 2 as she will make more money each week starting the 15th week (if students calculate a running total, option 2 will result in more total money starting the 18th week).

4. **Recursive:** \( a_n = a_{n-1} + 250 \)  
   **Explicit:** \( a_n = 250 + 250(n - 1) \) or \( a_n = 250(n) \)  
   It would be the 16th day so \( a_n = 250(16) = \$4,000 \)

5. **Recursive:** \( a_n = a_{n-1} \times 1.02 \) where \( a_0 \) is the initial year where the population is 5,000.  
   **Explicit:** \( a_n = 5000(1.02)^{(n)} \)  
   The population of Smallville will be 5,306 people after 3 years and 13,458 people after 50 years.

6. \[
   \frac{205 - 145}{18 - 13} = \frac{60}{5} = 12 \text{ widgets in one day. So to solve for the original amount, set up an explicit equation such that } a_n = a_0 + 12(n - 1) \text{ and select one of your data points to plug in and solve for } a_0. \text{ For example }
   \begin{align*}
   a_{18} &= a_0 + 12(18 - 1) \\
   205 &= a_0 + 12(17) \\
   205 &= a_0 + 204 \\
   1 &= a_0
   \end{align*}
   \]
   Hence, \( a_{30} = 1 + 12(30 - 1) = 349 \text{ widgets after 30 days.} \)

7. \[
   50 = a_1 \times \left(\frac{1}{2}\right)^{(6-1)}
   \]
   \[
   50 = a_1 \times \frac{1}{32}
   \]
   \[
   1,600 \text{ mg} = a_1
   \]
8. Geometric sequence
   a. Recursive: $a_n = a_{n-1} \times 3$
   b. Explicit: $a_n = \frac{2}{3} \times 3^{(n-1)}$
   c. Next two terms: 54; 162
   d. $10^{th}$ term: 13,122

9. Neither
   a. Formula: $a_n = \frac{1}{n}$
   b. Next two terms: $\frac{1}{5}; \frac{1}{6}$
   c. $10^{th}$ term: $\frac{1}{10}$

10. Geometric sequence
    a. Recursive: $a_n = a_{n-1} \times (-1)$
    b. Explicit: $a_n = 5 \times (-1)^{(n-1)}$
    c. Next two terms: 5; -5
    d. $10^{th}$ term: -5

11. Geometric and Arithmetic
    a. Recursive: $a_n = a_{n-1} + 1$ or $a_n = a_{n-1} + 0$
    b. Explicit: $a_n = 3$
    c. Next two terms: 3; 3
    d. $10^{th}$ term: 3

12. Arithmetic sequence
    a. Recursive: $a_n = a_{n-1} - 3$
    b. Explicit: $a_n = 7 - 3(n - 1)$
    c. Next two terms: -5; -8
    d. $10^{th}$ term: -20

13. Neither
    a. Formula: $a_n = 3 + (-1)^n$ or $a_n = 2$ if $n$ is odd and 4 if $n$ is even
    b. Next two terms: 2; 4
    c. $10^{th}$ term: 4
<table>
<thead>
<tr>
<th>Day</th>
<th>Amount of Medicine</th>
<th>(Formula to enter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1600</td>
<td>B3*2</td>
</tr>
<tr>
<td>2</td>
<td>800</td>
<td>B4*2</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
<td>B5*2</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>B6*2</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>B7*2</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>
Linear Connections

**Student/Class Goal**
Given data from a chart, students want to be able to create a graph (and equations) in order to draw conclusions easily from the information.

**Outcome** *(lesson objective)*
Students will take everyday situations and create linear equations.
Students will solve linear equations.
Students will create graphs using data from contextual situations.

**Time Frame**
2 Hours

**Standard** *Use Math to Solve Problems and Communicate*
(Primary benchmarks in bold.)

<table>
<thead>
<tr>
<th><strong>Number Sense</strong></th>
<th><strong>Benchmarks</strong></th>
<th><strong>Geometry &amp; Measurement</strong></th>
<th><strong>Benchmarks</strong></th>
<th><strong>Algebra &amp; Patterns</strong></th>
<th><strong>Benchmarks</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Connect number words</td>
<td>Identify/apply basic geometric concepts</td>
<td></td>
<td></td>
<td>Patterns/sequences</td>
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<td>Evaluate/solve expressions/equations 5.16</td>
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<tr>
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<td>Use of correct units</td>
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<td>Solving equations using algebra/graphs</td>
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**Data Analysis & Probability**

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<th><strong>Solve problems</strong></th>
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<td>Connect concepts 3.27, 6.36</td>
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<tr>
<td>Probability</td>
<td></td>
<td>Mathematical performance 4.35</td>
</tr>
</tbody>
</table>

**Materials**
*Linear Connections Tasks* handout
Slide N’ Measure Compass (can be used as a straightedge/ruler)
Giant Graph Paper (or some way to present graphs large enough for the class to see)
SmartPal kit (SmartPal sleeves, wipe off cloths, dry erase markers) – inserting a blank sheet of paper into the sleeves will give students a reusable sheet of paper that they can quickly try answers out on and erase without using up a pencil eraser. It’s quicker as well.

**Learner Prior Knowledge**
Students should be able to plot points on an X-Y plane, translate word problems into linear equations, solve algebraic equations, and have a basic concept of slope.

**Instructional Activities**

**Part 1: (I do)**
To start the lesson, write the three main forms of linear equations on the board, with their generic formula, and an example (see Teacher Answer Sheet). Then go through the steps of graphing each example. Start with the Standard Form example and create a t-table to list possible points.

- The easiest way to solve for two points is to plug in a 0 for x and solve for y (you get the point (0,2)) and 0 in for y and solve for x (you get the point (3,0)).
- Plotting the two points on a coordinate plane and connecting the dots will create the graphical representation of the equation of the line.

It may be a good idea to give further information about the particular points we found. When we plug in 0 for x and solve for y, the point we find is known as the y-intercept since it crosses the y-axis there. If we plug 0 in for y and solve for x, we get the x-intercept as that is where the graph crosses the x-axis. Choose two other points on the line, say (−3,4) and (9, −4), to demonstrate that all points on the line are solutions to the equation. Moving to the Point-Slope Form example, right from the equation, we know the point (−1,3) is a point on the line as the x is added by 1 (or
subtracted by -1) and the y is subtracted by 3. Also, since the slope is 2, which can be written as $\frac{2}{1}$ to display rise-over-run, our next point can be found by going up two and right one. Connecting the points with a line will be the graphical representation of the equation.

Again, choose two other points to demonstrate that all points on the line are solutions to the equation. For the Slope-Intercept Form example, since the constant term is -1, we know the y-intercept is the point (0, -1) and the slope is $-\frac{2}{3}$ because it is the coefficient of the x. Hence plotting the y-intercept and going down two and right three will provide two points to create the line. Again, choose two more points to demonstrate that all points on the line are solutions to the equation.

Part 2: (We do/ You do)

Hand out the Linear Connections Tasks worksheet to students and read question one.

- Ask your students if anyone has an idea of where to start (since question one is given to you in standard form, you will need to solve for two points in order to graph the equation).
- Ask your students if they have any idea of how to solve for two points (setting up a t-table and setting each variable equal to zero one at a time gives you two points, see teacher answer sheet).
- After solving for each point, have a student plot them on a graph (be sure to label your scale). Using a ruler, connect the two points with a line (be sure to continue your line long enough to demonstrate that it goes beyond the two points).

Once completed, have students work individually on question 2. Once all students have finished, have one student present their solution while explaining each step. Give students the opportunity to ask questions and correct their solution if needed.

For question 3, you will lead the discussion. Since the problem gives a point and the slope, discuss the easiest form to write the equation and what the equation would be. The most common places for mistakes are forgetting to put parenthesis around the x variable and mixing up the addition and subtraction.

Once the equation is correct, have a student plot the point on a graph, use the slope to find another point, and connect the two points with a line.

Have students work individually on question 4. Once all students have finished, have one student present their solution while explaining each step. Give students the opportunity to ask questions and correct their solution if needed. Again, for question 5, ask students since it is in slope-intercept form, what do we know about the equation? (Slope of -3/4 and the point (0,-5).) Have a willing student plot the y-intercept, use the slope to find another point, and connect the points with a line. Have students work individually on question 6.

Once all students have finished, have one student present their solution while explaining each step. Give students the opportunity to ask questions and correct their solution if needed.

Part 3: (I do)

- Read question 7 out loud.
- Since the problem involves the perimeter of a rectangle, we know the problem translates to the equation $x + y + x + y = 36$ or $2x + 2y = 36$, which is standard form.
- Using the same method of setting each variable equal to zero one at a time and solving for the other will give you two points.
- Plot the points on a graph and connect them with a line. (If you are using a 10 by 10 graph, you will need to change your scale in order to plot your points. Since the intercept of each axis is 18, making each line on the graph represent 2 spaces instead of one will allow your points to fit on your graph.) Make sure to discuss what happens when the line extends to values where x or y are negative. Since this is a contextual situation, make sure your students understand why that part of the graph, while technically correct to graph, does not make sense here. This will hold true for all graphs on this worksheet.
Part 4: *(We do)*
- Have a student read question 8 out loud.
- Ask students how to graphically represent the fact that in 2006 there were 12 hybrid cars. (e.g., the point (6, 12)). Ask them how to represent the 20 new models produced every 3 years. (e.g., a line with the slope is $\frac{20}{3}$).
- After the point and slope have been announced, ask if any students are able to come up with a formula to represent the problem (should be in point-slope form).
- Once a correct equation has been found, have a different student attempt to graph it with help of his/her peers if needed. *An important first step will be discussing an appropriate scale* (see teacher answer sheet for one example).

Part 5: *(You do)*
Have students work individually on the remaining problems. Walk around the room silently monitoring the students’ progress. When you see them run into difficulties, try not to answer their questions directly; instead, remind them of similar situations from the earlier tasks.

Part 6: As students finish, have them pair up and share their answers; allow them time to revise their answers if needed. After, go through each problem having pairs of students present their collective solution. Allow peers to check their answers and verify the presented solution.

**Assessment/Evidence** *(based on outcome)*
Parts 2, 4, 5, and 6 will provide evidence of students’ understanding and mastery of the concepts. During part 5, the teacher should look and listen for signs of understanding and misconceptions that students may have.

Exit slip:
Have students create equations for the following problems. Then, have them solve them algebraically and graph their equations.

1. Danielle and Chad are starting a business tutoring students in math. They rent an office for $400 per month and charge $20 per hour per student. If they have 15 students each for one hour per week how much profit do they make together in a month? (assume 4 weeks per month)
   Answer: $400 + 20x = y$

   For 15 students, they would make $700 in one month.
2. Adam's Bikes rents bikes for $10 plus $5 per hour. Shane paid $45 to rent a bike. For how many hours did he rent the bike?

Answer: \[ 10 + 5x = y \]
Shane rented the bike for 7 hours.

<table>
<thead>
<tr>
<th>Teacher Reflection/Lesson Evaluation</th>
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<tbody>
<tr>
<td>Not yet completed</td>
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</table>

<table>
<thead>
<tr>
<th>Next Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Have students use the equations and graphs to solve for certain situations (points).</td>
</tr>
<tr>
<td>2. Use the concepts associated with graphing different linear functions to graph non-linear functions including using base functions as a reference to determine other functions.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Technology Integration</th>
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<tbody>
<tr>
<td>If internet is available, graphs can be created using GeoGebra (a free software). GeoGebra can also be used to check solutions. <a href="http://www.geogebra.org/webstart/geogebra.html">http://www.geogebra.org/webstart/geogebra.html</a></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Purposeful/Transparent</th>
</tr>
</thead>
<tbody>
<tr>
<td>After the teacher presents key terms an example of each form, the teacher will lead a class discussion of each form to help build students understanding and independence in translating different equations into graphical representations. Contextual situations are added to provide motivation and rationale for mastering concepts and their relation to the world around us.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Contextual</th>
</tr>
</thead>
<tbody>
<tr>
<td>This lesson explores different real life situations from stock growth and earnings based on commission to building fences and meeting weight restrictions to see how linear equations and graphs can be used to represent and better understand a given situation.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Building Expertise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student will build on their knowledge of linear equations and graphs to represent and model real life situations.</td>
</tr>
</tbody>
</table>
**Point-Slope Form**  
The form \( y - y_1 = m(x - x_1) \), where \((x_1, y_1)\) is a point on the line, one of three forms of linear equations.

**Standard Form**  
The form \( Ax + By = C \), one of three forms of linear equations.

**Slope-Intercept Form**  
The form \( y = mx + b \), where \( m \) is the slope and \( b \) is the y-intercept, one of three forms of linear equations.

**Slope**  
The constant rate of change of output to input and is most commonly denoted by the letter \( m \). Sometimes referred to as rise-over-run as it is calculated by the difference in the y-values over the difference in the x-values or \( m = \frac{y_1 - y_2}{x_1 - x_2} \) where \((x_1, y_1)\) and \((x_2, y_2)\) are two points.

**T-table**  
A table used to organize points where the x-values are listed in one column and y-values are listed in another column and each row corresponds to a point.
1. Graph: \( x - 3y = 6 \)

2. Graph: \( -11x + 8y = 88 \)

3. Write and graph an equation through \((3, -5)\), slope = \( \frac{5}{3} \)

4. Write and graph an equation through \((-2, 5)\), slope = \( -\frac{1}{2} \)
5. Graph \( y = -\frac{3}{4}x - 5 \)  

6. Graph \( y = \frac{7}{4}x + 2 \)

7. Tina has 36 feet of fencing and wants to build a rectangular garden, but doesn’t know what dimensions she should use to erect her fence. Write an equation to represent the problem (using \( x \) to represent the length and \( y \) to represent the width) and graph the possible solutions.

8. In 2006, there were 12 different models of hybrid cars and they say 20 new models are introduced every 3 years. Write and graph an equation that represents the number of different models of hybrid cars (\( y \)) in each year after 2006.

9. Rosie plants a 3 cm tall rosebush that grows 1.5 cm every week. Write and graph an equation that represents the height (\( y \)) of the rosebush as time (\( x \)) passes.

10. To calculate the number of people and packages a small cargo plane can hold, airlines assume that the average person weighs 180 pounds and each package weighs 90 pounds. If a plane can hold 900 pounds, write and graph an equation that represents possible number of passengers (\( x \)) and packages (\( y \)) the airplane can carry.

11. Carrie makes a base salary of $20,000 and gets 30% commission. Write and graph an equation that represents her earnings (\( y \)) and her total sales (\( x \)).

12. Joey is training to be a competitive eater. He can eat 1.5 burritos per minute and there is one burrito left on his plate after 4 minutes. Write and graph an equation that represents the amount of burritos left on his plate (\( y \)) and the time (\( x \)) that passes.

13. Dianne has $180 to spend on some new clothes. If a pair of jeans costs $30 and shirts cost $20, write and graph an equation to represent the different combinations of jeans (\( x \)) and shirts (\( y \)) that Dianne can purchase.

14. The price of a certain stock is $4. If analysts claim that the stock will rise $2 every 6 months, write and graph an equation which represent this situation using time as the \( x \) value and the stock price as the \( y \) value.

15. Tom is running across the country to fulfill a lifelong dream. If Tom was running down a straight highway one mile outside of town (headed towards town) an hour ago and runs at a steady 6 miles per hour, write and graph an equation that represents his distance from the town (\( y \)) at any given time (\( x \)).
From Lesson Plan:

<table>
<thead>
<tr>
<th>Form</th>
<th>Generic Formula</th>
<th>Example</th>
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</thead>
<tbody>
<tr>
<td>Standard Form</td>
<td>$Ax + By = C$</td>
<td>$2x + 3y = 6$</td>
</tr>
<tr>
<td>Point-Slope Form</td>
<td>$y - y_1 = m(x - x_1)$, where $(x_1, y_1)$ is a point on the line</td>
<td>$y - 3 = 2(x + 1)$</td>
</tr>
<tr>
<td>Slope-Intercept Form</td>
<td>$y = mx + b$, where $m$ is the slope and $b$ is the y-intercept</td>
<td>$y = -\frac{2}{3}x - 1$</td>
</tr>
</tbody>
</table>

**Graph of Standard Form example**

$2x + 3y = 6$

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
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<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

**Graph of Point-Slope Form example**

$y - 3 = 2(x + 1)$
Graph of Slope-Intercept Form example

\[ y = -\frac{2}{3}x - 1 \]

From Linear Connection Tasks:

1.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

2.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>-8</td>
<td>0</td>
</tr>
</tbody>
</table>
3. \( y + 5 = \frac{5}{3}(x - 3) \)
4. \( y - 5 = -\frac{1}{2}(x + 2) \)
5. \( y = -\frac{3}{4}x - 5 \)
6. \( y = \frac{7}{4}x + 2 \)
7. \(2x + 2y = 36\)

8. \(y = \frac{20}{3}x + 12\)

9. \(y = 1.5x + 3\) or \(y = \frac{3}{2}x + 3\)

10. \(180x + 90y = 900\)
11. \( y = .3x + 20,000 \)

12. \( y - 1 = -\frac{3}{2} (x - 4) \)

13. \( 30x + 20y = 180 \)

14. \( y = 2x + 4 \) where \( x \) is time in years

15. \( y - 1 = -6x \)

where \( y \) is the distance in miles and \( x \) is the time in hours

\( y - 1 = -\frac{1}{10} x \)

where \( y \) is the distance in miles and \( x \) is the time in minutes

(since 6 miles per hour means he runs 1 mile per 10 minutes)

\( x \)-axis: each line represents 1 minute
\( y \)-axis: each line represents \( \frac{1}{10} \) of a mile
## Base Function Fun

**Student/Class Goal**
Given a situation of population growth (or decay, or other nonlinear situation), students want to be able to visualize what is happening with a graphical representation and draw conclusions based on that.

**Outcome (lesson objective)**
- Students represent contextual situations using functions.
- Students analyze functions to draw conclusions about real-life situations.

**Time Frame**
2 hours

**Standard**
*Use Math to Solve Problems and Communicate*
(Primary benchmarks in bold.)

| NRS EFL Levels | 4-6
---|---

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<tr>
<th><strong>Number Sense</strong></th>
<th>Benchmarks</th>
<th><strong>Geometry &amp; Measurement</strong></th>
<th>Benchmarks</th>
<th><strong>Algebra &amp; Patterns</strong></th>
<th>Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connect number words</td>
<td>Identify/apply basic geometric concepts</td>
<td>Solve problems using computations</td>
<td>Connect graphical and algebraic representations</td>
<td>Evaluate/solve expressions/equations</td>
<td>5.16</td>
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<tr>
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<td>Perimeter/area/volume</td>
<td>Connect relationships to representations</td>
<td>Graphing</td>
<td>5.18, 6.18</td>
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<td>Solving equations using algebra/graphs</td>
<td>5.19</td>
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<td>Right triangle trigonometry</td>
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<td>Solve problems</td>
<td>Processes</td>
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<td>Central tendency</td>
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<td>Connect concepts</td>
<td>5.25, 6.36</td>
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<td>Probability</td>
<td></td>
<td>Mathematical performance</td>
<td>4.35</td>
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</table>

**Materials**
- SmartPal kit (SmartPal sleeves, wipe off cloths, dry erase markers) – inserting a blank sheet of paper into the sleeves will give students a reusable sheet of paper that they can quickly try answers out on and erase without using up a pencil eraser. It’s quicker as well.
- Calculator
- Base Function Tasks handout

**Learner Prior Knowledge**
- Students should be familiar with plotting points on an X-Y plane, point-slope form and slope-intercept form of linear equations, solving linear equations, and translating points.

**Instructional Activities**
- **Part 1:** To begin the lesson, review the concept of point-slope form with students. Start with the basic equation \( y = x \) and graph it on a large piece of graph paper or on GeoGebra. Ask students what it would look like in point-slope form: \( y - 0 = 1(x - 0) \).
- Break the students into two groups (down the middle of the class). Ask one group of students what would the equation of a line be that was “moved” or translated 3 points to the right: \( y - 0 = 1(x - 3) \), and ask the other group of students what would be the equation of the line that was moved down 2 points: \( y + 2 = 1(x - 0) \).
- Ask students to find the equation of each graph in slope-intercept form (see Teacher Answer Sheet). Ask students if they notice any patterns between the original equation and graph, \( y = x \), and each of the other equations and graphs.
- For example, the equation \( y + 2 = 1(x - 3) \) can be thought of as taking the equation down two and over three, or in slope-intercept form, \( y = x - 5 \) just down 5 or right 5 (graph moves opposite than one might originally guess; subtracting 3 from the x means right 3). It would be worth mentioning to students that you can think of what would give you (0,0) or our original origin. For the point-slope form, you would have to plug 3 in for x and -2 in for y to get the original origin and for the slope-intercept form you would have to plug in a 5 in for the x only to get the original origin.
Then ask students what the graph $y = 3x$ looks like in comparison to the original equation $y = x$. Since (0,0) doesn’t change, ask students what happened to the point (1,1). There are two possible answers to this question, one way is that it tripled to (1,3) or it was cut by a factor of 3 to $(1/3, 1)$. So if you think in terms of x not moving, the y is tripled, but if you think of the y not moving, the x has to be cut by a factor of 3 to keep the equation balanced.

Part 2: (I do) Write the equations $y = x^2$ and $y = x^3$ on the board. Introduce students to the terms “quadratic” and “cubic” to describe each of the equations. Then make a three column t-table to find key points and graph each equation (see Teacher Answer Sheet for example). Write the equation $y = 3(x - 2)^3 + 1$ on the board. Explain that since the highest power of x is 3, we know it is a cubic, the 3 is going to stretch the graph vertically by a factor of 3, the 2 is going to move the graph 2 points to the right, and the 1 is going to move the graph one point up. From that information and the reference graph of $y = x^3$, we have everything we need to graph the new equation. Making a table can help visualize how each “move” changes your points to help graph your new equation (see Teacher answer sheet). It is important when getting the new y values, that we follow the order of operations, the stretch of 3 comes before the shift up of 1. Plot your new points on a graph to construct your new quadratic equation.

Part 3: (We do) Pass out the Base Equation Tasks handouts to students and ask students to read question 1. After giving students time to read the question, ask them what type of equation is the question asking you to graph and how you know (cubic, highest power of x is 3). Then ask students about what each non-variable number represents in the equation. Starting with the negative sign (or negative one) it “stretches” the y-values by a factor of negative one, the two in the parenthesis moves the points to the left two points, and the two outside the parenthesis means moves points up two.

Part 4: (You do) Have students work in pairs to complete the remaining problems. Provide students with graph paper to make sketches of each graph and if available, allow students to use GeoGebra or other graphing software to check their answers. As pairs finish, have the partners split up and find new partners to compare their answers.

Part 5: Go through each of the problems having students present their solutions and justify their answers. Allow time for students to check their solutions and make corrections if needed. For the word problems, make sure students understand the context. Discuss which parts of the graph make sense for the given context.

**Assessment/Evidence (based on outcome)**

Parts 4 and 5 serve as evidence of student mastery. During part 4, the teacher should actively listen to partner discussions for signs of understanding or of misconceptions. When students are working, the teacher should have students speak out loud as they solve the problems.

Exit slip: Given the following information, graph the function $y = -[(x - 3)^2] - 1$

**Given:**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = -(x^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-4</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-4</td>
</tr>
</tbody>
</table>

**Answer:**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = -[(x - 3)^2] - 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-5</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>-2</td>
</tr>
<tr>
<td>5</td>
<td>-5</td>
</tr>
</tbody>
</table>

**Graph:**

[Graph showing points (-2,-4), (-1,1), (0,0), (1,1), (1,5), (2,4), (2,-4), (3,1), (4,-2), (5,-5)]
**Teacher Reflection/Lesson Evaluation**

**Not yet completed**

**Next Steps**
After students master the transformations of base functions, introducing students to finding the roots of functions would help students build their understanding of functions and increase their capacity to construct graphical representations of functions.

**Technology Integration**
This lesson lends itself to the plotting and graphing applications of the free online software GeoGebra. Excel can also be used to calculate the translated points.

**Purposeful/Transparent**
This lesson starts with a review of point-slope and slope-intercept form to help build students understanding of how thinking of the equation $y = x$ as a base function for linear functions can offer yet another way of understanding how to transform and graph functions. After review, students are introduced to quadratic and cubic functions and their attributes as well as a step by step procedure to transform key points to graph other functions. After a doing a similar example as a class, students are paired up to work through another example and then move into contextualized problems using the concepts to find key points and information that is vital for making decisions.

**Contextual**
This lesson used equations and graphs to represent everyday situations from volume of objects, money, and objects in motion, to the growth of bacteria at different temperatures.

**Building Expertise**
Students build on their knowledge of graphing functions and translating points to find important information about real life situations.
Vocabulary Sheet

**Cubic Equation** □ Any equation of the form $ax^3 + bx^2 + cx + d = 0$ and $a \neq 0$ □ the highest power of □ is 3.

**Linear Equation** □ an equation for which the average rate of change between any pair of points remains the same and is written $ax + by = c$ in standard form □ the highest power of □ or y is 1 (though it could also be 0 □

**Quadratic Equation** □ Any equation of the form $Ax^2 + Bx + C = y$ where $A \neq 0$ □ the highest power of □ is 2.

**Translation** □ a type of transformation where you move a figure along a straight line in a given direction and distance.

**T-table** — a table used to organize points where the □-values are listed in one column and y-values are listed in another column and each row corresponds to a point.
Base Function Tasks

1. Graph the function \( y = -(x + 2)^3 + 2 \).

2. Graph the function \( y = 2(x + 3) - 1 \).

3. The formula for the amount of air needed to pump up a balloon is given by the equation \( y = \frac{4}{3}\pi(x - 1)^3 \) where \( x \) is the radius of the balloon. Using 3.14 as an estimation of pi, graph the function and use it to estimate the radius of the balloon when the volume of air in the balloon is approximately 30 cubic inches.

4. Dianne is a terrible gambler. She loses $35 an hour and runs out of money after six and a half hours at the slots machine. Determine the equation and graph of the amount of money Dianne has in her possession and determine the amount of money she started with.

5. The number of bacteria in a refrigerated piece of meat is given by the equation \( y = 20(x - 1)^2 + 53 \) where \( x \) is the temperature in degrees Celsius. Graph this function and determine the minimum number of bacteria in a piece of meat. At what temperature does this minimum occur?

6. A cube of side length \( x \) is set on top of a box with the known volume of 45 cubic inches. Determine an equation and graph for the total volume of the cube and box.

7. The height, in feet, of an object above the ground is given by \( y = -16(x - 4)^2 + 254 \), where \( x \) is the time in seconds. Graph this function and determine what would be the maximum height of the object above the ground.

8. A t-shirt company charges $5 a t-shirt plus a setup fee. If having 15 t-shirts made costs $95, determine and graph the equation that the company uses to charge its customers and determine what the company charges for a setup fee.
In the following answers, the gray columns represent the original values before the graph was shifted/scaled. The blue columns represent the points that appear on the graph after the shift/scale.

Table for graphing \( y = 3(x - 2)^2 + 1 \)
From Base Functions Tasks:

1. 

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2. 

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5. 

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<tr>
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6. 

\[ y = x^3 + 45 \]

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### 8.

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**Questioning Quadratics**

**Student/Class Goal**
Given a situation of throwing a ball or other object, students want to be able to find when the ball hits the ground.

**Time Frame**
2 hours

**Outcome (lesson objective)**
- Students use factoring to solve quadratic equations.
- Students graph quadratic equations to find solutions.
- Students will “complete the square” to solve quadratic equations.
- Students use the quadratic formula to find solutions to quadratics.

**Standard** *Use Math to Solve Problems and Communicate*
(Primary benchmarks in bold.)

<table>
<thead>
<tr>
<th>Number Sense</th>
<th>Benchmarks</th>
<th>Geometry &amp; Measurement</th>
<th>Benchmarks</th>
<th>Algebra &amp; Patterns</th>
<th>Benchmarks</th>
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<tbody>
<tr>
<td>Connect number words</td>
<td>Identify/apply basic geometric concepts</td>
<td></td>
<td>Patterns/sequences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solve problems using computations</td>
<td>Connect graphical and algebraic representations</td>
<td></td>
<td>Evaluate/solve expressions/equations</td>
<td>4.16, 5.16</td>
<td></td>
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<tr>
<td>Order of operations</td>
<td>Perimeter/area/volume</td>
<td></td>
<td>Connect relationships to representations</td>
<td>6.17</td>
<td></td>
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<tr>
<td>Compare/order numbers</td>
<td>Graphical representations</td>
<td></td>
<td>Graphing</td>
<td>5.18</td>
<td></td>
</tr>
<tr>
<td>Estimate &amp; compute to solve problems</td>
<td>Use of correct units</td>
<td></td>
<td>Solving equations using algebra/graphics</td>
<td>5.19, 6.20</td>
<td></td>
</tr>
<tr>
<td>Evaluate using roots and exponents</td>
<td>Right triangle trigonometry</td>
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**Data Analysis & Probability**

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<th>Solve problems</th>
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<td>Measurement conversions</td>
<td>Communicate ideas</td>
</tr>
<tr>
<td>Create and display data</td>
<td>Rounding</td>
<td>Reason mathematically</td>
</tr>
<tr>
<td>Central tendency</td>
<td></td>
<td>Connect concepts</td>
</tr>
<tr>
<td>Probability</td>
<td></td>
<td>Mathematical performance</td>
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</tbody>
</table>

**Materials**
- SmartPals
- Algebra Tiles
- Graph paper
- Calculators (optional)
- Questioning Quadratics worksheet

**Learner Prior Knowledge**
Students should be familiar with solving linear equations, and graphing linear and quadratic equations.

**Instructional Activities**

**Part 1: (I do)**

To start the lesson, review with students the base function \( y = x^2 \) also known as a “quadratic” as the highest power of \( x \) is two. We call \( y = x^2 \) the base function as it is as simple as a quadratic function gets. Introduce students to the concept of roots of a function, also known as the zeros of a function. Tell students that the roots of a function can represent running out of money, when two functions are the same if another function is used to represent their difference, when a projectile will reach the ground, or just as another way to find points that can help in plotting a function. This information can be introduced now or once you’ve done a few problems. The important thing is that students understand the usefulness of this type of problem.

Write the standard notation of a quadratic equation on the board, \( ax^2 + bx + c = 0 \), and remind students that \( a, b, \) and \( c \) are just numbers. For example, the base function in standard form would look like \( 1x^2 + 0x + 0 = 0 \).

To demonstrate how to solve quadratics, write the equation \( x^2 + 4x = 5 \) and the four ways of solving the equation (graphing, factoring, completing the square, and the quadratic formula) on the board. Starting with graphing, make a table to help graph the functions \( y = x^2 + 4x \) and \( y = 5 \) (both sides of the original equation).

Graph the functions on the same X-Y plane, and the solutions are the x-values of where the two intersect (this can also be
For the other methods, students may find a manipulative helpful. Give each student a set of the Algebra Tiles. For using the factoring method, be sure to emphasize writing the equation in standard form (setting it equal to 0 and combining like terms), \(x^2 + 4x - 5 = 0\). Review with students how to multiply binomials using a generic form and the Algebra Tiles (see Teacher Answer Sheet). Since we have to multiply to get -5 and add to positive 4, the numbers must be -1 and 5. Hence \((x - 1)(x + 5) = 0\). So to get 0 as our answer we know by the multiplication property of 0, \((x - 1)\) or \((x + 5)\) must be equal to 0. Setting them both equal to 0 we get \(x = 1\) or \(-5\).

Move to completing the square, again emphasizing writing the equation in standard form first and review with students how to square a binomial with generic terms and the Algebra Tiles (see Teacher Answer Sheet). Since we have the term 4\(x\), by dividing the coefficient by two, we get 2. Squaring that 2 gives us 4. So in order to have a squared binomial we would need to have 4 added in the equation, so if we add and subtract 4 from the equation we will get our squared binomial. Simplifying and solving we get our solutions of \(x = 1\) or \(-5\).

Finally, write the quadratic formula on the board, \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\), and remind students that the \(a\), \(b\), and \(c\) are the same as in standard form. Plug in the numbers from the example and show students that you still get the solutions of \(x = 1\) or \(-5\) (see Teacher Answer Sheet).

Part 2: \((\text{We do})\)
Distribute the Quadratic Equations Tasks handout to students and read the first question out loud. Ask students what it means that the ball will hit the ground (set the equation equal to 0). Suggest to students dividing each term by -4.9 to get a coefficient of the \(x^2\) term equal to one.

Tell students, that you are going to solve the equation using each method. Ask for a volunteer to help solve the equation using the graphing method. With input from fellow students and yourself, walk the student through the graphing process to solve the problem. Then ask for a volunteer to help solve the equation using the factoring method.

Again have fellow students help the volunteer complete the task. Then ask for a volunteer to help solve the problem by completing the square. Again have fellow students help the volunteer complete the task.

Finally, ask all students to take out their calculators. Using the original equation, have half the class calculate the solution using \(x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}\) and have the other half calculate the solution using \(x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}\).

Part 3: \((\text{You do})\)
Have students work in pairs to complete the remaining problems. Provide students with graph paper to solve graphically if so desired and if available, allow students to use GeoGebra or other graphing software to check their answers. As pairs finish, have the partners split up and find new partners to compare their answers and discuss the methods they used to solve each problem.

Part 4: Go through each of the problems having students present their solutions and justify their answers. Allow time for students to check their solutions and make corrections if needed.

Assessment/Evidence \((\text{based on outcome})\)
Parts 3 and 4 serve as evidence of student mastery. During part 4, the teacher should actively listen to partner discussions for signs of understanding or of misconceptions. When students are working, the teacher should have students speak out loud as they solve the problems.

Exit slip:
1. Find the roots of the function \(y = x^2 + 15x + 56\). (Answers: -7 and -8)
2. Solve the equation \(26 = 2x^2 - 2x + 14\) (Answers: 3 and -2)

Teacher Reflection/Lesson Evaluation
Not yet completed

Next Steps
Introducing students to similar problems where the coefficient of \(x^2\) cannot be factored out easily, imaginary roots, or when one method is better to use than the others can be very helpful for students to see the importance of knowing multiple methods of solving quadratics.
<table>
<thead>
<tr>
<th>Technology Integration</th>
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<tbody>
<tr>
<td>This lesson incorporates the use of calculators along with graphing software if available to solve real life situations. <a href="http://www.khanacademy.org/math/algebra/quadtratics">Khan Academy - Videos for working with quadratics</a></td>
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</table>

<table>
<thead>
<tr>
<th>Purposeful/Transparent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Following a review of quadratics and an introduction to the different methods of solving quadratic equations, the teacher shows how to solve the same example using all four methods demonstrating that no matter which method students choose to use, they should get the same answer. Following, students and the teacher work through a similar example and each of the methods to build their understanding of each of the methods. Then students will work in pairs to further their understanding of the methods and learn when one method may be more appropriate to use than another.</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Contextual</th>
</tr>
</thead>
<tbody>
<tr>
<td>This lesson uses quadratic equations to model everyday situations involving projectile motion, area and volume of objects, and money.</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Building Expertise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students build on their knowledge of quadratic equations and solving linear equations to solve equations and find the roots of functions.</td>
</tr>
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</table>
Vocabulary Sheet

**Binomial** — an expression with two terms.

**Completing the Square** — a method of solving for the roots of a quadratic equation using the process of forming a trinomial square from a binomial of the form $x^2 + bx$.

**Factoring** — a method of solving for the roots of a quadratic equation using the process of finding the individual factors of a product.

**Quadratic Equation** — Any equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$.

**Quadratic Formula** — a method of solving for the roots of a quadratic equation in standard form,

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

**T-table** — a table used to organize points where the $x$-values are listed in one column and $y$-values are listed in another column and each row corresponds to a point.

**Trinomial** — an expression with three terms.
Quadratic Equations Tasks

Solve each of the following using the method of your choice. Use another method to check your answer.

1. If a ball is thrown from a 58.8 meter tall platform at 19.6 meters per second, the equation of the ball’s height above the ground \( y \) at time \( x \) seconds after it is thrown is \( y = -4.9x^2 + 19.6x + 58.8 \). When will the ball hit the ground?

2. During World War I, the army was firing mortar shells at a target 144 ft. above their position. If the initial velocity of the mortars was 160 feet per second, the equation for the height of the shell above the target was \( y = -16x^2 + 160x - 144 \). How long did it take for the mortar shell to strike its target?

3. Sarah has 50 feet of fencing to make a rectangular garden and plans on using the side of her house as one edge of the garden. What dimensions will give her an area of 200 square feet? Use \( x \) to denote the side of the garden that is not opposite of the house wall (see diagram below).

4. The amount of money, \( A \), in an account with an interest rate \( r \) compounded annually is given by the equation \( A = P(1 + r)^t \) where \( P \) is the initial principal and \( t \) is the number of years the money is invested. If a $10,000 investment grows to $11,664 after 2 years, find the interest rate.

5. The volume of a box with a square bottom and a height of 4 inches is given by \( y = 4x^2 \) where \( x \) is the sides of the bottom of the box. If the volume of the box is 324 square inches, find the dimensions of the box.

6. A cliff diver jumps of a 320 foot high cliff at an initial speed of 16 feet per second. If the equation for his height above the water is \( y = -16x^2 + 16x + 320 \), when will he hit the water?

7. Tina runs a pizza parlor and figured out that the profit for selling pizzas is given by the equation \( y = -\frac{1}{8}x^2 + 5x \) where \( x \) is the number of pizzas sold in one hour. How many pizzas does she need to sell in an hour to make a profit of $32 and how many pizzas does she need to sell to not turn a profit at all?

8. John’s mom made him a quilt that was 5 feet by 7 feet and wants to use a 64 square foot piece of material to make a border of uniform width. What should the width of the border be?
Teacher Answer Sheet

From lesson plan:

**Graphing:**

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<th>$y = x^2 + 4x$</th>
<th>$y = 5$</th>
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**Factoring:**

*Generic binomial multiplication*

$$(x + a)(x + b) = x^2 + bx + ax + ab = x^2 + (a + b)x + ab$$

*Example of binomial multiplication*

$$(x + 3)(x + 2) = x^2 + 2x + 3x + 2 + 3 = x^2 + (3 + 2)x + 6 = x^2 + 5x + 6$$

*Solving example:*

$$x^2 + 4x = 5$$
$$x^2 + 4x - 5 = 0$$
$$(x - 1)(x + 5) = 0$$
$$x - 1 = 0 \text{ or } x + 5 = 0$$
$$x = 1 \text{ or } x = -5$$

**Completing the square:**

*Generic squaring a binomial*

$$(x + a)^2 = (x + a)(x + a) = x^2 + ax + ax + a^2 = x^2 + 2ax + a^2$$

*Example of squaring a binomial*

$$(x + 5)^2 = (x + 5)(x + 5) = x^2 + 5x + 5x + 5^2 = x^2 + 10x + 25$$
Solving example:

\[ \begin{align*}
& x^2 + 4x = 5 \\
& x^2 + 4x - 5 = 0 \\
& (x^2 + 4x + 4) - 4 - 5 = 0 \\
& (x + 2)^2 - 9 = 0 \\
& (x + 2)^2 = 9 \\
& x + 2 = \pm 3 \\
& x + 2 = 3 \quad \text{or} \quad x + 2 = -3 \\
& x = 1 \quad \text{or} \quad x = -5
\end{align*} \]

Quadratic Equation:

\[ \begin{align*}
& x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-5)}}{2(1)} \\
& x = \frac{-4 \pm \sqrt{16 + 20}}{2} \\
& x = \frac{-4 \pm \sqrt{36}}{2} \\
& x = \frac{-4 \pm 6}{2} \\
& x = \frac{-4 + 6}{2} \quad \text{or} \quad x = \frac{-4 - 6}{2} \\
& x = 2 \quad \text{or} \quad x = -5 \\
& x = 1 \quad \text{or} \quad x = -5
\end{align*} \]
From Quadratic Equations Tasks:

1. Equation after dividing by -4.9: $0 = x^2 - 4x - 12$

Graphing method:

So $x = 6$ or $-2$ but -2 doesn’t make sense for the problem, so the answer is the ball will hit the ground 6 seconds after it is thrown.

**Note:** Since we divided by -4.9 at the beginning, this equation no longer follows the path of the ball. Due to gravity, the ball would eventually fall. However, the graph above would have the ball rising endlessly, which would not make sense given our context. This is an important note to make to students. If they want their graph to show the height of the ball at a given time, they need to keep it in its original form: $y = -4.9x^2 + 19.6x + 58.8$.

Factoring method:

$$0 = x^2 - 4x - 12$$
$$0 = (x - 6)(x + 2)$$
$$x - 6 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 6 \quad \text{or} \quad x = -2$$

Completing the square method:

$$0 = x^2 - 4x - 12$$
$$0 = (x^2 - 4x + 4) - 4 - 12$$
$$0 = (x - 2)^2 - 16$$
$$16 = (x - 2)^2$$
$$\pm 4 = x - 2$$
$$x - 2 = 4 \quad \text{or} \quad x - 2 = -4$$
$$x = 6 \quad \text{or} \quad x = -2$$
2. Equation after dividing by -16: \( 0 = \frac{x^2 - 10x + 9}{-16} \)

Graphing method:

So \( x = 1 \) or 9 but 1 doesn’t make sense for the problem as the shell would still be on its way up, so the answer is the mortar will hit its target 9 seconds after it is fired.

Factoring method:

\[
0 = x^2 - 10x + 9 \\
0 = (x - 1)(x - 9) \\
x - 1 = 0 \quad \text{or} \quad x - 9 = 0 \\
x = 1 \quad \text{or} \quad x = 9
\]

Completing the square method:

\[
0 = x^2 - 10x + 9 \\
0 = (x^2 - 10x + 25) - 25 + 9 \\
0 = (x - 5)^2 - 16 \\
16 = (x - 5)^2 \\
\pm 4 = x - 5 \\
x - 5 = 4 \quad \text{or} \quad x - 5 = -4 \\
x = 9 \quad \text{or} \quad x = 1
\]
3. Equation:

\[200 = x(50 - 2x)\]
\[200 = 50x - 2x^2\]
\[0 = -2x^2 + 50x - 200\]
\[0 = x^2 - 25x + 100\]

Graphing method:

So \( x = 5 \) or 20 since both answers make sense, she has two options: make the garden 5 feet by 40 feet where the side opposite of the wall is 40 feet or 20 feet by 10 feet where the side opposite of the wall is 10 feet.

Factoring method:

\[0 = x^2 - 25x + 100\]
\[0 = (x - 20)(x - 5)\]
\[x - 20 = 0 \quad \text{or} \quad x - 5 = 0\]
\[x = 20 \quad \text{or} \quad x = 5\]

Completing the square method:

\[0 = x^2 - 25x + 100\]
\[0 = (x^2 - 25x + 156.25) - 156.25 + 100\]
\[0 = (x - 12.5)^2 - 56.25\]
\[56.25 = (x - 12.5)^2\]
\[\pm 7.5 = x - 12.5\]
\[x - 12.5 = 7.5 \quad \text{or} \quad x - 12.5 = -7.5\]
\[x = 20 \quad \text{or} \quad x = 5\]
4. Equation:

\[ 11,664 = 10,000(1 + r)^2 \]
\[ 1.1664 = (1 + r)^2 \]
\[ 1.1664 = r^2 + 2r + 1 \]
\[ 0 = r^2 + 2r - .1664 \]

Graphing method:

So \( x = -2.08 \) or 0.08, but since -2.08 doesn’t make sense, the interest rate is 0.08.

Factoring method:

\[ 0 = r^2 + 2r - .1664 \]
\[ 0 = (r + 2.08)(r - 0.08) \]
\[ r + 2.08 = 0 \quad \text{or} \quad r - 0.08 = 0 \]
\[ r = -2.08 \quad \text{or} \quad r = 0.08 \]

Completing the square method:

\[ 0 = r^2 + 2r - .1664 \]
\[ 0 = (r^2 + 2r + 1) - 1 - .1664 \]
\[ 0 = (r + 1)^2 - 1.1664 \]
\[ 1.1664 = (r + 1)^2 \]
\[ \pm 1.08 = r + 1 \]
\[ r + 1 = 1.08 \quad \text{or} \quad r + 1 = -1.08 \]
\[ r = 0.08 \quad \text{or} \quad r = -2.08 \]
5. Equation:

\[
324 = 4x^2 \\
0 = 4x^2 - 324 \\
0 = x^2 - 81
\]

Graphing method:

So \( x = -9 \) or 9, but since -9 doesn’t make sense, the dimensions of the box are 9 inches by 9 inches by 4 inches.

Factoring method:

\[
0 = x^2 - 81 \\
0 = (x + 9)(x - 9) \\
x + 9 = 0 \text{ or } x - 9 = 0 \\
x = -9 \text{ or } x = 9
\]

Completing the square method:

\[
0 = x^2 - 81 \\
81 = x^2 \\
\pm 9 = x \\
x = 9 \text{ or } x = -9
\]
6. Equation after dividing by -16: \(0 = x^2 - x - 20\)

Graphing method:

\[
\begin{align*}
0 &= x^2 - x - 20 \\
0 &= (x - 5)(x + 4) \\
x - 5 &= 0 \quad \text{or} \quad x + 4 = 0 \\
x &= 5 \quad \text{or} \quad x = -4
\end{align*}
\]

So \(x = -4\) or 5 but -4 doesn’t make sense for the problem, so the answer is he will hit the water 5 seconds after he jumps.

Factoring method:

Completing the square method:

\[
\begin{align*}
0 &= x^2 - x - 20 \\
0 &= (x^2 - x + .25) - .25 - 20 \\
0 &= (x - .5)^2 - 20.25 \\
20.25 &= (x - .5)^2 \\
\pm 4.5 &= x - .5 \\
x - .5 &= 4.5 \quad \text{or} \quad x - .5 = -4.5 \\
x &= 5.5 \quad \text{or} \quad x = -3.5
\end{align*}
\]
7. For profit of $32

Equation in standard form after multiplying by -8: \( 0 = x^2 - 40x + 256 \)

Graphing method:

So \( x = 8 \) or 32 and since both make sense, the answer is either 8 or 32 pizzas.

Factoring method:

\[
\begin{align*}
0 &= x^2 - 40x + 256 \\
0 &= (x - 8)(x - 32) \\
x - 8 &= 0 \quad \text{or} \quad x - 32 = 0 \\
x &= 8 \quad \text{or} \quad x = 32
\end{align*}
\]

Completing the square method:

\[
\begin{align*}
0 &= x^2 - 40x + 256 \\
0 &= (x^2 - 40x + 400) - 400 + 256 \\
0 &= (x - 20)^2 - 144 \\
144 &= (x - 20)^2 \\
\pm 12 &= x - 20 \\
x - 20 &= 12 \quad \text{or} \quad x - 20 = -12 \\
x &= 32 \quad \text{or} \quad x = 8
\end{align*}
\]
For no profit

Equation in standard form after multiplying by -8: \(0 = x^2 - 40x\)

Graphing method:

![Graph of the equation](image)

So \(x = 0\) or 40 and since both make sense, the answer is either 0 or 40 pizzas.

Factoring method:

\[
0 = x^2 - 40x \\
0 = x(x - 40) \\
x = 0 \quad \text{or} \quad x - 40 = 0 \\
x = 40
\]

Completing the square method:

\[
0 = x^2 - 40x \\
0 = (x^2 - 40x + 400) - 400 \\
0 = (x - 20)^2 - 400 \\
400 = (x - 20)^2 \\
\pm 20 = x - 20 \\
x - 20 = 20 \quad \text{or} \quad x - 20 = -20 \\
x = 40 \quad \text{or} \quad x = 0
\]
Equation:

\[ 64 = (2x + 5)(2x + 7) - 35 \]
\[ 64 = 4x^2 + 14x + 10x + 35 - 35 \]
\[ 64 = 4x^2 + 24x \]
\[ 0 = 4x^2 + 24x - 64 \]
\[ 0 = x^2 + 6x - 16 \]

Graphing method:

So \( x = -8 \) or 2, but since -8 doesn’t make sense, the answer is the border should be 2 feet in width.
Factoring method:

\[ 0 = x^2 + 6x - 16 \]
\[ 0 = (x + 8)(x - 2) \]
\[ x + 8 = 0 \quad \text{or} \quad x - 2 = 0 \]
\[ x = -8 \quad \text{or} \quad x = 2 \]

Completing the square method:

\[ 0 = x^2 + 6x - 16 \]
\[ 0 = (x^2 + 6x + 9) - 9 - 16 \]
\[ 0 = (x + 3)^2 - 25 \]
\[ 25 = (x + 3)^2 \]
\[ \pm 5 = x + 3 \]
\[ x + 3 = 5 \quad \text{or} \quad x + 3 = -5 \]
\[ x = 2 \quad \text{or} \quad x = -8 \]
# TABLES, GRAPHS, AND CHARTS, OH MY!

<table>
<thead>
<tr>
<th>Student/Class Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>After seeing a graph on TV or while reading a newspaper or magazine, students want to be able to read and interpret what appears on the graph.</td>
</tr>
</tbody>
</table>

## Outcome (lesson objective)
Students will develop understanding of vocabulary pertaining to tables, graphs, and charts. Students will make connections between tables, graphs, and charts involving everyday situations and data.

## Time Frame
2 Hours

## Standard
Use Math to Solve Problems and Communicate

### NRS EFL
Level 3-6

<table>
<thead>
<tr>
<th>Number Sense</th>
<th>Geometry &amp; Measurement</th>
<th>Algebra &amp; Patterns</th>
<th>Processes</th>
<th>Benchmarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connect number words</td>
<td>Identify/apply basic geometric concepts</td>
<td>Patterns/sequences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solve problems using computations</td>
<td>Connect graphical and algebraic representations</td>
<td>Evaluate/solve expressions/equations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Order of operations</td>
<td>Perimeter/area/volume</td>
<td>Connect relationships to representations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compare/order numbers</td>
<td>Graphical representations</td>
<td>Graphing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimate &amp; compute to solve problems</td>
<td>Use of correct units</td>
<td>Solving equations using algebra/graphs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Evaluate using roots and exponents</td>
<td>Right triangle trigonometry</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data Analysis &amp; Probability</th>
<th>Measurement applications</th>
<th>Solve problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpret data</td>
<td>3.16, 4.20, 5.20</td>
<td>Communicate ideas</td>
</tr>
<tr>
<td>Create and display data</td>
<td>3.17, 4.21</td>
<td>Reason mathematically</td>
</tr>
</tbody>
</table>

### Materials
- Slide N’ Measure Compass (can double as a straightedge or ruler)
- Big Minnow Lake Fishing Data
- Average Household Spending Task
- The Growing Trashpile Task
- The Name Game Task

### Learner Prior Knowledge
Students should be familiar with an X-Y plane and how to solve equations.

### Instructional Activities

#### Step 1:
To help students get prepared for the lesson, make a table with three columns and write each of the following on the board in the first column as ways to organize and display data: tables, line graph, bar graph, pictograph, histogram, and pie chart (see Teacher Answer Sheet). Then distribute Big Minnow Lake Fishing Data handout to students and go through each of the examples explaining the attributes of each display type. Starting with the table, be sure to call attention to the title, column headings, columns, and rows. For the line graph, bar graph, pictograph, and histogram, be sure to call attention to the title, axis labels, and scales of each display. For the pie chart, call attention to the title, the segment labels, and how size of each segment is proportional to the percentage of each category compared to the total.

#### Step 2:
Lead a discussion on the advantages and disadvantages of each visual display, writing down responses in the corresponding columns (see Teacher Answer Sheet). Ask students what they like/dislike about each display and how they compare to each other as they all display the same data.

#### Step 3: (I do)
Teacher leads the Average Household Spending Task using the Think Aloud technique. Pass out the Task 1 (Average Household Spending Task) handout to each student and give the students approximately 2 minutes to look over the graphs, chart, and table. Starting with question 1, instructor explains out loud what each display visually represents and the reasoning for selecting the display of choice. For question 2, explain that you just need to add the two percentages displayed by the Pie Chart in order to find the percentage for both. For question 3, rereading the wording of the instructions may be helpful to understand why the 34% is not of total income but of just the categories listed in table 2.1. For question 4, just like the percentages, we need to add the average spending of each from Table 2.1. For question 5, besides discussing each graph separately, be sure to
compare how the graphs show the same data differently (see Teacher Answer Sheet).

Step 4: (We do) Teacher and class work through The Growing Trashpile Task together. After giving students time to look over the task, create your graphs using a ruler to make sure your lines are straight and your scale is evenly distributed. Let students decide height of each bar (be sure to keep space between bars). This can also be done using Excel or other statistical software if available. Before moving to the remaining questions, make sure each student has correctly created each graph (depending on your class dynamics, you may want to have students check another student’s work). Students should be able to provide the answer to question two without help, but be sure to show how the difference can be calculated and visualized in each of the different displays (including the table). For questions 3 through 6, allow students time to come up with an answer, have students pair up and discuss possible answers. If no one is willing or able to answer, provide students with hints without giving away the answer. For question 3, you can suggest that students look forward to question 6 for a hint. For question 4, suggest students try to approximate how many of the 1960 trashcans can fit in the 2000 trashcan. For question 5, ask students who would benefit from making it look like the average trash discarded grew more than it actually did. For question 6, if students don’t know where to start, ask them to refer back to Polya’s four steps and suggest that they try to write an algebraic equation that represents the question.

Step 5: (You do) Students independently work through The Name Game Task. Students should work individually on this task, but may need some explanation of the instructions. After passing out the handouts, walk around the room silently monitoring the students’ progress. When you see them run into difficulties, try not to answer their questions directly; instead, remind them of definitions from the introduction or similar situations from the first two tasks.

Step 6: Have each student (or pair) share their visual displays for Task 3 with the entire group. Lead a discussion on the difference between qualitative and quantitative data (and their relation to the different types of displays), hopefully based on the students’ findings from Task 3.

Assessment/Evidence (based on outcome)
Steps 5 and 6 will serve as evidence of student mastery. During Step 5, the teacher should walk about the room asking students to explain their actions and look for signs of misunderstandings or misconceptions. In step 6, allow students the opportunity to modify their solutions based on what they learn from listening and watching others present their solutions.

Exit Slip: For each of the following, determine which type of display would best display the data.
1. The change in temperature throughout the year. (Line Graph)
2. The amount of time outside of school a child spends watching TV, playing video games, exercising, doing school work, and sleeping. (Pie Chart)
3. The number of home runs hit by the participants of the Home Run Derby. (Bar Graph or Pictograph)
4. The number of students that scored 0-9, 10-19, 20-29, 30-39, and 40-50 points on a test. (Histogram)

Teacher Reflection/Lesson Evaluation
Not yet completed

Next Steps
Have students collect and organize a variety of data types using tables and displays. You can also introduce students to displays of more than one variable.

Technology Integration
If available, Excel can be used to create graphs and charts. If more explanation is need for the differences between bar graphs and histograms, check out: http://illuminations.nctm.org/LessonDetail.aspx?id=L812

Purposeful/Transparent
This lesson starts with the introduction to different types of displays and the differences between them. Then the teacher goes through a task providing students with the opportunity to gain understanding and build their vocabulary before giving the students the chance to explore the concepts on their own.

Contextual
This lesson uses data from real life situations and requires students to read and make judgments about different types of displays that are commonly found throughout the media and can be utilized in nearly every facet of life.

Building Expertise
Students will build their understanding of different types of displays allowing them to make sense of displays found throughout their everyday lives and be critical of those that are used to persuade them to feel a certain way.
**Bar Graph** — a display type where the number of data values falling in a category is represented by the height or length of a rectangle (a bar).

**Histogram** — a display type similar to a bar graph, but without spaces between categories to show connection between categories.

**Interpolate** — estimate the y-values for all x's in between the minimum and maximum values.

**Line Graph** — a display type which connects points with line segments to better show a trend with categories listed on the x-axis and values listed on the y-axis.

**Pictograph** — a display type similar to a bar graph, but the height or length of the bars is replaced with symbols or pictures to represent the number of data values falling into each category.

**Pie Chart** — a display type that displays the relative number of data values in each category by the size of the corresponding sector or slice of a circle where the whole circle represents 100% or all of the data values.

**Qualitative** — data that is expressed using a means other than numbers. Quantitative data can include pictures, words, and sound.

**Quantitative** — data that is expressed using numbers and frequencies. Lists of people's ages, heights, or weights are all examples of quantitative data.
Big Minnow Lake Fishing Trip

<table>
<thead>
<tr>
<th>Day</th>
<th>Number of fish caught</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>20</td>
</tr>
<tr>
<td>Tuesday</td>
<td>25</td>
</tr>
<tr>
<td>Wednesday</td>
<td>15</td>
</tr>
<tr>
<td>Thursday</td>
<td>13</td>
</tr>
<tr>
<td>Friday</td>
<td>12</td>
</tr>
<tr>
<td>Saturday</td>
<td>15</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
</tr>
</tbody>
</table>

Line Graph:

Bar Graph 1 (in order of number of fish caught):

Bar Graph 2 (in order of date):

Pictograph:

Histogram:

Pie Chart:
Average Household Spending Task

<table>
<thead>
<tr>
<th>Category</th>
<th>Avg Spending</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transportation</td>
<td>$8,758</td>
</tr>
<tr>
<td>Entertainment</td>
<td>$2,698</td>
</tr>
<tr>
<td>Food</td>
<td>$6,133</td>
</tr>
<tr>
<td>Alcohol/Tobacco</td>
<td>$780</td>
</tr>
<tr>
<td>Insurance</td>
<td>$5,336</td>
</tr>
<tr>
<td>Healthcare</td>
<td>$2,853</td>
</tr>
<tr>
<td>Housing</td>
<td>$16,920</td>
</tr>
</tbody>
</table>

Table 2.1 Average Spending per household unit

For the following questions, refer to the data in Table 2.1 and Figures 2.1–2.3. Table 2.1 gives the breakdown of select categories of spending for the average family unit in 2008 (an average family unit is defined as 2.5 people, so a household of 5 people would be considered as 2 units). Note that the original data set has several more categories of spending not included in this data.

1. How much money did the average unit spend on food and transportation?

2. According to the chart, what percent of money was spent on food and transportation?

3. Return to Question #2. Is the answer you gave the percentage of total income that is spent on food and transportation? If not, what is this percent out of?

4. What are the advantages and disadvantages of each of the three figures in conveying the information?

5. Which of the three figures do you think best conveys the information shown in Table 2.1?

---

The Growing Trashpile Task

Average Units of Trash Discarded by Household

<table>
<thead>
<tr>
<th>Year</th>
<th>Trash Discarded</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>100</td>
</tr>
<tr>
<td>1980</td>
<td>175</td>
</tr>
<tr>
<td>2000</td>
<td>250</td>
</tr>
</tbody>
</table>

Table 2.2 Average units of trash discarded per household

Figure 2.4 Pictograph

For the following questions, refer to the data in Table 2.2.

1. Use the space above to construct a bar graph and a line graph for the data.
2. What was the increase in average trash discarded between 1980 and 2000?
3. What meaning do the lines in the line graph have?
4. How might Figure 2.4 cause the data to be misinterpreted? Is this same problem present in your bar graph or line graph? How might you prevent this problem?
5. Based on your answer to #4, who might stand to benefit from distorting the data like Figure 2.4 does?
6. Use the line graph to interpolate the units of trash discarded per household in 1985. What assumption does this estimate make?
The Name Game Task

For this task, you will need to collect and organize your own data.

1. Choose five letters of the alphabet. Now take 30 seconds per letter and write down as many people you know as possible whose first name begins with that letter (two and a half minutes total). Tally the number of names per letter and then create a bar graph to represent your data (there should be five bars).

2. Return to all the names you wrote down in question one and place them each in a five-year age bracket. For example 15-19, 20-24, 25-29, etc. You may have to estimate some ages if you are not sure about them. Create a histogram for these data.

3. Try to create a histogram for #1. What problems do you run into?

4. What differences do you notice between a bar graph and a histogram?

5. What types of information would be best represented by a bar graph? By a histogram?
From Lesson Plan:

<table>
<thead>
<tr>
<th>Display Type</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table</td>
<td>Organizes data neatly (great for all types of data).</td>
<td>Hard to compare between categories visually.</td>
</tr>
<tr>
<td>Line graph</td>
<td>Shows connection between categories as category values increase. Needs to be quantitative data (numbers).</td>
<td>Data must be ordered and connected.</td>
</tr>
<tr>
<td>Bar graph</td>
<td>Visually displays differences between categories. Can display quantitative data as well as frequency of qualitative data (words).</td>
<td>Has gaps between categories when sometimes it isn’t appropriate.</td>
</tr>
<tr>
<td>Pictograph</td>
<td>Same as bar graph, but makes connection between what the data represents.</td>
<td>More to translate in order to understand the display and partial images hard to translate.</td>
</tr>
<tr>
<td>Histogram</td>
<td>Shows connection between bars as categorical value increases.</td>
<td>Categorical value determines placement of data in display and cannot be reordered.</td>
</tr>
<tr>
<td>Pie Chart</td>
<td>Makes for easy comparison between categories and categorical amounts in relation to total. Great for qualitative data.</td>
<td>Loss of visual totals and order of categorical values.</td>
</tr>
</tbody>
</table>

Task 1:

1. $14,891
2. 34%
3. No, it is 34% of the spending listed in table 2.1 as opposed to all possible spending. There could be categories left off of the table.
4.  

<table>
<thead>
<tr>
<th>Figure</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line Graph</td>
<td>Shows difference between categories nicely.</td>
<td>Shows inappropriate connection between categories.</td>
</tr>
<tr>
<td>Pie Chart</td>
<td>Shows good comparison between all categories</td>
<td>No idea about the actual value of each of the categories.</td>
</tr>
<tr>
<td>Bar Graph</td>
<td>Shows total of each category and keeps each category separate.</td>
<td>Hard to see compare some categories, such as entertainment and healthcare. It is also difficult to tell exactly how much was spent in each category.</td>
</tr>
</tbody>
</table>

5. Pie Chart as it allows for easy comparison between all categories listed and displays percentage of spending compared to other expenses included. However, we do need to keep in mind that the percentages listed are not the percentage of total spending. Rather, it is just the percentage of spending reported by the table.
2. 75 units of trash
3. An approximation of the number of units of trash between data points.
4. The amount of trash increased by 150%, but because the size of the trash can was increased in width as well as height, it looks like it increased much more than that. This problem doesn’t exist in the bar graph.
5. Anyone who wants to make it seem that the amount of trash has grown more per household than it really has (environmentalists).
6. $f(1985) = 150 + \frac{75}{20}(5) = 150 + 18.75 = 168.75$ units of trash. It assumes that the growth in units is linear.
Task 3:

Answers for 1 and 2 will vary between students. Make sure age groups in 2 are distributed evenly.

3. No connection between the different letters.
4. Histograms: order matters and no separation between categories.
   Bar Graphs: order doesn’t matter and categories are separated.
5. Bar graphs are best for categories that are distinct from each other and order doesn’t matter such as frequency of responses to favorite color or gender. Histograms are best for data that are ordered and connected such as age groups or test scores.
CENTRAL TENDENCIES AND SPREAD WITH MONEY

Student/Class Goal
Given a set of data from a corporation, students want to interpret the meanings of mean, median, and mode so that they better understand what the data is telling them.

Outcome (lesson objective)
Students will compare and contrast the mean, median, and mode in contextual situations. Students will verbally justify which measure of center is best for a given situation.

Time Frame
4 hours (can be split into two 2-hour lessons)

Standard
Use Math to Solve Problems and Communicate
(Primary benchmarks in bold.)

<table>
<thead>
<tr>
<th>NRS EFL Levels 3-6</th>
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</table>

<table>
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<tr>
<th>Number Sense</th>
<th>Benchmarks</th>
<th>Geometry &amp; Measurement</th>
<th>Benchmarks</th>
<th>Algebra &amp; Patterns</th>
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<tr>
<td>Connect number words</td>
<td>Identify/apply basic geometric concepts</td>
<td>Perimeter/area/volume</td>
<td>Graphical representations</td>
<td>Evaluate/solve expressions/equations</td>
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<tr>
<td>Solve problems using computations</td>
<td>Connect graphical and algebraic representations</td>
<td>Graphical representations</td>
<td>Graphing</td>
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<tr>
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<td>Use of correct units</td>
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<th>Benchmarks</th>
<th>Measurement applications</th>
<th>Benchmarks</th>
<th>Processes</th>
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<td>Rounding</td>
<td>Reason mathematically</td>
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<tr>
<td>Create and display data</td>
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<tr>
<td>Probability</td>
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<td></td>
<td>Connect concepts</td>
<td>3.27, 4.34, 5.35, 6.36</td>
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</tbody>
</table>

Materials
Centimeter Cubes
Calculators
Paying Bills Task Handout
Savings Account Task Handout
Teacher Answer Sheet
Vocabulary Sheet

Learner Prior Knowledge
Students should be able to perform accurate calculations for exponential equations using order of operations. Students should be able to plot a coordinate pair on the X-Y plane, and then interpolate between points.

Instructional Activities
Step 1: As you hand out the vocabulary sheet to each student, begin a discussion about what the word “average” means and some common uses of averages (sports, school grades, income/salary, prices). Conclude the discussion with the definition that an average is a single number used to describe the center of a set of data. Explain to students that they will be exploring three such averages: mean, median, and mode (write each on the board with its definition). Move discussion toward the meaning of each (mean is what we typically think of as average “what we mean by average,” median is the middle section splitting a highway, and mode is like the common saying “their mode of operation” or the thing someone does most often). Provide students with a quick example like test averages. If a student scored 82, 94, 74, 82, and 73 on five tests, his average score would be calculated by \( \frac{82+94+74+82+73}{5} = 81 \). To find the median, first order the sample from least to greatest (73, 74, 82, 82, 94) and then find the number in the middle (82). For the mode, the number that appears the most often is 82 as it appears twice while the rest of the numbers appear only once. Following that example, write the following numbers on the board: 1, 4, 5, 13, 3, 3, 7, 5, 4, 5 and show students how to find the mean, median, and mode of this set. Explain that these are all different ways of describing the center of a set of numbers (formally known as central tendencies or measures of center). When finding the median, be sure to
emphasize the need to order the data from smallest to largest. Also, with the median, make sure the distinguish between finding the median of a data set with an even number of terms and a data set with an odd number of terms. For mode, make sure to clarify that while all examples so far have had only one mode, it is possible to have more than one mode. For example, if we took away a 5 from our data set, we would have 1, 4, 5, 13, 3, 3, 7, 5, 4. In this set, 3, 4, and 5 all appear twice while the other numbers appear only once. Since 3, 4, and 5 appear more than once and all tie for the most appearances, they are all considered modes of the data set.

Step 2: Pass out 50 centimeter cubes to each student. To get them familiar with using the cubes, ask them to model the measures of center from Step 1. In other words, how could they use the cubes to figure out the mean, median, and mode of the set? There are two main types of methods the students might use (see Teacher Answer Sheet, Method 1 and Method 2); make sure they see them both (if they only come up with one, show them the other one also). Method 1: Each cube represents 1 unit (one cube would represent 1, four cubes stacked together would represent 4, and thirteen cubes stacked together would represent 13). Using this method, the students will need all 50 cubes to model the problem. To find the median and mode, they must first arrange their stacks in order of height. To find the mean, take cubes from the tallest stacks and place them onto the smaller stacks until all the stacks are as equal as possible (for this problem, they will all equal 5). Method 2: Each cube will represent one number from the data set. So we will use 10 cubes in total with a cube representing each number in the data set (use one cube to represent the 1, two cubes to represent the two 4’s, and one cube to represent the 13, and so on). In this case, they will only need 10 cubes, but they will need some way to hold a place for each value (like a meter/yard stick or a simple number line), even the missing values of 2 and 6 (see teacher answer sheet). Using this method, the mode is the highest stack (5), the median is between the 5th and 6th cube from the left (4.5), and they can find the mean by taking a cube from any two stacks and placing both cubes in the middle until all blocks are at the same number (5). There are many ways to do this; one way to do so is: Take a block from 3 and the block at 13 and place them both at 8. Take the block from 1 and the block from 3 and place them at 2. Take a block from 2 and one block from 4 and place them both at 3 (then repeat). Take a block from 3 and one block from 5 and place them both at 4 (then repeat two more times). Take a block from 4 and one block from 8 and place them both at 6 (then repeat). Take a block from 3 and one block from 7 and place them both at 5. Take a block from 2 and one block from 4 and place them both at 3 (then repeat until all blocks are at 5).

Step 3: (I do) Teacher models the solution process. Pose this problem to students, “Find a set of five numbers for which (a) the median is 3, (b) the mode is 3, and (c) the mean is 5.” Walk students through the process of creating such a data set using the cubes to visualize your process. Be sure to emphasize what is needed both visually and computationally to satisfy all three conditions. There are an infinite number of solutions and you may want to show more than one solution. (Solution 1, using method 2) You can start by assigning one cube the number 1, one cube the number 2, one cube the number 3, one cube the number 4, and one cube the number 5 to satisfy the condition (a). Then point out that the mode should be 3, so you can choose any of the non-three cubes and move it to three so you have two cubes at 3 making your mode and median 3. Using the definition of mean and that you have five numbers, you know the sum of your numbers has to be 25 in order to get your mean of 5, but you must do so without changing the median or the mode. So by increasing your maximum number until you get a sum of 25, you will get your mean of 5 without changing your median or mode. (Solution 2, using method 1) Then using another set of 5 blocks, you can also start with all the blocks at 3 and point out that conditions (a) and (b) are satisfied but the mean needs to be increased by 5 and moving one cube to 13 would satisfy all conditions. (Solution 3, using method 1) Lastly, using 25 cubes, make 5 stacks of 5 that would represent having 5 numbers and a mean of 5. Then to satisfy the condition of your mode being 3, take two cubes from two of stacks, then rearrange the other three stacks in such a way that none of them have the same number of cubes. To make sure that the median is 3, you have to make sure that at least one and no more than two of the non-three stacks smaller than 3; placing the removed cubes onto any of the other non-three stacks in such a way as not to have any other stacks (other than your two stacks of three) have the same amount of cubes.

Step 4: (we do) Teacher and students collaboratively work through the problem. Pose this problem to students, “Find a set of 7 numbers for which (a) the median is 6, (b) the mode is 2, and (c) the mean is 5.” Have students discuss aloud as to which method they want to use (either one is fine), and work your way through the problem. You may want to start with the mode and ask yourself out loud what it means to “keep the mode the same.” When you feel you have an answer, make sure you go back through and check whether the median, mode, and mean of your modified set meet the requirements of the problem. Be sure to comment that this is not the only correct solution. Then pose the question “How could I change this set of numbers, so that (a) the median stays the same, (b) the mode stays the same, and (c) the mean decreases?” Again, there are multiple solutions to this question, but the easiest way is by decreasing any data point besides a two or six while being sure the order has remained the same to keep 6 the median and the mode is still 2. For example, consider the data set 1, 2, 2, 6, 7, 8, 9; the only number that you can decrease without changing the mode or median is the 1; making it 0 would decrease your mean and keep the median 6 and mode 2. Another example would be 2, 2, 2, 6, 7, 8, 8; then you would not be able to decrease any of the two’s as there are 2 eights, so you have two options, change the 7 to a six, or change one of the eights to either a 7 or a six. If you decrease the number to anything lower than 6, you will change your median.
Step 5: (you do) Students independently work through the problem. Pose this problem to students, “Change your current set of numbers, so that (a) the median decreases, (b) the mode stays the same, and (c) the mean increases.” Allow students to choose a starting point for the problem, but if they struggle to decide, suggest that they begin with the mode again. The key learning point in Step 5 is that the mean is affected by how far each block is from the center, whereas the median is only impacted by whether a block is to the right or the left of the center.

Teacher note: If you need to split this lesson into two parts, this would be a natural spot to split. Provide students with task 3. If you have level 3/4 students in the class, you may want to save the calculations of variance and standard deviation in steps 6–8 for a later lesson, or skip them in exchange for more time with the other calculations. When calculating variance and standard deviation, it may be most helpful to set up a chart where the columns are labeled as follows: Original data points, Deviation from mean (subtract the mean from each point), and Squared deviations (square each number from the previous column). The variance is then the mean of the numbers in the final column. The standard deviation is the square root of the variance. For this level, it is sufficient to understand how to carry out the calculation procedure (Excel is an excellent resource for these calculations if available; see spreadsheet in teacher resources).

Step 6: A second group of measurements are called measures of spread. Examples are range, five-number summary, interquartile range (IQR), variance, and standard deviation of this set. If there are any terms that the students are not familiar with, give them the definition from the Vocabulary Sheet. Using the original set of numbers (1, 4, 5, 13, 3, 3, 7, 5, 4, 5), work through the calculation of each of these five measures. The five-number summary will probably be new to many students. It is important to note that it is easiest if the numbers are placed in order from smallest to largest. List the five numbers next to the set of numbers in order (min, Q1, med, Q3, and max). It would be worth mentioning the fact that the minimum is also known as Q0 (starting point), the median is known as Q2 (second quarter point), and the maximum is also known as Q4 (fourth quarter point). It may be easiest to start by finding the median, and then visualizing it as a wall that separates two sets of data (1, 3, 3, 4, 4 on one side and 5, 5, 5, 7, 13 on the other). Now find the median of the first set of numbers (this number is Q1, quartile one, or the number that sits at the one quarter point of the data set) and then the median of the second set of numbers (this is Q3, quartile three or the number that sits at the three quarter point of the data set). Everything else can be calculated directly from the definitions (five-number summary: 1, 3, 4.5, 5, 13; range: 12; IQR: 2; variance: 9.4; standard deviation: 3.07). It may be helpful to explicitly call attention to and discuss the relationship and difference between variance and standard deviation.

Step 7: Take away one of the 5’s from the set of numbers in Step 6 and ask students to calculate the three measures of center and the five measures of spread (mean: 5; mode: 3, 4 and 5; median: 4; five-number summary: 1, 3, 4, 6, 13; range: 12; IQR: 3; variance: 10.44; standard deviation: 3.23). If students seem proficient at calculating the six measures, move on to the application in Step 8. If not, ask them to find the six measures on the following set of numbers: 1, 1, 1, 2, 5, 6, 7, 7, 10, 10 (mean: 5; mode: 1, median: 5.5; five-number summary: 1, 1, 5.5, 7, 10; range: 9; IQR: 6; variance: 11.6; standard deviation: 3.41).

Step 8: (you do). Handout the Expected Salaries and Paying Bills tasks. Depending on your class dynamics, either partner students together or have them work individually. Before you pass out the task, explain that you want the students to tackle these problems as independently as possible, using their partner to check answers after they have produced a solution. After passing out the handouts, walk around the room silently monitoring the students’ progress. When you see them run into difficulties, try not to answer their questions directly; instead, remind them of similar situations from the examples given earlier.

Step 9: Have each student (or pair) share both the process they used and their final comparisons. When students disagree, do not immediately provide the correct answer; allow each student or pair to try to convince the other first.

Assessment/Evidence (based on outcome)
Steps 8 and 9 will serve as evidence of student mastery. During Step 8, the teacher should actively listen to partner discussions for signs of understanding or of misconceptions. If students are working alone, the teacher should have students speak out loud as they solve the problem. During Step 9, allow students the opportunity to modify their solutions based on what they learn from watching others present their solutions.

Exit Slip: For the exit slip problems, use the following set of numbers: {3, 2, 3, 6, 7, 3, 4}
1. Find the mean, median, and mode. (Mean = 4, Median = 3, Mode = 3)
2. Calculate the range, five-point summary, IQR, variance, and standard deviation. (Range = 5, five-point summary= (2, 3, 3, 6, 7), IQR =3, Variance = 2.86, St. Dev. = 1.69)
3. Which of these eight measures (from exit slip #1 and #2) would stay the same if the 7 were replaced with a much larger number? (Median, Mode, Minimum, and Q1)

Teacher Reflection/Lesson Evaluation
Not yet completed
**Next Steps**
Following this lesson, students can be introduced to a box and whisker plot (most common way of graphing five-number summaries) and explore how different types of graphical displays (bar graphs, line graphs, box and whiskers, etc.) can display the same data differently and help downplay or emphasize each of the measures discussed in this lesson. Another possibility is exploring the advantages and disadvantages of each measure and when some are more appropriate to use than others.

**Technology Integration**
This lesson incorporates calculators and students can make use of Microsoft Excel to help organize, order, and calculate all eight measures explored in this lesson.

**Purposeful/Transparent**
This lesson starts with simple visualizations (interlocking cubes) of measures associated with central tendencies and spread, and progresses to critical everyday situations such as salaries, bills, and financial planning. Starting with simple visualizations and calculations will allow students to gain understandings of the similarities and differences between each of the measures allowing them to grasp the importance of each in relation to real life situations.

**Contextual**
This lesson centers on two important ways of describing financial data: central tendency and spread. For any member of society, and presumably all ABLE learners, this is an important topic for gaining financial understanding and avoiding misperceptions of published data.

**Building Expertise**
Students will build on their simple understanding of calculating the mean, median, and mode to understanding how each of them is affected by individual data points. Moreover, they will learn to make decisions based on this understanding of central tendency.
Vocabulary Sheet

**Average** – one number used to describe the center of a set of numbers. "Average" commonly refers to the arithmetic mean.

**Conjecture** – an educated guess based on incomplete evidence.

**Five Number-Summary** – minimum, Q1, median, Q3, and maximum.

**Interpolate** – to estimate the y-values for all x's in between the minimum and maximum x-values.

**Interquartile Range (IQR)** – Q3 minus Q1.

**Mean** – the sum of a set of values, divided by the number of elements in the set.

**Median** – the middle number in an ordered set (if the number of elements is odd) or the mean of the two middle numbers of an ordered set (if the number of elements is even).

**Mode** – the element(s) in a set which occur with the highest frequency. Note that there may be more than one mode (for example, the set 1, 2, 2, 3, 3, 4, 4 has three modes: 2, 3, and 4).

**Quartiles** – the minimum is the smallest number in a set, Q1 is the number at the one quarter point of an ordered data set (i.e., the median of all the numbers before the median), the median of the set, and Q3 is the number at the three quarter point of an ordered data set (i.e., the median of all the numbers after the median) and the maximum is the largest number in the set.

**Range** – maximum minus minimum.

**Standard Deviation** – a measure of how far, on average, data points are from the center. Standard deviation is measured in the original units (for example, a set of heights measured in inches would have a standard deviation in inches also). Calculated by taking the square root of the variance.

**Variance** – the average squared distance from the mean. Calculated by (a) subtracting each data point from the mean, (b) squaring this difference, (c) adding up all the squared differences, and (d) dividing by the amount of data points in the set.
<table>
<thead>
<tr>
<th>Original data points</th>
<th>Deviation from mean</th>
<th>Squared deviations</th>
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</thead>
<tbody>
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<td>16</td>
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<tr>
<td>5</td>
<td>0</td>
<td>0</td>
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</table>

**Variance**: 9.4

**Standard Deviation**: 3.07

Copy and paste the above (without parentheses) into each cell.

**Formulas**

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<th>Original data points</th>
<th>Deviation from mean</th>
<th>Squared deviations</th>
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<td>G2^2</td>
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</table>

**Variance**: 9.4

**Standard Deviation**: 3.07

Copy and paste the above (without parentheses) into each cell.
Joe has received an associate’s degree and is now looking for a workplace. He is considering three companies—Company A, Company B, and Company C. Each of the companies has published some salary information about their staff (See Table 1). Use this table to answer the following questions.

<table>
<thead>
<tr>
<th>Staff Salaries at Three Companies in 2010 (in thousands of dollars per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Minimum</td>
</tr>
<tr>
<td>Q1</td>
</tr>
<tr>
<td>Median</td>
</tr>
<tr>
<td>Q3</td>
</tr>
<tr>
<td>Maximum</td>
</tr>
</tbody>
</table>

1. What is the range and interquartile range (IQR) for each company’s staff salaries?

2. In each of the companies, the median salary is lower than the mean salary. What does this indicate?

3. Create a graph that effectively displays and compares salaries from the three companies.

4. In which company does the management probably get paid the best? Justify your answer.

5. Why could it be misleading for the companies to only publish the mean salary?
Paying Bills

Roberta has been receiving offers in the mail to enroll in a fixed-rate billing program. She has collected her monthly bill totals in a spreadsheet (See Table 2) and is now reviewing the offers. Use this table to answer the following questions.

Table 2

<table>
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<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sept</th>
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<th>Nov</th>
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<td>43</td>
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<td>48</td>
<td>55</td>
<td>63</td>
<td>75</td>
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<td>18</td>
<td>22</td>
<td>24</td>
<td>38</td>
<td>55</td>
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<tr>
<td>Water/Sewer</td>
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<td>30</td>
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<td>15</td>
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<td>30</td>
<td>15</td>
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</tr>
<tr>
<td>Cable/Internet</td>
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<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
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<td>55</td>
<td>55</td>
<td>55</td>
</tr>
</tbody>
</table>

1. What is the five-point summary, range, interquartile range (IQR), variance, and standard deviation for each of the four types of services?

2. Offer 1 allows Roberta to pay the mean price for each service each month. Without calculating, decide whether she would save money in comparison with her current payment plan. Justify your answer and check by actually calculating the amount paid.

3. Offer 2 allows Roberta to pay the median price for each service each month. Calculate how much she would pay for each service.

4. Should Roberta choose one of the offers or just pay the amount of each service every month? Use your answers from questions 2 and 3 to justify your answer.
From the Lesson Plan:

Step 2

**Expected Salaries Task Answers**

1. Company A: median = $30,000; range = $21,000; IQR = $11,000
   Company B: median = $27,000; range = $39,000; IQR = $5,000
   Company C: median = $33,000; range = $25,000; IQR = $6,000

2. This suggests that the salaries are not equally distributed; in other words, more than half of the employees earn less than the mean salary. (Simply put: The high management salaries are distorting the mean.)

3.

4. Company B. Management would likely be the highest few salaries in the company and Company B has the highest maximum salary (plus, because Company B has the highest mean and yet is less than the other two companies on minimum, Q1, median, and Q3, we know that the top 25% of salaries must be very high).

5. In an asymmetric distribution (like with salaries), the mean is not representative of the typical salary.
Paying Bills Answers

1. Electric: Five-point summary = {40, 43.5, 47, 66.5, 78}, range = 38, IQR = 23, Var = 172.8, St. D = 13.1
   Gas: Five-point summary = {18, 22.5, 31, 57.5, 66}, range = 48, IQR = 35, Var = 331.6, St. D = 18.2
   Water/Sewer: Five-point summary = {15, 15, 15, 22.5, 30}, range = 15, IQR = 7.5, Var = 42.2, St. D = 6.5
   Cable/Internet: Five-point summary = {55, 55, 55, 55, 55}, range = 0, IQR = 0, Var = 0, St. D = 0

2. Paying the mean price each month would be the same as paying the actual price each month as it is just the sum divided by 12 and making 12 payments of the mean price would give you the sum ($650 for Electric, $463 for gas, $225 for water/sewer, and $660 for cable/internet).


4. Roberta should choose option 2 as paying each month and option 1 both costs a total of $1998 and option 2 only costs a total of $1776.
**Outcome** (lesson objective)
Students will compare and contrast the ideas of permutation and combination.
Students will compute possible number of outcomes based on a contextual situation.

**Student/Class Goal**
As poker is a popular card game, and casinos are opening up across the state, students want to be more informed about the game’s odds. They want to be able to calculate the total number of possible hands and their possible winning hands to decide their chances of winning.

**Time Frame**
2 hour

**Standard**  *Use Math to Solve Problems and Communicate*
(Primary benchmarks in bold.)

<table>
<thead>
<tr>
<th>Number Sense</th>
<th>Benchmarks</th>
<th>Geometry &amp; Measurement</th>
<th>Benchmarks</th>
<th>Algebra &amp; Patterns</th>
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</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>Order of operations</td>
<td>Perimeter/area/volume</td>
<td></td>
<td>Connect relationships to representations</td>
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<td>Compare/order numbers</td>
<td>Graphical representations</td>
<td></td>
<td>Graphing</td>
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<tr>
<td>Estimate &amp; compute to solve problems</td>
<td>Use of correct units</td>
<td></td>
<td>Solving equations using algebra/graphs</td>
<td></td>
<td></td>
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<tr>
<td>Evaluate using roots and exponents</td>
<td>Right triangle trigonometry</td>
<td></td>
<td></td>
<td></td>
<td>Processes</td>
</tr>
</tbody>
</table>

**Data Analysis & Probability**  
**Benchmarks**
Measurement applications
Communicate ideas
Reason mathematically

**NRS EFL**
Levels 3-6

<table>
<thead>
<tr>
<th>Processes</th>
<th>Benchmarks</th>
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<tr>
<td></td>
<td>4.25, 4.27, 5.25, 5.27, 6.28, 6.29</td>
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<td>3.24, 4.31, 5.30, 5.31, 6.31, 6.32</td>
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<td>3.27, 4.34, 5.35, 6.36</td>
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<tr>
<td></td>
<td>4.35, 5.36, 6.37</td>
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</tbody>
</table>

**Materials**
Calculator
Decks of Playing Cards
*Combinations and Permutations Tasks* handout

**Learner Prior Knowledge**
Students should be familiar with basic mathematical operations

**Instructional Activities**
Part 1: *(I do)* Start the lesson by defining combinations and permutations on the board (both with formulas and calculator representation), and giving examples of both. Mention that although both are arrangements of a group, order does not matter with combinations, but it does with permutations. For example, with combinations the word “FAMILY” is the same as “LIFYMA” but they are different when considering permutations. If students are unaware of what the factorial is, provide a couple of examples such as 3! = 3*2*1 = 6 and 6! = 6*5*4*3*2*1 = 720. For a contextualized example, consider a little league baseball roster with 15 members on the team (see Teacher Answer Sheet). Since there are only 9 players allowed to start the game, the coach wants to know how many different ways he can select who starts the game. Since order doesn’t matter (he just has to select 9 players), this is a combination problem. To start, list three different starting rosters where two of them have exactly the same players and explain that since order doesn’t matter, these rosters are the exact same (on the Teacher Answer Sheet the last two are exactly the same since order doesn’t matter and they are composed of the same players). To compute by hand, from the formula on the board, the number of starting rosters is equal to \( \frac{15!}{(15-9)!9!} = \frac{15!}{6!9!} = 5,005 \). Also show students how to use the calculator’s “nCr” button (if using a graphing calculator or the Casio). By changing the problem just slightly to how many different lineups are possible, you now have a permutation problem; the same starting roster in a different batting order constitutes a different lineup. Now the number of possible lineups is equal to \( \frac{15!}{(15-9)!} = \frac{15!}{6!} = 1,816,214,400 \). Double check this total with...
the calculator’s “nPr” button. It’s a wonder that most little league coaches are not paid.

Part 2: (We do) For this part of the lesson you may want to have a deck of playing cards as reference. Explain to your students the idea of five card stud poker. (In this game, players are dealt one card face down and one card face up. They are then dealt three more face up cards with rounds of betting in between the dealing of each face-up card. More information on this game can be found in the technology integration section.) Then tell your students you would like to know how many possible five card hands are possible. Ask your students if this would be a combination or permutation problem and have them provide reasoning. Then ask if any students are able to calculate the answer. If you have a volunteer, have them provide reasoning. If they used the nCr button on their calculator, check their answer by using the formula and vice versa. Now ask your students how many different ways a dealer can flip over five cards (order does matter). Again ask for a student to provide an answer using one method and check the answer using the other method. Another popular poker game is Texas Hold’em where each player only holds two cards. Ask if any of your students are able to calculate the number of possible hands in Texas Hold’em. Again be sure to demonstrate both methods of calculation. Now ask your students how many different ways a dealer can flip over two cards (order does matter). Again, be sure to demonstrate both methods of calculation.

Part 3: (You do) Handout the Combinations and Permutations Tasks to your students. Have your students work individually through the problems. Once they have finished, have them pair up and check their answers.

Part 4: Go through each of the problems having a different student present their solution for each problem. As students present their solutions, make sure they provide reasoning for solving the problem as a combination or permutation.

Assessment/Evidence (based on outcome)
Part 4 allows students the chance to demonstrate the mastering of the concepts. The teacher should actively listen to students during part 3 for signs of understanding or misconceptions.

Exit Slip:
1. How many 7 card hands are possible in a game of rummy using a standard deck of 52 playing cards? (133,784,560)
2. How many different four letter acronyms are possible that have no repeating letters? (358,800)

Teacher Reflection/Lesson Evaluation
Not yet completed

Next Steps
Combinations and permutations are great as a lead into probability. Now that students are able to calculate the total number of ways something can happen, they can take the number of things they want to have happen and divide it by the total number of possible outcomes (e.g., the probability of being dealt “Pocket Aces”, or two aces to start with, in Texas Hold’em).

Technology Integration
If available, Excel spreadsheets can be used to organize and show different permutations and combinations along with calculations.
http://poker.oddschecker.com/poker-games-reviews/5-card-stud-rules.html
Five card stud rules
Texash Hold’em rules

Purposeful/Transparent
This lesson begins with the definitions and examples of combinations and permutations. The teacher then uses a contextualized example to demonstrate how to determine what formula to use and how to use the calculator. Then students will help walk through a series of tasks using playing cards and wrap up working individually and in pairs to gain understanding of and independence when dealing with the concepts.

Contextual
This lesson uses multiple different everyday situations that involve combinations and permutations, such as baseball lineups, playing cards, and lotteries.

Building Expertise
Students will build upon their understanding of how the number of options available and the number of those selected are related to apply them to real life situations.
Factorial □ For an integer greater than 0, call it \(n\), it is the product of all natural numbers less than or equal to \(n\). Denoted by an exclamation point (\(n! = n(n - 1)(n - 2) \ldots (1)\). For example \(6! = 6(5)(4)(3)(2)(1) = 720\). We read \(n!\) as \(n\) factorial. □

Combination □ Collection of objects selected from a larger or as large collection, where the order of selection is not important. Denoted by \(nC_r\) where \(n\) stands for the number of objects in the collection to be selected from and \(r\) stands for the number of objects to be selected, and is equal to \(\frac{n!}{(n-r)!r!}\). For example, if you select 2 people out of 8 to win a prize, the number of possible combinations is equal to \(8C_2 = \frac{8!}{(8-2)!2!} = \frac{8(7)(6)(5)(4)(3)(2)(1)}{6(5)(4)(3)(2)(1)(2)(1)} = \frac{8(7)}{2} = 28\).

Permutation □ Collection of objects selected from a larger or as large collection, where the order of selection is important. Denoted by \(nP_r\) where \(n\) stands for the number of objects in the collection to be selected from and \(r\) stands for the number of objects to be selected, and is equal to \(\frac{n!}{(n-r)!}\). For example, if you want to find how many ways 8 people can finish first and second in a race, the number of possible permutations (since finishing first is different than finishing second) is equal to \(8P_2 = \frac{8!}{(8-2)!} = \frac{8(7)(6)(5)(4)(3)(2)(1)}{6(5)(4)(3)(2)(1)} = 8(7) = 56\).
Combinations and Permutations Tasks

1. Josie is hanging pictures in her new room. She has 6 pictures but only 4 spots to hang her pictures. How many different choices does she have to hang her pictures?

2. Patty is packing for a three day business trip and has to choose between 12 different outfits. If she needs to pack 3 different outfits, how many different selections can she make?

3. The lottery has 40 numbered balls and picks 5 balls. How many different ways can the numbers be picked if order does not matter?

4. The chess club has 14 members. Due to new rules, the club must elect a president, vice president, secretary, and treasurer. How many different ways can they fill these positions if no person can hold two positions?

5. There are five women and six men in a group. From this group a committee of 4 is to be chosen. How many different ways can a committee be formed that contains three women and one man?

6. The game of euchre uses only 24 cards from a standard deck of cards. How many different 5 card euchre hands are possible?

7. How many different ways can 8 people fit around a circular table? (*Hint: The seat that the first person occupies is unimportant.)

8. In a co-ed softball league, lineups consist of 10 players of which 5 are men and 5 are women. The batting order must alternate gender. How many different lineups are possible if there are 7 women on the team and 9 men?
Teacher Answer Sheet

From Lesson Plan:

Roster of baseball team:

1. Al
2. Bob
3. Chris
4. Dan
5. Ed
6. Fred
7. Greg
8. Hal
9. Isaac
10. Jack
11. Ken
12. Luke
13. Mike
14. Nick
15. Omar

Possible lineups:

2. Ed 2. Chris 2. Isaac
5. Omar 5. Isaac 5. Ken
7. Fred 7. Mike 7. Ed

Number of five card hands:

\[
\binom{52}{5} = \frac{52!}{(52-5)!5!} = \frac{52!}{47!5!} = 2,598,960 \text{ possible hands}
\]

Number of ways to deal five cards:

\[
\binom{52}{5} = \frac{52!}{(52-5)!} = \frac{52!}{47!} = 311,875,200 \text{ possible ways}
\]

Number of two card hands:

\[
\binom{52}{2} = \frac{52!}{(52-2)!2!} = \frac{52!}{50!2!} = 1,326 \text{ possible hands.}
\]

Number of ways to deal to cards:

\[
\binom{52}{2} = \frac{52!}{(52-2)!} = \frac{52!}{50!} = 2,652 \text{ possible hands.}
\]

From *Combinations and Permutations Tasks*:

1. Permutation; (6 options, choose 4) 360
2. Combination; (12 options, choose 3) 220
3. Combination; (40 options, choose 5) 658,008
4. Permutation; (14 options, choose 4) 24,024
5. Combination; (5 women, choose 3; 6 men, choose 1) \( \binom{5}{3} \cdot \binom{6}{1} = 60 \)
6. Combination; (24 options, choose 5) 42,504
7. Permutation; (8 options, choose 8, however, since we have a circular table, there are 8 configurations that are actually all the same; we are not worried about where that first person sits, so we divide 40,320 by 8) 5,040
8. Permutation; (7 women, choose 5; 9 men, choose 5; Start lineup with a man or with a woman) \( \binom{7}{5} \cdot \binom{5}{5} \cdot 2 = 76,204,800 \)
# Probable Problems

**Student/Class Goal**
As poker is a popular card game, and casinos are opening up across the state, students want to be more informed about the game’s odds. They want to be able to calculate their chances of winning in poker and other casino games.

<table>
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<tr>
<th>Outcome (lesson objective)</th>
<th>Time Frame</th>
</tr>
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<tbody>
<tr>
<td>Students will recognize the difference among multiple types of probability. Students will use proper formulas to calculate probabilities of contextual situations.</td>
<td>2 hour</td>
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**Standard** *Use Math to Solve Problems and Communicate*  
(Primary benchmarks in bold.)

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**Data Analysis & Probability**

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<th>Processes</th>
</tr>
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</table>
| Measurement applications | Solve problems  
3.21, 4.25, 4.27, 5.25, 5.27, 6.26, 6.27 |
| Measurement conversions | Communicate ideas  
3.24, 4.31, 5.30, 5.31, 6.31, 6.32 |
| Rounding | Reason mathematically  
3.27, 4.34, 5.35, 6.36 |
| Probability | Mathematical performance  
4.35, 5.36, 6.37 |

**Materials**
Standard deck of playing cards  
Standard six-sided dice  
Cubes (optional to use for the battery/marble questions on the task handout)  
Fraction dice (optional to use if there is extra time for more review)  
*Probability Tasks* handout

**Learner Prior Knowledge**
Students should be able to compute combinations and permutations.

**Instructional Activities**
Part 1: Start the lesson by handing out the vocabulary sheet; to make sure students are aware of the definitions of each key word, go over each key word. Make sure to distinguish between theoretical and experimental probabilities. When students are asked to find the probability in a given situation, the problem is often asking for theoretical probability. However, it is possible for the actual, observed outcome to be different than the calculated probability.

Once students have definitions, you should review/go over simple probabilities as well as the addition and multiplication rules. An example of a simple probability would be drawing the Queen of Hearts from a standard deck of playing cards \( \frac{1}{52} \), getting a six when rolling a single die \( \frac{1}{6} \), or getting a tails when flipping a coin \( \frac{1}{2} \). The addition rule comes into play when we consider the probability of getting outcome A or outcome B. For example, the probability of rolling a one or a two when rolling a single die would be \( \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3} \). If you want to consider subsequent events, that is when we need to decide if the events are...
dependent or independent.

Part 2: (I do) Pass out the Probability Tasks handout to students and have a standard deck of playing cards ready. After reading the first problem and all of its parts, explain how you determine if the events are independent or dependent and how to calculate each probability (see Teacher Answer Sheet).

Part 3: (We do) Read question 2 of the Probability Tasks handout. After giving students some time to think, ask if anyone is able to determine if the events are independent or dependent. If a student is willing to answer, be sure to ask them to provide their reasoning. If no one is willing to provide an answer, ask students “If you select one battery out of the drawer that you know is good, has the probability of selecting a defective battery changed?” This should lead students to notice that the events are dependent. Allow students to provide the probability of picking a defective battery first as it comes straight from the problem. Then ask if someone is able to calculate the probability of picking a defective battery if the first battery was defective. Continue until all 5 probabilities are calculated and have students calculate the overall probability of drawing 5 defective batteries. For the probability of no defective batteries, ask if any students are able to calculate the probability that the first battery you pick will work. If no one is able to calculate remind students that there are a total of 50 batteries and only 8 are defective. After it is discovered that the initial probability of picking a working battery is 42/50, allow time for students to compute the remaining probabilities. Ask for a volunteer to announce each probability and give justifications for each. For the probability of at least one battery being defective, ask students if they have any ideas of calculating the probability. Since there are so many different permutations that will allow for at least one defective battery, the easiest way to calculate the probability is noticing that the only way not to get at least one defective battery is when every battery works which was just calculated. Hence the probability is the complement of that event.

Part 4: (You do) Give students regular dice and cubes to work on the rest of the problems. Walk around the room to help students work through the problems, being sure not to give them the answers. You may redirect student questions to other students that seem to have a better grasp of the concepts ("Ask Jill how she figured out that one"). If students are unfamiliar with the game of rock, paper, scissors, you will need to explain to them the concept that you get the choice of rock, paper or scissors and rock beats scissors, scissors beats paper, paper beats rock, and if each player throws the same thing, is a tie. After two or three draws, students should be able to get the concept of the game.

Part 5: After all students have completed the problems, go over each problem, having a different student present each solution. Interject often to have other students confirm the solution and provide reasoning.

Note: If there is remaining time, you can utilize the fraction dice. For example, you could make a scenario where two dice are rolled. The fraction that comes up on the first die is the probability that you will inherit a large sum of money and the fraction on the second die is the probability that you will be elected president. Based on that, what is the probability that you will be both rich and powerful?

**Assessment/Evidence** *(based on outcome)*

Parts 4 and 5 will provide the teacher the opportunity to assess students understanding of the concepts and look for signs of misconceptions.

Exit slip:

1. If two dice are thrown, what is the probability that at least one of them shows a number greater than 3?

   Answer: \( \frac{27}{36} = \frac{3}{4} = .75 \)

2. The following table shows the percentage of Americans with each blood type:

   3.

<table>
<thead>
<tr>
<th>Blood Type</th>
<th>Percentage of Americans</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>43%</td>
</tr>
<tr>
<td>A</td>
<td>40%</td>
</tr>
<tr>
<td>B</td>
<td>12%</td>
</tr>
<tr>
<td>AB</td>
<td>5%</td>
</tr>
</tbody>
</table>

   a. What is the probability that two people getting married both have blood type O?

   Answer: Here we want to take the probability of person 1 having type O blood and person 2 having type O blood. These are independent events, thus we have \( 0.43 \cdot 0.43 = 0.1849 \approx 18.5\% \)
b. What is the probability that two people getting married both have the same blood type?

Answer: This time, we want to do something similar to the previous problem, but we also want to consider two people with type A, two with type B, and two with type AB. We will then be looking for the probability of person 1 having type O blood and person 2 having type O blood or the probability of person 1 having type A blood and person 2 having type A blood or the probability of person 1 having type B blood and person 2 having type B blood or the probability of person 1 having type AB blood and person 2 having type AB blood. In mathematical terms, this is:

\[
(0.43 \cdot 0.43) + (0.40 \cdot 0.40) + (0.12 \cdot 0.12) + (0.05 \cdot 0.05)
= (0.1849) + (0.16) + (0.0144) + (0.0025)
= 0.3618 \approx 36.2\%
\]

Teacher Reflection/Lesson Evaluation

Not yet completed

Next Steps

After student master these basic probability concepts, introducing them to the concepts of conditional probability and weighted probabilities would follow nicely.

Technology Integration

Excel spreadsheets can be used to build sample spaces of possible incomes as in the charts provided in the Teacher Answer Sheet if available and time permits.

Online probability calculators can also be found. One such calculator is located at:

http://stattrek.com/online-calculator/probability-calculator.aspx

Purposeful/Transparent

After introducing students to key words, definitions, and examples, the teacher will demonstrate several types of probability problems. The teacher will then lead students through a trio of related problems where students will explore the concepts of dependent events and complements. Students will then work individually on variety of similar problems gaining experience in working with probability.

Contextual

This lesson uses playing cards and dice along with different everyday situations and games to help students get a better idea of how probabilities work throughout our society.

Building Expertise

Students will build upon the definitions of key words to compute probabilities related to common games to better understand the world around them.
Vocabulary Sheet

**Complement** — the probability of an event not happening equal to one minus the probability of the event happening. For example, if the probability of being selected for a job is 0.23, the probability of not being selected for a job, or its complement, is $1 - 0.23 = 0.77$.

**Conditional Probability** — the probability that event $B$ happens given that event $A$ has already taken place. The formula for this type of probability is $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$

We read $P(B|A)$ as “the probability of $B$ given $A$.”

**Dependent Events** — events $A$ and $B$ are called dependent if the outcome of $A$ does affect the outcome of $B$. For example, if you have a standard deck of playing cards and draw an Ace on the first trial and do not put the Ace back in the deck, the chance of getting an Ace in the second trial will be different than it was for the first trial.

**Experimental Probability** — also known as observational probability because the probability of an event is calculated based upon the amount of times an event is observed compared to the total number of trials of an experiment. For example, if you rolled a standard 6-sided die 30 times and rolled a 4 six times, the experimental probability of rolling a 4 is $\frac{6}{30} = \frac{1}{5} = 0.2$.

**Independent Events** — events $A$ and $B$ are called independent if the outcome of $A$ does not affect the outcome of $B$. For example, if you flip a coin and get heads on the first trial, the probability of getting heads on the second trial is still 0.5.

**Probability** — the extent to which something is likely to happen represented by a number between 0 and 1 where 0 means impossible to happen, 0.5 means equally likely to happen as to not happen, and 1 means certain to happen.

**Theoretical Probability** — the probability of an event happening based on mathematical theory and found by using formulas. For example, if rolling a standard 6-sided die, the theoretical probability of rolling a 4 is $\frac{1}{6} \approx 0.1667$ and the theoretical probability of rolling a number less than or equal to 4 is $\frac{4}{6} = \frac{2}{3} \approx 0.6667$.

**Trial** — a particular performance of a random experiment. For example, the drawing of one card from a standing deck of playing cards.
Probability Tasks

1. Using a standard deck of playing cards calculate each of the following:
   a. The theoretical probability of being dealt a pair in a five-card poker hand. (Note, we just want a pair. No better hands: three-of-a-kind, four-of-a-kind, two pairs, full house.)
   b. The theoretical probability of flipping over two of the face cards (including aces) in a row off the top of a deck if you do not replace the first card you flip. (Face cards consist of: Jacks, Queens, Kings, and Aces.)
   c. The theoretical probability of flipping over three spades in a row if the card is replaced and the deck is shuffled after each flip.

2. If 8 out of 50 batteries in a drawer are defective and you grab 5 batteries, what is the probability that all of the batteries will be defective? What is the probability that none of the batteries will be defective? What is the probability that at least one battery will be defective?

3. Use standard six-sided dice to calculate each of the following:
   a. The theoretical probability of rolling an even number.
   b. The theoretical probability of rolling two dice and getting a pair.
   c. The theoretical probability of rolling two dice whose product is greater than or equal to 20.
   d. The theoretical probability of rolling two dice whose product is less than 20.
   e. The theoretical probability of rolling two dice whose product is greater than 40.

4. If there are 30 marbles in a bag, 21 blue marbles and 9 green marbles, what is the probability of picking two blue marbles with replacement? What is the probability of picking two green marbles without replacement?

5. In the classic game of rock, paper, scissors, what is the probability of winning three rounds in a row (ties count as a draw)? What is the probability of not losing three rounds in a row? Pair up, play 10 three-round games of rock, paper, scissors, and find the experimental probability of each of the questions above.
1. a. This is a dependent event. We want our hand to have a pair and nothing better. Therefore, our card choices depend on what was previously put into our hand. First off, we need to know the total number of possible hands. As the order in which we are dealt the cards does not matter, we have permutations. Since we are being dealt five cards from a standard deck of cards, we have 52 total cards. Our total number of possible five-card hands is \( \binom{52}{5} = 2,598,960 \). This is the same as \( \frac{52!}{5!(52-5)!} \). In order to find the total number of hands involving a pair (and nothing better), we can follow the following thought process:

- In order to the pair, we must pick a value for the pair (i.e., there are 13 possible face values for our pair - 2 through Ace - and our pair would only be one of those 13 possibilities). This gives us \( \binom{13}{1} = 13 \).
- After setting the value of our pair, we have four possible cards of that value - one of each suit. Out of those four cards, we must choose two: \( \binom{4}{2} = 6 \).
- Now that we have our pair, we need three more cards. They all need to be of different values as we do not want a hand better than just a pair. We have 12 remaining face values and we need to choose three of those: \( \binom{12}{3} = 220 \).
- Once the value for each of our final three cards is chosen, we have the same scenario as before: there are four of each value. We need only one of each value this time, though we need to do it three times, so we have: \( \binom{4}{1}^3 = 4^3 = 64 \).
- To find our possible number of hands with a single pair, we multiply all of our values. Thus: \( 13 \cdot 6 \cdot 220 \cdot 64 = 1,098,240 \).
- If we wanted to write this using shorthand notation, it would be \( \frac{13}{1} \cdot \frac{4}{2} \cdot \frac{12}{3} \cdot \frac{4}{1}^3 = 1,098,240 \).

To find our overall probability, we take our possible number of hands with a pair and divide it by our possible number of five-card poker hands:

\[
\frac{1,098,240}{2,598,960} \approx 0.423 = 42.3\% 
\]

b. Since we have four different values for face cards and there are four of each (one for each suit), we have a total of 16 different face cards. These are dependent events as the chance of your first card being a face card is \( \frac{16}{52} \) and the chance your second card is a face card given that you got a face card on the first flip is \( \frac{15}{51} \). Hence the probability is \( \frac{16}{52} \cdot \frac{15}{51} \approx 0.0905 \).

c. Since the card is being replaced each time, the events are independent and the probability of getting a spade stays the same each time at \( \frac{1}{4} \). Hence the theoretical probability is \( \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{64} \approx 0.0156 \). Below is another way to show how to calculate the probability of getting three spades in a row that you can use to visualize this probability.
Along the top we have the suit of our first card. Along the left side we have the suit of our second card. If we look at the intersection of our first two cards, we have the four suits again as an option. Since there are 64 different squares and only one represents all three cards being spades, you end up with a 1 out of 64 chance of flipping over three spades, or approximately a 1.6% chance.

2. The probability of your first battery being defective is 8/50, the probability of your second battery being defective given your first battery was defective is 7/49, and so on. Hence your final probability is \( \frac{8}{50} \times \frac{7}{49} \times \frac{6}{48} \times \frac{5}{47} \times \frac{4}{46} \approx 0.000026. \)

For the second part of the question, the probability of your first battery not being defective is \( \frac{42}{50} \times \frac{41}{49} \times \frac{40}{48} \times \frac{39}{47} \times \frac{38}{46} \approx 0.4015. \)

For the third part of the question, we can use the idea of a complement. If we want at least one defective battery that is the opposite of having all five be working. Thus, the probability is equal to the complement of the probability of all batteries working, which is \( 1 - 0.4015 = 0.5985. \)
3.

a. 2, 4, and 6 are all even numbers, so we have 3 possibilities on our die. Thus, \( \frac{3}{6} = 0.5 \)

b. The chart below shows we have 36 possible outcomes from rolling 2 dice. 6 of those outcomes result in a pair, thus \( \frac{6}{36} \approx 0.1667 \)

<table>
<thead>
<tr>
<th>First Die</th>
<th>Second Die</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

c. \( \frac{8}{36} \approx 0.2222 \) based on the green-shaded boxes below.

<table>
<thead>
<tr>
<th>First Die</th>
<th>Second Die</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3.png" alt="Diagram" /></td>
<td><img src="image4.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

d. \( \frac{28}{36} \approx 0.7778 \) or \( 1 - 0.2222 = 0.7778 \) based on the non-shaded boxes above.

e. \( \frac{0}{36} = 0 \)

4. Two blue with replacement: \( \frac{21}{30} \times \frac{21}{30} = 0.49 \), two green without replacement: \( \frac{8}{30} \times \frac{8}{29} \approx 0.0828 \).

5. Three wins in a row: \( \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \approx 0.037 \); three non-losses in a row: \( \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \approx 0.2963 \); answers will vary for experimental probability.
### STATS PROJECT

**Student/Class Goal**
Now that they can read/interpret graphics, students want to be able to create their own for data that matters to them so they can share their results with their families/peers.

**Outcome** (lesson objective)
Students will calculate probabilities of outcomes based on collected data. Students will construct charts and graphs to represent their data.

**Time Frame**
2 hours

**Standard** *Use Math to Solve Problems and Communicate*
(Primary benchmarks in bold.)

**NRS EFL**
Levels 4-6

<table>
<thead>
<tr>
<th>Number Sense</th>
<th>Geometry &amp; Measurement</th>
<th>Algebra &amp; Patterns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connect number words</td>
<td>Identify/apply basic geometric concepts</td>
<td>Evaluate/solve expressions/equations</td>
</tr>
<tr>
<td>Solve problems using computations</td>
<td>Connect graphical and algebraic representations</td>
<td>Connect relationships to representations</td>
</tr>
<tr>
<td>Order of operations</td>
<td>Perimeter/area/volume</td>
<td>Graphing</td>
</tr>
<tr>
<td>Compare/order numbers</td>
<td>Graphical representations</td>
<td>Solving equations using algebra/graphs</td>
</tr>
<tr>
<td>Estimate &amp; compute to solve problems</td>
<td>Use of correct units</td>
<td></td>
</tr>
<tr>
<td>Evaluate using roots and exponents</td>
<td>Right triangle trigonometry</td>
<td></td>
</tr>
</tbody>
</table>

**Processes**

<table>
<thead>
<tr>
<th>Data Analysis &amp; Probability</th>
<th>Measurement applications</th>
<th>Solve problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpret data</td>
<td>4.20, 5.20, 6.21</td>
<td>6.26, 6.27, 6.28, 6.29</td>
</tr>
<tr>
<td>Create and display data</td>
<td>4.21</td>
<td>6.30, 6.31, 6.32</td>
</tr>
<tr>
<td>Central tendency</td>
<td>6.23</td>
<td>Reason mathematically</td>
</tr>
<tr>
<td>Probability</td>
<td>6.25</td>
<td>Connect concepts</td>
</tr>
</tbody>
</table>

**Materials**
*Stats Project Task Handout*
*Stats Project Example Handout*
Computer lab (optional, but will be useful in data collection and putting it all together via Excel)
Any materials that can be used by students to collect data (playing cards, spinners, dice, etc.)
Slide N’ Measure Compass (can be used as a straightedge/ruler)

**Learner Prior Knowledge**
Reading and interpreting charts and graphs
Creating graphs based on a set of data
Choosing which graph is most appropriate for a set of data
Calculating combinations, permutations, and probabilities

**Instructional Activities**
Part 1: (*I do*) Begin the lesson by handing out the *Stats Project Task* to students and going over the directions. Then hand out the *Stats Project Example* to students to give them an idea of what is expected from them. The “Fish Caught During Fishing Trip” table displays the type and weight of fish caught along with the day they were caught on a fishing trip. To help build the different graphs and charts, the “Types of Fish Caught,” “Number of Fish Caught by Weight,” and the “Number Fish Caught per Day” tables display the frequencies (and average weight for the “Types of Fish Caught” table) of the different categories of interest. The “Descriptive Statistics of Weight” table lists all the requested descriptive statistics. The line graph displays the amount of fish caught per day as the week went on. A histogram was used to show the amount of fish that were caught by weight. A bar graph was used to display the average weight of each type of fish. Finally, a pie chart was used to display the percentage that each type of fish made up of the total amount caught.

For the probabilities, the different types of fish were used. It is worth mentioning that this is not the only way experimental probabilities could have been computed (for example the probability a fish weight under 2 pounds, between 2 and 5 pounds, between 5 and 10 pounds, and over 10 pounds). Each experimental probability was calculated by taking the amount of each type of fish caught and dividing it by the total amount of fish caught (53). It is worth noting that relation between the experimental probabilities and the pie chart. For the two calculated probabilities, the probability that one could catch five fish with none being bluegill as well as all of them being a bass, pike, or walleye was calculated using the experimental probabilities. To calculate the
probability of no bluegill five times, the complement of catching a bluegill was used and raised to the fifth power. To calculate the probability of catching all bass, walleye, or pike, the four probabilities were added together and then raised to the fifth power.

Part 2: *(You do)* Have students work individually and give them up to 5 minutes to come up with a project that you approve. Be sure that the students have an idea of how they will use their data to complete the rest of their task. Your students may work in pairs if your class is really large and you will not have time for them all to present their findings. Once all students have an approved project, allow them 20 minutes to collect their data. If computers are available, allow students to collect data using the internet and use Excel or other statistical software to build their displays. Be sure to walk around and help students who may be struggling collecting their data. Once all their data has been collected give students up to 45 minutes to prepare the rest of their project; again, walk around helping students who are struggling to work through the task.

Part 3: Once all students have completed their project, have students take turns presenting their findings. Have students explain their graphs and walk through the calculated probabilities. After each student presents their findings, allow other students to ask questions.

**Assessment/Evidence (based on outcome)**
Part 3 allows students to present their understanding of how to design, analyze, and display a statistical project. During part 2, the teacher should actively observe students to diagnose misconceptions and look for signs of understanding.

**Exit Slip:**
1. Have students turn in their calculated probabilities as well as their reasoning for choosing those particular probabilities to calculate.
2. Have students create two additional graphs based on their collected data and give the pros and cons of using each graph.

**Teacher Reflection/Lesson Evaluation**
*Not yet completed*

**Next Steps**
Teachers could cover more advanced graph types that have not previously been covered. They could also do more on comparisons of graphs and why companies/publishers choose to use certain types of graphs over others.

**Technology Integration**
This lesson lends itself to multiple opportunities to integrate technology. If available, students can use the internet to collect data and Excel or other statistical software to organize and present their data.

**Purposeful/Transparent**
This lesson begins by explaining to students what is expected from them along with an example for them to get a better idea as to what is expected from them. Then students will design, organize, analyze, and present their own statistical projects.

**Contextual**
As students will be creating their own statistical projects, the contextual possibilities are endless.

**Building Expertise**
Students will build on the understanding of statistical concepts to design and carry out statistical projects of their own creation.
Stats Project Task

For this task you will need to collect and analyze at least two sets of data; one quantitative and one qualitative. Each data set must include at least 10 data points. Your project must include but is not limited to:

a) a table organizing all of your data;

b) the descriptive statistics (mean, mode, minimum, Q1, median, Q3, maximum, standard deviation, range, and interquartile range) of your quantitative data;

c) two calculated probabilities;

d) and at least two types of graphs.

You may use internet searches or use a survey to collect your data. A possible project would involve surveying 10 people to determine their favorite type of food and how many times a month they have it.
Stats Project Example

Fish Caught During Fishing Trip

<table>
<thead>
<tr>
<th>Fish Caught</th>
<th>Weight</th>
<th>Day</th>
<th>Fish Caught</th>
<th>Weight</th>
<th>Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Mouth Bass</td>
<td>2</td>
<td>Monday</td>
<td>Pike</td>
<td>12</td>
<td>Thursday</td>
</tr>
<tr>
<td>Bluegill</td>
<td>0.25</td>
<td>Monday</td>
<td>Pike</td>
<td>7</td>
<td>Thursday</td>
</tr>
<tr>
<td>Bluegill</td>
<td>0.5</td>
<td>Monday</td>
<td>Crappie</td>
<td>0.7</td>
<td>Thursday</td>
</tr>
<tr>
<td>Small Mouth Bass</td>
<td>1.5</td>
<td>Monday</td>
<td>Small Mouth Bass</td>
<td>4</td>
<td>Thursday</td>
</tr>
<tr>
<td>Walleye</td>
<td>13</td>
<td>Monday</td>
<td>Large Mouth Bass</td>
<td>2.5</td>
<td>Thursday</td>
</tr>
<tr>
<td>Bluegill</td>
<td>0.25</td>
<td>Monday</td>
<td>Crappie</td>
<td>0.8</td>
<td>Thursday</td>
</tr>
<tr>
<td>Large Mouth Bass</td>
<td>3</td>
<td>Monday</td>
<td>Small Mouth Bass</td>
<td>1.5</td>
<td>Thursday</td>
</tr>
<tr>
<td>Small Mouth Bass</td>
<td>3.2</td>
<td>Monday</td>
<td>Small Mouth Bass</td>
<td>4</td>
<td>Thursday</td>
</tr>
<tr>
<td>Pike</td>
<td>10</td>
<td>Tuesday</td>
<td>Pike</td>
<td>10</td>
<td>Thursday</td>
</tr>
<tr>
<td>Bluegill</td>
<td>0.5</td>
<td>Tuesday</td>
<td>Crappie</td>
<td>0.2</td>
<td>Thursday</td>
</tr>
<tr>
<td>Bluegill</td>
<td>0.4</td>
<td>Tuesday</td>
<td>Small Mouth Bass</td>
<td>5</td>
<td>Thursday</td>
</tr>
<tr>
<td>Walleye</td>
<td>7</td>
<td>Tuesday</td>
<td>Large Mouth Bass</td>
<td>5</td>
<td>Thursday</td>
</tr>
<tr>
<td>Small Mouth Bass</td>
<td>4</td>
<td>Tuesday</td>
<td>Walleye</td>
<td>18</td>
<td>Friday</td>
</tr>
<tr>
<td>Bluegill</td>
<td>0.3</td>
<td>Tuesday</td>
<td>Walleye</td>
<td>8</td>
<td>Friday</td>
</tr>
<tr>
<td>Bluegill</td>
<td>0.2</td>
<td>Tuesday</td>
<td>Walleye</td>
<td>4</td>
<td>Friday</td>
</tr>
<tr>
<td>Crappie</td>
<td>0.8</td>
<td>Tuesday</td>
<td>Small Mouth Bass</td>
<td>1</td>
<td>Friday</td>
</tr>
<tr>
<td>Crappie</td>
<td>0.4</td>
<td>Tuesday</td>
<td>Bluegill</td>
<td>0.25</td>
<td>Friday</td>
</tr>
<tr>
<td>Crappie</td>
<td>0.3</td>
<td>Tuesday</td>
<td>Small Mouth Bass</td>
<td>4</td>
<td>Friday</td>
</tr>
<tr>
<td>Bluegill</td>
<td>0.5</td>
<td>Tuesday</td>
<td>Pike</td>
<td>13</td>
<td>Friday</td>
</tr>
<tr>
<td>Bluegill</td>
<td>0.2</td>
<td>Tuesday</td>
<td>Walleye</td>
<td>5</td>
<td>Friday</td>
</tr>
<tr>
<td>Pike</td>
<td>8</td>
<td>Wednesday</td>
<td>Walleye</td>
<td>8</td>
<td>Friday</td>
</tr>
<tr>
<td>Walleye</td>
<td>12</td>
<td>Wednesday</td>
<td>Crappie</td>
<td>0.2</td>
<td>Friday</td>
</tr>
<tr>
<td>Crappie</td>
<td>0.25</td>
<td>Wednesday</td>
<td>Walleye</td>
<td>4</td>
<td>Friday</td>
</tr>
<tr>
<td>Bluegill</td>
<td>0.5</td>
<td>Wednesday</td>
<td>Bluegill</td>
<td>0.8</td>
<td>Friday</td>
</tr>
<tr>
<td>Large Mouth Bass</td>
<td>1</td>
<td>Wednesday</td>
<td>Small Mouth Bass</td>
<td>4</td>
<td>Friday</td>
</tr>
<tr>
<td>Large Mouth Bass</td>
<td>6</td>
<td>Wednesday</td>
<td>Crappie</td>
<td>0.5</td>
<td>Friday</td>
</tr>
<tr>
<td>Large Mouth Bass</td>
<td>4.5</td>
<td>Thursday</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Types of Fish Caught

<table>
<thead>
<tr>
<th>Fish Caught</th>
<th>Number Caught</th>
<th>Average Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pike</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Walleye</td>
<td>9</td>
<td>8.78</td>
</tr>
<tr>
<td>Large Mouth Bass</td>
<td>7</td>
<td>3.43</td>
</tr>
<tr>
<td>Small Mouth Bass</td>
<td>10</td>
<td>3.02</td>
</tr>
<tr>
<td>Crappie</td>
<td>9</td>
<td>0.46</td>
</tr>
<tr>
<td>Bluegill</td>
<td>12</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Number of Fish Caught by Weight

<table>
<thead>
<tr>
<th>Weight</th>
<th>Number Caught</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to .9</td>
<td>21</td>
</tr>
<tr>
<td>1 to 1.9</td>
<td>4</td>
</tr>
<tr>
<td>2 to 2.9</td>
<td>3</td>
</tr>
<tr>
<td>3 to 3.9</td>
<td>2</td>
</tr>
<tr>
<td>4 to 4.9</td>
<td>7</td>
</tr>
<tr>
<td>5 to 5.9</td>
<td>3</td>
</tr>
<tr>
<td>6 to 6.9</td>
<td>1</td>
</tr>
<tr>
<td>7 to 7.9</td>
<td>2</td>
</tr>
<tr>
<td>8 to 8.9</td>
<td>3</td>
</tr>
<tr>
<td>9 to 9.9</td>
<td>0</td>
</tr>
<tr>
<td>10+</td>
<td>7</td>
</tr>
</tbody>
</table>

Number of Fish Caught per Day

<table>
<thead>
<tr>
<th>Day</th>
<th>Number Caught</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>8</td>
</tr>
<tr>
<td>Tuesday</td>
<td>12</td>
</tr>
<tr>
<td>Wednesday</td>
<td>6</td>
</tr>
<tr>
<td>Thursday</td>
<td>13</td>
</tr>
<tr>
<td>Friday</td>
<td>14</td>
</tr>
</tbody>
</table>

The table above shows the number of fish caught of each type, along with their average weight. The second table provides the number of fish caught by weight, with the third table listing the number of fish caught per day.
### Descriptive Statistics of Weight

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>3.81132</td>
</tr>
<tr>
<td>Mode</td>
<td>4</td>
</tr>
<tr>
<td>Minimum</td>
<td>.2</td>
</tr>
<tr>
<td>Q1</td>
<td>.5</td>
</tr>
<tr>
<td>Median</td>
<td>2</td>
</tr>
<tr>
<td>Q3</td>
<td>5.5</td>
</tr>
<tr>
<td>Maximum</td>
<td>18</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.2157</td>
</tr>
<tr>
<td>Range</td>
<td>17.8</td>
</tr>
<tr>
<td>IQR</td>
<td>5</td>
</tr>
</tbody>
</table>

### Fish Caught per Day

![Graph showing the number of fish caught per day](image)

- The graph illustrates the number of fish caught each day from Monday to Friday.
- The highest number of fish was caught on Wednesday.

### Number of Fish Caught by Weight

![Bar chart showing the number of fish caught by weight](image)

- The x-axis represents the weight of the fish in lbs, categorized into different intervals.
- The y-axis represents the number of fish caught.
- The highest number of fish was caught in the weight category of 0 to .9 lbs.
<table>
<thead>
<tr>
<th>Fish</th>
<th>Experimental Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bluegill</td>
<td>.2264</td>
</tr>
<tr>
<td>Crappie</td>
<td>.1698</td>
</tr>
<tr>
<td>Large Mouth Bass</td>
<td>.1321</td>
</tr>
<tr>
<td>Pike</td>
<td>.1132</td>
</tr>
<tr>
<td>Small Mouth Bass</td>
<td>.1887</td>
</tr>
<tr>
<td>Walleye</td>
<td>.1698</td>
</tr>
</tbody>
</table>

Given the experimental probabilities, the probability of catching 5 fish and none of them being blue gill is 
\[(1 - .2264)^5 = (.7736)^5 = .2771\] and the probability of all the fish being a bass, walleye, or pike is 
\[(.1321 + .1132 + .1887 + .1698)^5 = (.6038)^5 = .0803\].