Planar Cable-Direct-Driven Robots, Part II: Dynamics and Control

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ABSTRACT

A hybrid parallel/serial manipulator architecture was introduced in a companion paper where the translational freedoms are provided by a cable-direct-driven robot (CDDR) and the rotational freedoms are provided by a serial wrist mechanism. While the companion paper presents kinematics and statics, the current paper presents a dynamics model and simulated control for planar CDDRs. Examples are presented to compare the planar 3-cable CDDR with one degree of actuation redundancy and the 4-cable CDDR with two degrees of actuation redundancy. It was found that the 4-cable tracking error was worse than for the 3-cable case, due to increased inertia with an additional actuator. Also, the controller architecture considers including and not including a feedforward reference acceleration term with the overall mass matrix; the performance of the controller with the feedforward term is clearly preferable.

1. INTRODUCTION

Cable-direct-driven robots (CDDRs) are a type of parallel manipulator wherein the end-effector link is supported in-parallel by n cables with n tensioning motors. Several CDDRs and cable-direct-driven haptic interfaces (CDDHIs) have been presented by other authors and are reviewed in the companion paper (Williams and Gallina, 2001). Roberts et al. (1998), present the inverse kinematics, cable-tension-optimization, and fault tolerance of Charlotte-type (Campbell et al., 1995) CDDRs, but no dynamics modeling is presented. Sklar and Tesar (1988) present the dynamics of hybrid serial manipulators consisting of parallel joints. This manipulator architecture is significantly different from the currently-proposed hybrid parallel/serial manipulator architecture where the translational freedoms are provided by a CDDR and the rotational freedoms by a serial wrist mechanism.

CDDRs are designed with actuation redundancy, i.e. with more actuated cables than wrench-exerting degrees-of-freedom in attempt to avoid configurations where certain wrenches require an impossible compression force in one or more cables. Despite actuation redundancy, there exist subspaces in the potential workspace where some cables can lose tension. This problem can be exacerbated by CDDR dynamics, hence the current paper studies dynamics and control of planar CDDRs.

This paper reviews the two candidate planar CDDRs from the companion paper, presents dynamics modeling resulting in a nonlinear, coupled dynamics model, followed by Cartesian trajectory control simulation employing Cartesian PD control for planar CDDRs with one and two degrees actuation redundancy. This paper focuses on translational motion and forces.

2. CABLE-DIRECT-DRIVEN ROBOTS (CDDRs)

A CDDR consists of a single end-effector point supported in parallel by n cables controlled by n tensioning actuators. Figures 1 and 2 show the planar 3-cable and 4-cable CDDR diagrams. These figures are similar to the kinematics diagrams in the companion paper (Williams and Gallina, 2001), but we have added the end-effector point mass m and the lumped motor shaft/cable pulley rotational inertias for each actuator $J_i$ ($i=1,...,n$). The cable pulley radius for each actuator is $r_i$ ($i=1,...,n$; shown in Fig. 3). We also include viscous damping coefficients $c_i$ ($i=1,...,n$) at each motor shaft to provide a linear model for the system friction.

For 2-dof planar translations there must be at least two cables. Since cables can only exert tension on the end-effector, there must be more cables to avoid configurations where the robot can be slack and lose control. Figure 1 represents one degree of actuation redundancy, i.e. three cables to achieve the two Cartesian degrees-of-freedom $X=x$, $Y=y$; the four-cable CDDR in Fig. 2 has two degrees of actuation redundancy. Figures 1 and 2 show the inertially-fixed reference frame $\{0\}$ whose origin is the centroid of the base polygon; the regular base polygon (triangle and square, respectively) has sides of fixed length $L_B$; each cable is attached to the ground link at $A_i=\{A_{ix}, A_{iy}\}$; the length of each cable is denoted as $L_i$, and the cable angles are $\theta_i$ ($i=1,...,n$).
This section presents dynamics modeling for planar CDDRs. Dynamics modeling is required for improved control (compared to using kinematics and statics modeling only) when CDDRs are to provide high velocities and accelerations in translational motion. Dynamics modeling is concerned with relating the Cartesian translational motion of the moving CDDR point to the required active joint torques. Due to the cable actuation, CDDR dynamics modeling is not very similar to in-parallel-actuated robot dynamics modeling (e.g. Tsai, 1999; Gosselin, 1996). Another complicating factor is that with CDDRs the joint space is overconstrained with respect to the Cartesian space due to the redundant actuation.

For the dynamics model derived in this section we assume that the CDDR cables are massless and perfectly stiff so we do not consider their inertias or spring stiffnesses. We further ignore the Coulomb friction and instead model linear viscous friction to account for the frictional losses. Despite these simplifications, the resulting model is coupled and nonlinear. We now present the Cartesian, actuator, and overall system dynamics models.

### 3.1 Cartesian Dynamics Model

The free-body diagram for the moving end-effector point is very simple and hence not shown. The 2-dof Cartesian dynamic model for the end-effector is given by:

\[
\mathbf{m} \ddot{\mathbf{X}} - \mathbf{F} = 0
\]

where the Cartesian mass matrix is \( \mathbf{m} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \), \( \mathbf{X} = \begin{bmatrix} x \\ y \end{bmatrix} \) is the end-effector position and \( \mathbf{F} \) is the resultant of all \( n \) cable forces acting on the end-effector.

### 3.2 Actuator Dynamics Model

We also take into consideration the dynamic behavior of the lumped motor shaft/cable pulley; the free-body diagram for the \( i^{th} \) motor shaft/cable pulley subsystem is shown in Fig. 3. The combined motor shaft/cable pulley dynamics equations are expressed by the relationship:

\[
\mathbf{J} \ddot{\boldsymbol{\theta}} + \mathbf{C} \dot{\boldsymbol{\theta}} = \mathbf{T} - \mathbf{r}\mathbf{T}
\]

where:

\[
\mathbf{J} = \begin{bmatrix} J_1 & 0 \\ 0 & J_n \end{bmatrix} \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} c_1 & 0 \\ 0 & c_n \end{bmatrix}
\]

are diagonal matrices with rotational inertia and rotational viscous damping coefficients on the diagonal, all cable pulley radii are identical (\( r_i = r \); \( i = 1, \ldots, n \)), \( \mathbf{T} \in \mathbb{R}^n \) is the vector of torques exerted by the motors, \( \mathbf{T} \in \mathbb{R}^n \) is the vector of cable tensions \( t_i \), and \( \boldsymbol{\theta} \in \mathbb{R}^n \) is the vector of pulley angles. Since the cables can only exert positive tensions (they cannot push), to express the cable tensions as a function of the motor torques and angular motion from (2), we obtain:

\[
\mathbf{T} = \text{pos}\left(\frac{1}{r} (\mathbf{T} - \mathbf{J} \ddot{\boldsymbol{\theta}} - \mathbf{C} \dot{\boldsymbol{\theta}})\right)
\]

where the symbol \( \text{pos}(\cdot) \) means we take the value of each vector component that is positive and we set to zero those components that were originally negative. Let us suppose that the torque on each motor is large enough to make all cables remain in tension at all times. Under this assumption:

\[
\mathbf{T} = \frac{1}{r} (\mathbf{T} - \mathbf{J} \ddot{\boldsymbol{\theta}} - \mathbf{C} \dot{\boldsymbol{\theta}})
\]

### 3.3 System Dynamics Model

We now derive the overall system dynamics model by combining the Cartesian and actuator dynamics equations of motion. From the companion paper (Williams and Gallina, 2001), the statics relationship between forces on the end-effector and cable tensions is given by:

\[
\mathbf{T} = \text{pos}\left(\frac{1}{r} (\mathbf{T} - \mathbf{J} \ddot{\boldsymbol{\theta}} - \mathbf{C} \dot{\boldsymbol{\theta}})\right)
\]
\[ F = ST \] (5)

where the 2n statics Jacobian matrix is:

\[
S = \begin{bmatrix}
-\cos \theta_1 & \cdots & -\cos \theta_n \\
-\sin \theta_1 & \cdots & -\sin \theta_n
\end{bmatrix}
\]

where \( \theta_i \) (i = 1, \ldots, n) are the cable angles (see Figs. 1 and 2).

We now need an inverse kinematics mapping relating the pulley angles \( \beta_i \) (i = 1, \ldots, n) expressed as functions of the end-effector position \( \mathbf{X} = \{x, y\}^T \). Let us define all \( \beta_i \) to be zero when the end-effector is located at the origin of frame \( \{0\} \). From this position, a right-handed positive angle \( \beta_i \) on one pulley will cause a negative change \( \Delta \mathbf{L}_i \) in cable length \( i \): \( \beta_i \mathbf{L} = -\Delta \mathbf{L}_i \). The change in cable length \( i \) is \( \mathbf{L}_i - \mathbf{L}_i - \mathbf{L}_j \) where \( \mathbf{L}_i = \sqrt{(x-A_{ix})^2 + (y-A_{iy})^2} \) is the general length for cable \( i \) from the inverse position solution (Williams and Gallina, 2001) and \( \mathbf{L}_0 = \sqrt{(A_{ix})^2 + (A_{iy})^2} \) is the initial length for cable \( i \). Therefore:

\[
\beta_i = \begin{bmatrix}
\beta_i(\mathbf{X}) \\
\beta_n(\mathbf{X})
\end{bmatrix} = -\frac{1}{r} \begin{bmatrix}
L_{01} - \sqrt{(x-A_{ix})^2 + (y-A_{iy})^2} \\
L_{0n} - \sqrt{(x-A_{ix})^2 + (y-A_{iy})^2}
\end{bmatrix}
\]

(7)

Successive time derivatives of (7) yield:

\[
\dot{\beta} = \frac{\partial \mathbf{B}}{\partial \mathbf{X}} \dot{\mathbf{X}}
\]

\[
\dot{\mathbf{B}} = \frac{d}{dt} \left( \frac{\partial \mathbf{B}}{\partial \mathbf{X}} \right) \mathbf{X} + \frac{\partial \mathbf{B}}{\partial \dot{\mathbf{X}}} \dot{\mathbf{X}}
\]

where:

\[
\frac{\partial \mathbf{B}}{\partial \mathbf{X}} = -\frac{1}{r} \begin{bmatrix}
(x-A_{ix}) & y - A_{iy} \\
-x - A_{ix} & y - A_{iy} \\
L_1 & L_1 \\
L_n & L_n
\end{bmatrix}
\]

and \( L_i = \sqrt{(x-A_{ix})^2 + (y-A_{iy})^2} \) is the length of cable \( i \), a function of \( \mathbf{X} = \{x, y\}^T \). By substituting (8) into (4) we obtain:

\[
\mathbf{T} = \frac{1}{r} \left( \mathbf{r} \dot{\mathbf{X}} - \mathbf{J} \frac{d}{dt} \left( \frac{\partial \mathbf{B}}{\partial \mathbf{X}} \right) \mathbf{X} + \frac{\partial \mathbf{B}}{\partial \dot{\mathbf{X}}} \dot{\mathbf{X}} - \mathbf{C} \frac{\partial \mathbf{B}}{\partial \dot{\mathbf{X}}} \dot{\mathbf{X}} \right)
\]

(10)

Finally, by combining (1), (5), and (10), we obtain the overall dynamics equations of motion, expressed in a standard Cartesian form for robotic systems (Lewis et al., 1993):

\[
\mathbf{M}_{eq}(\mathbf{X}) \ddot{\mathbf{X}} + \mathbf{N}(\mathbf{X}, \dot{\mathbf{X}}) = \mathbf{S}(\mathbf{X}) \mathbf{r}
\]

where:

\[
\mathbf{M}_{eq}(\mathbf{X}) = \mathbf{J} \mathbf{J}^T + \frac{1}{r} \mathbf{C} \frac{\partial \mathbf{B}}{\partial \dot{\mathbf{X}}} \dot{\mathbf{X}} \]

\[
\mathbf{N}(\mathbf{X}, \dot{\mathbf{X}}) = \frac{1}{r} \mathbf{r} \dot{\mathbf{X}} - \mathbf{J} \frac{d}{dt} \left( \frac{\partial \mathbf{B}}{\partial \mathbf{X}} \right) \mathbf{X} + \frac{\partial \mathbf{B}}{\partial \dot{\mathbf{X}}} \dot{\mathbf{X}} - \mathbf{C} \frac{\partial \mathbf{B}}{\partial \dot{\mathbf{X}}} \dot{\mathbf{X}}
\]

\[
\mathbf{S}(\mathbf{X}) = \mathbf{J} \frac{\partial \mathbf{B}}{\partial \dot{\mathbf{X}}} \dot{\mathbf{X}} + \frac{\partial \mathbf{B}}{\partial \dot{\mathbf{X}}} \dot{\mathbf{X}} - \mathbf{C} \frac{\partial \mathbf{B}}{\partial \dot{\mathbf{X}}} \dot{\mathbf{X}}
\]
\[
F_Y - M_{eq}(X_R)\ddot{X}_R + K_P \epsilon + K_D \dot{\epsilon}
\] (14)

The control architecture (shown in the block diagram of Fig. 4) is made up of three different parts: the PD controller, the feedforward term, and the virtual-Cartesian-force-input-to-real-actuator-torque calculation. In this paper the PD controller gains are determined via trial-and-error using a Matlab Simulink simulation to achieve reasonable performance. The matrix gains \(K_P, K_D\) are 2x2 diagonal matrices, which means that the PD control is accomplished independently for the \(x\) and \(y\) motions, even though the dynamics model is coupled. The feedforward term \(M_{eq}(X_R)\ddot{X}_R\) is composed of the overall position-dependent Cartesian mass matrix and the reference Cartesian acceleration components. The virtual-to-real calculation has the problem to invert the matrix \(S(X)\) that is non-square and such that only positive cable tensions result. This problem is solved in the companion paper (Williams and Gallina, 2001).

We do not generally have access directly to Cartesian position \(X\) feedback via a sensor. Instead, we must calculate this feedback using the encoder feedback for each cable pulley angle \(\beta_i\) to determine the cable lengths \(L_i\); these lengths are then used as the inputs to the forward position kinematics solution (Williams and Gallina, 2001) to calculate Cartesian position \(X\) for feedback in the control architecture. This feedback scheme will work well only if sufficient tension is maintained on all cables at all times.

![Figure 4. Control Architecture for Planar CDDRs](image)

5. EXAMPLES

This section presents controls simulation examples using the dynamics equations of motion for the planar 3-cable CDDR with one degree of actuation redundancy and for the planar 4-cable CDDR with two degrees of actuation redundancy. The circle-tracing task is identical to that presented in the examples of the companion paper (Williams and Gallina, 2001), but now dynamics and control is considered. The base equilateral triangle side is \(L = 1\) m and the base square side is \(L_n = 0.6580\) m.

The simulated task is for the CDDR end-effector point \(X = \{x, y\}^T\) to trace a circle in the plane. Unlike the kinematics and statics example task in the companion paper, the constant Cartesian force \(F_R\) on the environment will be set to zero. The identical task will be performed by both 3-cable and 4-cable CDDRs, both without and with the feedforward term, and the results will be compared. The circle is centered at the base polygon centroid (the origin of \(0\)) and the circle radius is arbitrarily chosen to be three-quarters of the shortest distance from the base triangle centroid to a triangle side: \(r = 0.2165\) m. Figures 5 and 6 show the simulated task to scale for the 3- and 4-cable cases, respectively, at the starting (and ending) point.

![Figure 5. Planar 3-Cable CDDR Example Task](image)

![Figure 6. Planar 4-Cable CDDR Example Task](image)

We define polar angle \(\phi\) as the independent parameter for the circle; it is measured using the right-hand from the right horizontal to the current circle radius; \(\phi\) is shown as 0 (and 360°) in Figs. 5 and 6. In the simulated task, one circle revolution is traced out in 1 sec. We provide a constant acceleration for the first half of the circle and an equal constant deceleration for the second half of the circle; the motion starts and ends at zero velocity. The kinematic task relationships for this desired reference motion are \(a = -\dot{\phi} = -g\pi, \ \omega = -\dot{\phi} = -\alpha\), and \(\phi = \alpha t^2 / 2\) for the first half of the circle; the second half is symmetric to the first. Figures 7a and 7b show the associated commanded (reference) Cartesian position \(X_R\) and Cartesian acceleration \(\ddot{X}_R\) for use in the feedforward term.

The parameters for the dynamics equations of motion (11) for both the 3- and 4-cable CDDRs are: point mass \(m = 1\) kg; rotational shaft/pulley inertias \(J_i = 0.0008\) kgm² (for all \(i = 1, \ldots, n\)); shaft rotational viscous damping coefficients \(c_i = 0.01\) Nms (for all \(i = 1, \ldots, n\)); and \(r_i = r = 5\) cm (for all \(i = 1, \ldots, n\)). The same Cartesian PD controller is used for both CDDRs, found by trial-and-error using the Simulink simulation: gain \(K_P\) is a 2x2 diagonal matrix with equal gains of \(K_P = 2000\) on the diagonal, and gain \(K_D\) is a 2x2 diagonal matrix with equal gains of \(K_D = 10\) on the diagonal.
Four control simulations of the dynamics model are presented in this section: the 3-cable and 4-cable CDDRs, both without and with the feedforward term in the block diagram of Fig. 4. Figure 8 shows the circle trajectory tracking error $e = X_R - X$ for the 3-cable CDDR without the feedforward controller term. Figure 9a shows the tracking error for the 3-cable CDDR with the feedforward controller term included.

The feedforward term has cut the tracking error nearly in half: without feedforward the peak errors were approximately ±4 cm, and with feedforward the peak errors were approximately ±2 cm. For both cases, the peak errors occurred after the halfway point in the circle, when the velocity is the greatest. This is expected since we ignore the nonlinear dynamics terms $N(X, \dot{X})$ for both cases. The tracking error is small for the non-feedforward case at the end of the circle motion (when the velocity is supposed to go back to zero); the tracking error is better, nearly zero, for the feedforward case at the end of the circle motion.

Since the tracking error with the feedforward term included is clearly preferable, we now show more control details for this case. Figure 9b shows the virtual Cartesian force input $F_V$, Fig. 9c shows the three cable tensions, and Fig. 9d shows the three actuator torques for the prescribed dynamic circular motion.

The virtual Cartesian force inputs, cable tensions, and actuator torques of Figs. 9b-9d show these variables for the feedforward case. The equivalent plots for the non-feedforward case are not shown to save space. However, the dependent axis ranges in Figs. 9b-9d were chosen such that the non-feedforward cases (not shown) just fit. From this we can conclude that the virtual Cartesian force inputs, cable tensions, and actuator torques for the control with feedforward are significantly less than for the same variables in the non-feedforward case. Therefore, due to better tracking error and reduced forces, tensions, and torques, the control architecture including the feedforward term is superior, despite the additional preparation and controller cost. These two paragraphs of results discussion for the 3-cable CDDR also extend to the four cable CDDR, as will be seen next.

The tracking error in these examples is rather large (several cm). Our intent is to validate the theoretical work via experimental results, and then develop improved control to reduce this tracking error. Thus, we need a sizable tracking error that we can measure in the lab.
For comparison purposes, the same types of results are now shown, for simulated control of the 4-cable CDDR. Figure 10 shows the tracking error $e = \mathbf{X}_n - \mathbf{X}$ for the 4-cable CDDR without the feedforward controller term. Figure 11a shows the tracking error for the 4-cable CDDR with the feedforward controller term included.

Again, the tracking error was cut nearly in half by using the feedforward term, so we now show more control details for this case. Figure 11b shows the virtual Cartesian force input $\mathbf{F}_V$, Fig. 11c shows the four cable tensions, and Fig. 11d shows the four actuator torques for the prescribed dynamic circular motion.
tensions and actuator torques approach more nearly the cases without the feedforward term (as seen by the scale on the independent axes, which were set for the non-feedforward cases) than the 3-cable CDDR did. It is still clear that the controller including the feedforward term is superior to the controller without the feedforward term.

6. CONCLUSION

This paper performs dynamics modeling and control simulation for a proposed hybrid parallel/serial manipulator architecture wherein the translational freedoms are provided by a parallel cable-direct-driven robot (CDDR) and the rotational freedoms are provided by a serial wrist mechanism. The motivation behind this work is to improve the serious cable interference and negative cable tensions possible with existing CDDRs that guide both translational and rotational freedoms. Dynamics modeling is required to achieve high velocities and accelerations in translational motion. A controls simulation example is presented comparing the dynamics of planar 3- and 4-cable CDDRs performing the same task.

Only the translational motion is considered in this paper. It was found that the tracking error of the 4-cable CDDR was higher than that of the 3-cable CDDR, due to the added inertia of an extra actuator shaft/cable pulley. In both CDDRs, the cable tensions became negative (slack) due to dynamics in small ranges of the motion despite the prescribed controller minimum of 1 N tension on each cable. From comparing simulated performance, the controller with the feedforward term proved to be superior to the controller without the feedforward term for both 3- and 4-cable CDDRs.

Our future work plans include more complete dynamics modeling (cable inertia and stiffness, Coulomb friction, among others), development of a method to avoid slack cables based on dynamics, improved nonlinear controller development, hardware implementation, and experimental validation of our results.

REFERENCES


