

A COMPUTER SIMULATION OF STIRLING  
CYCLE MACHINES

Israel Urieli

A COMPUTER SIMULATION OF STIRLING CYCLE MACHINES

Israel Urieli

A Thesis Submitted to the Faculty of Engineering,  
University of the Witwatersrand, Johannesburg, for  
the Degree of Doctor of Philosophy.

Johannesburg, February 1977



DECLARATION

I, Israel Urieli, hereby declare that this thesis is my own work and that the material herein has not been submitted by me for degree purposes at any other University.

A handwritten signature in black ink, appearing to read 'Israel Urieli', is written over a horizontal line.

## ABSTRACT

This thesis describes the development of a computer program to accurately simulate the performance and detailed behaviour of Stirling cycle machines. The program can be used both as a development tool to predict the performance characteristics of particular machines, as well as a research tool to study the inter-related gas dynamic, thermodynamic and heat transfer behaviour of such machines.

Other simulation methods published to date are based on oversimplified pressure drop, friction and heat transfer relationships.

The approach adopted in the present study was to subdivide the machine into a finite number of one-dimensional cells. Complete differential equations of continuity, momentum and energy of the working gas, as well as energy of the regenerator matrix and heat exchanger walls, are developed. In particular the energy equation of the working gas also includes kinetic energy terms whilst the momentum equation includes the effects of working gas acceleration.

The resulting set of non-linear partial differential equations is solved numerically by the 'method of lines', due regard being taken of the local instantaneous values of dynamic viscosity, Reynolds number, friction factor and heat transfer coefficient, which are all non-linear empirical functions of the system geometry and fluid properties.

A unique method of accelerating convergence of the solution to cyclic steady state is used. The effect of the number of cells of the system subdivision on the accuracy and consistency of the results has also been investigated.

Simulation results of a hypothetical test engine are presented. These include, inter alia, efficiency and indicated power versus rotational speed using air, helium and hydrogen as the working gas. Three-dimensional plots showing incremental temperature, flow and pressure profiles through a complete cycle are presented. The results show the detailed behaviour of the working fluid as influenced by the various machine parameters and working fluid properties, and as such help to provide a new insight into the complex behaviour of Stirling cycle machines.

## ACKNOWLEDGEMENTS

I would like to express my particular appreciation to the following:

Costa J Rallis, my supervisor, for his unflinching encouragement and active support throughout all phases of the project. His spontaneous assistance and guidance has contributed substantially to the successful conclusion of this work, for which I am deeply grateful.

Dave M Berkowitz, a postgraduate student in Mechanical Engineering who has worked with me on this research topic and who many times has helped me to clarify various salient topics.

Jim Archbold, of the NITR, who continually provided the important liaison with the CSIR computing center. The staff of the CSIR Computer Centre, without whose help I could not have been able to do the quite considerable computer runs required in this work. In particular I wish to thank Francois van der Merwe, Norma Whiteley, and Ann Hulme for their cheerful assistance.

The members of staff of the NITR, who encouraged me throughout this study.

Fernanda Andrade, who had the unenviable task of typing this Thesis.

Finally, to Nili - Thank you.

## TABLE OF CONTENTS

|   |        |
|---|--------|
| DECLARATION   | (i)    |
| ABSTRACT  | (ii)   |
| ACKNOWLEDGEMENTS  | (iv)   |
| TABLE OF CONTENTS   | (v)    |
| LIST OF FIGURES   | (ix)   |
| LIST OF TABLES  | (xii)  |
| NOTATION  | (xiii) |
| <br>  |        |
| 1. <u>INTRODUCTION</u>                                    | 1      |
| 1.1 General   | 1      |
| 1.2 The Stirling Cycle                                    | 2      |
| 1.3 Problem Areas   | 6      |
| 1.4 Computer Simulation                                   | 7      |
| 1.5 Purpose of this Study                                 | 8      |
| 1.6 Equations   | 10     |
| 1.7 References  | 10     |
| <br>  |        |
| 2. <u>REVIEW OF STIRLING CYCLE MACHINE ANALYSIS</u>       | 12     |
| 2.1 Introduction  | 12     |
| 2.2 Schmidt Cycle Analysis                                | 12     |
| 2.3 Basic Analysis  | 16     |
| 2.4 Non-Isothermal Compression and Expansion<br>Processes | 17     |
| 2.5 Comprehensive Analysis including Pressure<br>Drop     | 18     |
| 2.6 Discussion  | 30     |
| 2.7 Statement of the problem                              | 34     |