Application of Gauss’s Law

- At a point \( x \), the electric field can be found as the charge enclosed, divided by the permittivity of the material ...

  caveats (warnings):

  (i) the field must be zero at the other side of the charged region

  (ii) the sign of the field can be found by keeping track of the +\( x \) direction and the one-dimensional equivalent of the “outward normal;” however, the best approach is to know the sign of the field from the distribution of charge in the problem

- Example: metal-oxide-silicon structure

  \[ \text{Find } E(x = -t_{ox}/2) \]

  \[ \text{Find } E(x = 0^+) \text{ ... just inside the silicon} \]
Boundary Condition on $E$ (cont.)

- Sketch $E(x)$ from $x = -t_{ox}$ to $x = X_d$

- Sketch $\phi(x)$ through the structure, given that $\phi(X_d) = 400$ mV
Potential and Carrier Concentration in Silicon

What is a convenient reference for the electrostatic potential in silicon?

**Thermal equilibrium:** no external stimulus --> must have:

\[ J_p = 0 \text{ and } J_n = 0. \]

\[ \therefore 0 = qn_o \mu_n E + qD_n \frac{dn_o}{dx} = qn_o \mu_n \left( -\frac{d\phi}{dx} \right) + qD_n \frac{dn_o}{dx} \]

\[ d\phi = \frac{D_n}{\mu_n} \left( \frac{dn_o}{n_o} \right) = \frac{kT}{q} \left( \frac{dn_o}{n_o} \right) = V_{th} \left( \frac{dn_o}{n_o} \right) \]

where we have used Einstein’s relation.

\[ \frac{D_n}{\mu_n} = \frac{kT}{q} = V_{th} = 25 \text{ mV at “cool” room temperature to } 26 \text{ mV at “warm” room temperature} \]

\( V_{th} \) is called the **thermal voltage**.
The Intrinsic Potential Reference

- By integrating the equation relating potential to the electron concentration from a position $x_a$ to position $x$, we find that:

$$\phi(x) - \phi(x_a) = V_{th} \ln \left( \frac{n_o(x)}{n_o(x_a)} \right)$$

We can choose any reference; one convenient choice is to set:

$$\phi(x_a) = 0 \text{ when } n_o(x_a) = n_i = 10^{10} \text{ cm}^{-3} \text{ at room temperature.}$$

- Using this reference, the potential in thermal equilibrium can be found, given the electron concentration:

$$\phi = V_{th} \ln \left( \frac{n_o}{n_i} \right) = (26 \text{ mV}) \ln(10) \log \left( \frac{n_o}{10^{10}} \right) = (60 \text{ mV}) \log \frac{n_o}{10^{10}}$$

Donor concentrations from $10^{13}$ to $10^{19} \text{ cm}^{-3}$ therefore correspond to potentials of $(60 \text{ mV}) \times 3 = 180 \text{ mV}$ to $(60 \text{ mV}) \times 9 = 540 \text{ mV}$ (at room temperature)
The hole concentration can also be related to the potential, by repeating the derivation starting with $J_p = 0$ or by substituting

$$p_o = n_i^2 / n_o$$

into the 60 mV rule for electrons. The result is:

$$\phi = V_{th} \ln \left( \frac{n_i}{p_o} \right) = (-26 \text{mV}) \ln(10) \log \left( \frac{p_o}{10^{10}} \right) = (-60 \text{mV}) \log \left( \frac{p_o}{10^{10}} \right)$$

$p_o$, equilibrium hole concentration (cm$^{-3}$)

$p_n$, equilibrium electron concentration (cm$^{-3}$)
pn Junctions

- ubiquitous IC structure -- pn junctions are everywhere!

- thermal equilibrium: no hole current, no electron current ... no voltage applied between metal interconnects (could short them together)
Diffusion Currents in Thermal Equilibrium

- huge gradients in hole and electron concentration --> assume a transition region between \(-x_{po}\) and \(+x_{no}\)

\(n_{o} = \frac{n_i^2}{N_d} = 10^{4} \text{ cm}^{-3}\)

\(p_{o} = N_a = 10^{16} \text{ cm}^{-3}\)

Example: \(N_a = 10^{16} \text{ cm}^{-3}\)
\(N_d = 10^{16} \text{ cm}^{-3}\)

\(\text{transition region } \{ -x_{po} < x < x_{no} \}\)

\(p_{o} = \frac{n_i^2}{N_d} = 10^{4} \text{ cm}^{-3}\)

\(n_{o} = N_d = 10^{16} \text{ cm}^{-3}\)

\(\text{transition region } \{ -x_{po} < x < x_{no} \}\)

\(\text{transition region } \{ -x_{po} < x < x_{no} \}\)

note: we don’t know how wide the transition region is (yet)
Drift and Diffusion in the Transition Region

- $J_{no} = 0$ and $J_{po} = 0$ due to equilibrium

  --> negative electric field in the transition region is needed ...
  
  where do + and - charges come from?

- Answer: the roll-off in electron concentration between $x = 0$ and $x_{no}$ means that

  \[ \rho_o(x) = q(-n_o(x) + N_d) > 0 \]

  since $n_o(x) < N_d$ on the n-side of the transition region

- On the p-side, the charge density is negative, since the hole concentration rolls off between $x = -x_{po}$ and $x = 0$. 
Qualitative Electrostatics in Equilibrium

- From the charge density, we can find the electric field and the potential

![Diagrams showing charge density, electric field, and potential](image)

\[ \rho_\sigma(x) \text{ (C/cm}^3\text{)} \]

\[ E_\sigma(x) \]

\[ \phi_\sigma(x) \]

\[ \phi_p = -360 \text{ mV} \]

\[ \phi_n = 360 \text{ mV} \]
Quantitative pn Junction in Thermal Equilibrium

The Depletion Approximation

- In the bulk regions far away from the junction, we can approximate

\[ \rho = 0 \]

- Near the junction, the charge density is non-zero. For example, on the n-side of the junction in the transition region, \(0 < x < x_{no}\):

\[ \rho = q (p_o + N_d - n_o - N_a) = q (N_d - n_o) \]

since there are no acceptors on this side \((N_a = 0)\) and the hole concentration is negligible \(\rightarrow p_o = 0\) (approx.)

The maximum positive value for charge density on the n-side is when there are no electrons present in equilibrium -- that is, when the silicon in the transition region is depleted of electrons.

For hand calculations, we will assume that

\[ \rho = \rho_{max} = q N_d \quad (0 < x < x_{no}) \]

\[ \rho = -q N_d \quad (-x_p < x < 0) \]

and proceed to find the width of the transition region, which we will rename the depletion region. The charge density is assumed to fall off abruptly from these values to zero in the bulk regions, where \(x < -x_{po} and x > x_{no}\)
One more time ...

- Bulk silicon is \textit{NEUTRAL}, to a good approximation
  
  $\text{region 1 is } \text{bulk:}$
  
  \[
  \rho = q(N_d + p_o - N_a - n_o) \equiv 0 \quad \rightarrow \quad p_o \equiv N_a
  \]

  $\text{region 4 is } \text{bulk:}$

  \[
  \rho = q(N_d + p_o - N_a - n_o) \equiv 0 \quad \rightarrow \quad n_o \equiv N_d
  \]

- Near the junction, the silicon is \textit{DEPLETED} of mobile carriers:

  $\text{region 2 is } \text{depleted:}$

  \[
  \rho = q(N_d + p_o - N_a - n_o) \equiv -qN_a
  \]

  $\text{region 3 is } \text{depleted:}$

  \[
  \rho = q(N_d + p_o - N_a - n_o) \equiv qN_d
  \]

---

Diagram:

```
       p
         1
        / \ 2
       /   \ 3
      /     \ 4
     /       \
-xp  xno
```

\( x_{no}, x_{po} \) are not known yet -- use boundary conditions to find them
pn Junction in Thermal Equilibrium:
Using the Depletion Approximation

For detailed calculations, see Section 3.4. Analysis is straightforward, but involved. Use the fact that:

> Charge in depletion region must sum to zero (why?)
> Electrostatic potential is continuous
Depletion Widths in Thermal Equilibrium

\[ x_{po} = \sqrt{\frac{2 \varepsilon_s \phi_B}{qN_a}} \left( \frac{N_d}{N_d + N_a} \right) \]

\[ x_{no} = \sqrt{\frac{2 \varepsilon_s \phi_B}{qN_d}} \left( \frac{N_a}{N_d + N_a} \right) \]

\[ X_{do} = x_{no} + x_{po} = \sqrt{\frac{2 \varepsilon_s \phi_B}{q}} \left( \frac{1}{N_a} + \frac{1}{N_d} \right) \]

- Asymmetric junctions: i.e., \( N_a \gg N_d \) or \( N_d \gg N_a \).

  >> most of depletion width is on the side with the lower doping, since

\[ \frac{1}{N_a} + \frac{1}{N_d} \approx \frac{1}{N_d} \quad (N_a \gg N_d) \]

\[ \frac{1}{N_a} + \frac{1}{N_d} \approx \frac{1}{N_a} \quad (N_d \gg N_a) \]

>> most IC pn junctions are highly asymmetric
pn Junction under Reverse Bias

- First, we must understand the complete structure of the pn junction-- starting in thermal equilibrium:

- How can $V_D = 0$ and the built-in potential barrier be $\phi_B = 1$ V (approx.)?
  
  Answer: look at the complete circuit ... including the potential barriers at the p-type silicon-to-metal ($\phi_{pm}$) and the metal-to-n-type silicon ($\phi_{mn}$) junctions.

- Kirchhoff’s Voltage Law:

  \[ 0 = \phi_{pm} + \phi_B + \phi_{mn} \]

  therefore, the built-in voltage is given by:

  \[ \phi_B = -\phi_{pm} - \phi_{mn} \]