Special cases of Min-Cost Flow Problem (cont.)

Shortest Path Problem
- one source & one sink
- distances on arcs
- Find the shortest path from source to sink

Q: How to formulate this as a min-cost flow problem.
- Assign $b_s = 1$, $b_t = -1$, $b_i = 0$ for all other nodes $i$
- costs = distances
- no upper bounds necessary

Note: This formulation gives a reasonable solution to Shortest Path problem based on Integrality Property.

Maximum Flow Problem
- source $s$ and sink $t$
- upper bound $U_{ij}$ on each arc $i \rightarrow j$
- Goal: Send as much flow as possible from $s$ to $t$

Formulating as Min-Cost Flow Problem
- Set $C_{ij} = 0$ for all existing arcs $i \rightarrow j$
- Set $b_s = F$, $b_t = -F$ where $F$ is a safe upper bound for max flow
- $b_i = 0$ for other nodes $i$
- Add an arc $s \rightarrow t$ with $C_{st} = M$, $U_{st} = +\infty$ where $M$ is a large #
- Since $C_{st} = M$, the Max-Flow Problem will send as much as possible by real arcs and the rest of the demand by artificial arc $s \rightarrow t$

Note: Integrality property will provide a reasonable (integer) solution to the problem.

All the special cases we discussed,
- Transportation Problem
- Assignment Problem
- Shortest Path Problem
- Max. Flow Problem

Can be solved in two ways:
1) Formulating as Min-Cost Flow and applying a Min-Cost Flow algorithm (e.g., Network Simplex)
2) Applying special-purpose algorithms designed for each of these cases

An example of a Network Model which is not a special case of Min-Cost Flow:

Min Spanning Tree Problem
Given undirected graph $G = (V, E)$, $|V| = n$
- cost function $c : E \rightarrow \mathbb{R}$
- Find min-cost spanning tree for $V$.

That is, find subset of arcs $E' \subseteq E$ which connects any two nodes of $V$ with minimal possible cost.

Some Terminology of Networks (before going to Min Span. Tree problem)

- A path between two nodes is a sequence of distinct arcs connecting these nodes.

- A path that begins and ends at the same node is called a cycle.

- Two nodes are connected if there is a path between them.
- A graph is connected if every pair of nodes is connected.
- A graph is acyclic if it doesn't have any cycle.
- A graph is called tree if it is connected and acyclic.