Objective for "min-cost flow problem": Find a feasible flow \( X \) on arcs such that cost is minimized.

Feasible means:
- supply & demand are satisfied at each node (flow conservation constraints)
- \( b \leq X \leq u \) (bound constraints)

- What's the corresponding LP model?
  decision variables \( X_{ij} \): flow to send from node \( i \) to \( j \).

LP formulation:

\[
\begin{align*}
\text{min} \quad & \sum_{(i,j) \in E} C_{ij} X_{ij} \\
\text{s.t.} \quad & \sum_{j : (j,k) \in E} X_{jk} - \sum_{i : (i,j) \in E} X_{ij} = b_j \quad \forall \text{ node } j \\
& \text{(flow conservation constraints)} \\
& \ell_{ij} \leq X_{ij} \leq u_{ij} \quad \forall \text{ arc } i \rightarrow j \\
& \text{(bound constraints)}
\end{align*}
\]

- Integer solutions property: If \( b_i \) and \( c_{ij}, u_{ij} \) are all integers, then in every BFS solution, all basic variables also have integer values.

Special Cases:

Transportation Problem

The set of nodes is divided into 2 groups: sources & sinks.
\[ V = S \cup T \cup \text{sink} \]
- each source is a supply node: \( b_j > 0 \)
- each sink is a demand node: \( b_j < 0 \)
- each arc of the graph goes from a sink to a source. 

\[ \ell_{ij} = 0, u_{ij} = \infty \]

Assignment Problem

A special case of transportation problem when \( |S| = |T| \)
- \( b_j = 1 \) for each supply node
- \( b_j = 1 \) for each demand node

Next time
- Other special cases
- How to solve network models.