General form of LP

\[
\begin{align*}
\text{max/min} & \quad \sum_{i=1}^{n} c_i x_i \\
\text{subject to} & \quad \sum_{i=1}^{n} a_{ij} x_i \leq b_j \quad \forall j \in \{1, \ldots, m\} \\
& \quad x_i \geq 0 \quad \forall i \in \{1, \ldots, n\}
\end{align*}
\]

This is the standard form of LP.
Can also have equality or \( \geq \) constraints.

Assumptions of LP

- proportionality (no discounts, \( c_i x_i \) not allowed)
- additivity (\( x_i \)s not allowed)
- divisibility (fractional values allowed)
- certainty (of demand, etc.)

(read more in H&L 3.3)

Other ways of writing LPs

1) \[
\begin{align*}
\text{max} & \quad \sum_{i=1}^{n} c_i x_i \\
\text{s.t.} & \quad \sum_{i=1}^{n} a_{ij} x_i \leq b_j \quad \forall j \in \{1, \ldots, m\} \\
& \quad x_i \geq 0 \quad \forall i \in \{1, \ldots, n\}
\end{align*}
\]

This summation form is used in AMPL.

2) matrix form

\[
c = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, \quad A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}
\]

\[
\begin{align*}
\text{max} & \quad c^T x \\
\text{s.t.} & \quad Ax \leq b \\
& \quad x \geq 0
\end{align*}
\]

Input in LP is \( (A, b, c) \) \( \rightarrow \) given data, parameters
Output \( \rightarrow \) is \( x \) \( \rightarrow \) variables

LP applications

- production planning (inventory models, \ldots)
- finance (portfolio management, \ldots)
- scheduling (airline, production, \ldots)
- networks (transportation, telecommunications, \ldots)

Goals in this class

- practise modelling process: given "real world problem", formulate as a MP problem (here mostly LP); become aware of underlying assumptions & limitations
- learn a general algorithm (solution procedure) for LP problems (Known as the "Simplex method")
- learn to view LP & the algorithm from different perspectives: geometry, algebra
- perform sensitivity analysis:
  what if data changes?
  how much will solution change, if at all?
- look into applications & special cases of LP
- learn to justify answers in a mathematically precise way
- introduction to nonlinear programming

- Programming = Planning in this context, origins go back to military logistics in WWII (1940s)
- In a survey of Fortune 500 firms, 85% of those responding said that they had used LP