Mathematical models in optimization in general, look like the following:

\[
\begin{align*}
\min \text{ or } \max \ f(x) & \quad \text{}``objective function'' \\
\text{subject to } \ a(x) & \geq 0 \quad \text{}``functional constraint'' \\
\end{align*}
\]

where set

\[
f: \mathbb{R}^n \rightarrow \mathbb{R}, \quad a: \mathbb{R}^n \rightarrow \mathbb{R}^m, \quad S \subseteq \mathbb{R}^n
\]

\(x\) is called the decision variable.

\(\nabla\) In words, the goal is to find a vector \(x\) that

- satisfies the constraints \((a(x) \geq 0)\)
- achieves \(\max(\min)\) objective function value.

\(\nabla\) In the general form, very hard to solve.

\(\nabla\) In this class a special case that

- is widely used in practice.
- there is an algorithm which will find an "optimal" (best) solution.

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### A simple first example

**Furniture manufacturer**

- production of 1 table requires 2 ft pine, 1 ft oak, 3 hr labor.
  - 1 chair: 1 ft pine, 2 hr labor.
  - 1 desk: 5 ft pine, 4 ft oak, 5 hr labor.
  - 1 bookcase: 12 ft pine, 1 ft oak, 10 hr labor.

- for 1 week have
  - 1,500 ft pine
  - 1,000 ft oak
  - 20 employees who can each work 40 hrs.

- market data:

<table>
<thead>
<tr>
<th></th>
<th>profit</th>
<th>demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>table</td>
<td>$12/\text{unit}</td>
<td>40</td>
</tr>
<tr>
<td>chair</td>
<td>$5/\text{unit}</td>
<td>130</td>
</tr>
<tr>
<td>desk</td>
<td>$15/\text{unit}</td>
<td>30</td>
</tr>
<tr>
<td>bookcase</td>
<td>$10/\text{unit}</td>
<td>-</td>
</tr>
</tbody>
</table>

Goal: find production schedule for 1 week that will maximize profit.

Production schedule = \# tables to be produced \(\rightarrow x_t\)
- \# chairs \(\rightarrow x_c\)
- \# desks \(\rightarrow x_d\)
- \# bookcases \(\rightarrow x_b\)

**ALWAYS DEFINE VARIABLES PROPERLY!**