The graph $H$ is made up of the graph $G$ with an edge connecting $a$ and $g$. Any attempt to color $H$ using three colors must follow the same reasoning as that used to color $G$, except at the last stage, when all vertices other than $g$ have been colored. Then, since $g$ is adjacent (in $H$) to vertices colored red, blue, and green, a fourth color, say brown, needs to be used. Hence, $H$ has a chromatic number equal to 4. A coloring of $H$ is shown in Figure 4.

**EXAMPLE 2** What is the chromatic number of $K_n$?

*Solution:* A coloring of $K_n$ can be constructed using $n$ colors by assigning a different color to each vertex. Is there a coloring using fewer colors? The answer is no. No two vertices can be assigned the same color, since every two vertices of this graph are adjacent. Hence, the chromatic number of $K_n = n$. (Recall that $K_n$ is not planar when $n \geq 5$, so this result does not contradict the Four Color Theorem.) A coloring of $K_5$ using five colors is shown in Figure 5.

**EXAMPLE 3** What is the chromatic number of the complete bipartite graph $K_{m,n}$, where $m$ and $n$ are positive integers?

*Solution:* The number of colors needed may seem to depend on $m$ and $n$. However, only two colors are needed. Color the set of $m$ vertices with one color and the set of $n$ vertices with a second color. Since edges connect only a vertex from the set of $m$ vertices and a vertex from the set of $n$ vertices, no two adjacent vertices have the same color. A coloring of $K_{3,4}$ with two colors is displayed in Figure 6.