Unraveling complex systems:
What do brains, the internet, and ant colonies have in common?

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Brains

Human brains are composed of billions of neurons that exhibit relatively simple firing patterns. Neurons are connected by dendrites and axons and communicate (receive and send signals) along these structures.

How can the firing patterns of the neurons give rise to perceptions, feelings, thoughts, and actions?

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How does Wikipedia, a reasonably reliable source of reference, emerge out of the largely uncoordinated creativity of its authors? How does it happen that the accumulating mass of web pages reshapes the way we work, retrieve information, shop, socialize, and spend our leisure time?
Ant colonies

Individual ants are capable of only a very limited set of simple behaviors. They communicate with their nestmates by using only a few distinct olfactory and tactile cues.

How can an ant colony build elaborate nests and even farm fungi, which amounts to creating a habitat for another species?

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What is a complex system? First approximation.

The microscopic level consists of many relatively simple agents of possibly several types. Agents interact. The structure of these interactions is called the connectivity of the system. At the macroscopic level the system interacts with the environment in complex ways. The system may even shape its environment.
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The stock market

Individual investors make decisions to buy or sell stocks largely based on self-interest and partial information about the state of the system. How do the individual decisions lead to market equilibrium most of the time and market crashes some of the time? Which mechanisms can prevent systemic failure?

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Cells

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Administrative structures

Administrators are fallible human beings with cognitive limitations common to our species. They make decisions based on enlightened self-interest. They communicate along institutional channels. How does this make institutions function, by and large, the way they are supposed to? When do things go wrong? Which mechanisms can prevent systemic failure?

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A pioneer in the study of complex systems

The complex systems we listed are the subject matter of a variety of sciences. Nevertheless, they have a lot in common. The first scientist to clearly discern and spell out these communalities was Herbert A. Simon (1916-2001). He studied administrative structures. And a lot of other things.

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He was not a mathematician, but used a lot of mathematics in his work. And he built on the work of others, including mathematicians Norbert Wiener (1894–1964) and Alfred J. Lotka (1880-1949).
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- Most complex systems have a **hierarchical** and **modular** structure. So there may be more than two levels.
- At the microscopic level agents may behave somewhat randomly and are prone to failure. Despite of this, the behavior of the system at the macroscopic level tends to be fairly robust.
A Boeing 787

An airplane is a complex system in the sense of the previous slide. But somehow it doesn't seem to fit with the theme of our talk. What makes a Boeing 787 different from our other examples?
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The distinction is not the same as between “natural” and “engineered” systems. Electrical power grids and transportation systems are complex and fairly adaptive. They are also a lot more robust to component failure than airplanes, but not as resilient as, say, ant colonies.
Is it even possible to study complex adaptive systems?

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Fortune cookie: Doing the impossible is kind of fun.

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What’s math got to do with it?

If scientists want to predict any future measurements they need to build mathematical models. Mathematics is much more than the science of numbers. It is the science of abstract patterns. Mathematics is uniquely suited to unravel the common features of complex adaptive systems that we have been marveling at.
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Dynamical systems models

Study the time courses (trajectories) of the variables either by simulations or mathematical deductions (proofs).

Stochastic models allow for some randomness in the trajectories, deterministic models assume that the current state uniquely determines the trajectory.

Continuous (e.g. ODE, PDE) models assume that time can take any (nonnegative) real values; discrete time (e.g. difference equation, Boolean) models assume that time moves in discrete steps, that is, only takes integer values.

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Complex dynamical systems models

Identify sets of microscopic variables and macroscopic variables.
Write down equations that model the change of the microscopic variables to the best of our knowledge.
If we want to model adaptive systems, these equations must be allowed to change over time in some ways.
Try to predict (by simulations or deductions) the trajectories of the macroscopic variables, and, if applicable, the change of the equations for the microscopic variables.

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When does a system qualify as complex?

Defining “complexity” is devilishly complex. A lot of different definitions can be found in the literature. For starters, see http://en.wikipedia.org/wiki/Complexity. On the next slide we will give (sort of) a definition that will do for this talk.
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Let us say that a dynamical system is complex if the dynamics of individual agents is relatively simple in the sense of involving few variables (that is, if agents are low-dimensional) and at the macroscopic level the dynamics is capable to satisfactorily solve all computational problems posed by the environment. Computational problems usually arise out of the "need" for the system to sustain itself or its progeny. Usually there is some objective function that complex adaptive systems "try to" optimize. Beware of anthropomorphisms! Systems really only behave as if they had "needs" or "tried to." Complex adaptive systems rarely find optimal solutions, only reasonably good ones. In Herbert Simon's terminology, they "satisfice."
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An ideal gas consists of $n$ moles of particles, which gives $\approx 6n \times 10^{23}$ agents who can be characterized by $\approx 36n \times 10^{23}$ microscopic variables and who interact according to the laws of (Newtonian) mechanics. This causes the macroscopic variables $P$, $V$, $T$ to obey the law

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Personally, I prefer to look at this example as a limiting case of complex systems. We can learn a lot from this example.
What can we learn from classical thermodynamics?

The variables in (1) represent averages. Systems in which macroscopic variables represent averages of microscopic variables are usually not considered complex. If the equations of the microvariables are linear, the macroscopic behavior represents averages. Most definitions of complex systems require nonlinear dynamics. Nonlinearity is necessary for truly complex dynamics, but it is not sufficient for avoiding averages. Do the particles in an ideal gas bump into one another in a linear way? We also can average if the interactions of agents are assumed to be independent of each other. Details (of the initial state, of the mechanics of the actual bumpings of particles into each other) don't matter all that much in the derivation of (1). (1) does not hold with certainty, only with probability very, very close to 1, and relies on the yet unproven ergodic hypothesis.
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What’s the mystery?

In complex systems, the dynamics at the macroscopic level somehow emerges as a result of the dynamics at the macroscopic level. This is difficult to comprehend. Try to intuit how you perceive the average kinetic energy of the molecules as temperature. Many complex systems spontaneously self-organize into hierarchical structures. This is an especially counterintuitive kind of emergence. While evolution is the primary mechanism by which biological systems adapt, emergence and self-organization also happen ins systems that do not evolve.
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While evolution is the primary mechanism by which biological systems adapt, emergence and self-organization also happen in systems that do not evolve.
What theorems?

What will the theorems of a "mathematics of complex systems" look like?

Theorem
If suitable assumptions about the internal dynamics of the agents, their connectivity, and their interactions are satisfied, then the dynamics of the macroscopic variables will exhibit such and such features with probability close to 1.

Exhibit A:
The ideal gas law.

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**Exhibit A:** The ideal gas law.
Exhibit B: Hurricanes

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- The dynamics of complex systems may or may not be chaotic. Chaotic systems may or may not be complex. Chaos theory is something very different from a theory of complex systems.
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*If cells respond to fairly general feedback mechanisms, then with substantial probability yeast cultures at certain cell densities in a well-stirred vat will spontaneously break up into several clusters that are near-synchronized for cell-cycle state and we will observe corresponding oscillations of oxygen levels.*
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Results along these lines are being obtained in an ongoing NIH-funded research project led by Prof. T. R. Young that involves a number of OU graduate and undergraduate students as well as several internal and external collaborators.
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- One such theorem was proved in D. Terman, S. Ahn, X. Wang, and W. Just; Reducing neuronal networks to discrete dynamics. *Physica D* 237(3) (2008) 324-338.
- A joint paper by W. Just, M. Korb, B. Elbert, and T. R. Young that will be submitted shortly proves another theorem of this kind.
Which areas of mathematics are important for the study of complex adaptive systems?

- Differential equations: ODEs, PDEs, differential delay equations
- Stochastic differential equations
- Stochastic processes
- More generally: dynamical systems (discrete, continuous, deterministic, stochastic)
- Theory of computation
- Finite automata
- Probability and statistics
- Statistical mechanics

... (you name it)
Which areas of mathematics are important for the study of complex adaptive systems?

- Differential equations: ODEs, PDEs, differential delay equations
- Stochastic differential equations
- Stochastic processes
- More generally: dynamical systems (discrete, continuous, deterministic, stochastic)
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- ... (you name it)
Will there be a “new mathematics” of complex systems?

Existing mathematical tools work pretty well for complex systems up to a point. Complex systems have certainly spurred interest in previously overlooked mathematical questions. The study of “scale-free networks” is a prime example. We will need some new tools (e.g., Exhibit D). We will need to adapt new points of view, for example, treat ODE systems as performing computations of sorts. This will require us to leave our comfort zones and cross (sub)disciplinary boundaries. We will need to avoid getting bogged down by the messy details of real physical examples. Will there be whole new branches of mathematics? Perhaps. Nobody knows. A candidate may be a branch of dynamical systems theory devoted to systems that adapt over time.
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