Learning Objectives
1. To operate a Global Cooling Model 100B Stirling Cooler
2. To apply principles of thermodynamics in analyzing the performance of refrigeration
3. To calculate the coefficient of performance, \( \dot{Q}_{\text{rejected}} \) and \( \dot{Q}_{\text{lifted}} \)

Experimental Objective
To determine the coefficient of performance of a Stirling cooler at four different operating temperatures and observe any changes with temperature

Apparatus
The equipment used for the Striling Cooler laboratory is as follows:
1. Global Cooling Stirling Cooler (Model M100B)
2. Cole-Parmer Flow meter (S/N 185799)
3. Type K Thermocouples, and J-type thermocouple
4. 2 0-110 volt auto transformers used to control the voltage into the Stirling Cooler and heater
5. Minco Resistance Heater (Part # HK5546 R8.9L12B)
6. Signal Transformer 36-6 Step down transformer (Part #36-6)
7. Omega heater temperature indicator/controller using a K-type thermocouple sensor (Model CN76020)
8. Digital Power Meter (Model WT110)
9. 2 Digital Multi-meters (Fluke)
10. Temperature Meters (Omega)
11. USB computer data acquisition system

Figure 1: Schematic of the Stirling Cooler Experimental Apparatus.
The model 100B free-piston Stirling cooler operates on a Stirling refrigeration cycle using a moving magnet linear motor. In theory, the Stirling cycle is among the most efficient energy conversion devices. The cooler consists primarily of a piston and a displacer. An AC linear motor is used to drive the piston, which compresses (and conversely, expands) the working gas. In the lab, the working gas is helium, but in theory could be any gas with a positive Joule-Thompson coefficient. The displacer shuttles the working fluid (gas) between the cold (heat acceptor) and warm (heat rejector) sides of the cooler. The linear motor is driven by a pulse width modulated (PWM) control unit which yields maximum piston displacement until the set temperature is reached, then the cooler is modulated to meet the cooling demands.

One of the main objectives of the Stirling cooler experiment is to understand and apply the principles of thermodynamics in analyzing the performance of a refrigeration system, specifically a Stirling cooler. A schematic of the apparatus is shown in Figure 1, and details of the Stirling cooler are shown in Figure 3.
The performance of the Stirling cooler (or any refrigeration system for that matter) can be quantified by the coefficient of performance, \( COP_R \), which is equal to the heat per unit time removed (or lifted) for a known heat source divided by the power required to operate the Stirling cooler. This relationship is given in Equation 1.

\[
COP_R = \frac{\dot{Q}_{\text{Lifted}}}{\dot{W}_{\text{Stirling}}}
\]  

(1)

where \( \dot{Q}_{\text{Lifted}} \) is heat per unit time removed or “lifted” from the known heat source (which is a heater located at the heat acceptor seen in Figure 3 for this experiment). \( \dot{W}_{\text{Stirling}} \) is the power (or work per unit time) required by the Stirling Cooler to maintain \( \dot{Q}_{\text{Lifted}} \).

**Transient Analysis**

To reduce the time required to obtain the data necessary to calculate \( COP_R \) at different test temperatures, a dynamic test is proposed. Consider the 1st law of thermodynamics as a rate equation in which the heat acceptor head is the control volume. Since there is no mass flow or work associated with this control volume, the heat lifted per unit time is equal to the change in internal energy of the heat acceptor head with respect to time. This is given by Equation 2.

\[
\dot{Q}_{\text{Lifted}} = m c_p \frac{dT}{dt}
\]

(2)

where \( m \) is mass (270 g for the copper acceptor head of this Stirling Cooler), \( c_p \) is heat capacity of the copper, \( T \) is temperature and \( t \) is time. If the unit is operated initially from room temperature with no
applied heat to the acceptor head, then from Equation 2, $\dot{Q}_{lifted}$ can be determined at a specific temperature if the instantaneous slope of the time-temperature curve is known at that temperature. Note that $c_p$ also changes with temperature and this is depicted in Figure 4 for copper, which is from what the acceptor head is made. To determine the $COP_R$, $\dot{Q}_{lifted}$ per Equation 2 can be divided by $\dot{W}_{Stirling}$ at that instant.

![Graph showing heat capacity as a function of temperature for copper](image)

**Figure 4: Heat capacity as a function of temperature for copper (from Global Cooling)**

Time-temperature data will be recorded with the USB data acquisition system (see data acquisition set-up instructions), and these data can be differentiated at the temperatures at which $COP_R$ is to be determined. A plot, similar to Figure 5, of temperature vs. time must then be created. Knowing that the relationship between temperature and time is an exponential fit, determine the slope of the curve will yield $dT/dt$. For this lab the $COP_R$ at 5°C, 0°C, -10°C and -23°C are to be determined.
You must manually record the power to the Stirling Cooler and the temperature of the water leaving the heat rejector head (for Carnot COP) when the acceptor head reaches these temperatures otherwise you will not be able to calculate the $COP_R$.

### Static Analysis

In order to support the experimental approach given above, an energy balance at equilibrium conditions must be performed. The dynamic test data should be performed to about -30°C, at which point the heater is to be activated and its variac adjusted such that the temperature of the acceptor head increases to about -20°C and remains stable. It is not important that this temperature is precisely achieved, but it is important that equilibrium is achieved. Equation 4 presents the equation that must balance in terms of energy per unit time.

$$\dot{W}_{\text{Stirling}} = \dot{Q}_{\text{rejected}} - \dot{Q}_{\text{Lifted}}$$  \hspace{1cm} (3)

where $\dot{Q}_{\text{Lifted}}$ is calculated with Equation 4.

$$\dot{Q}_{\text{Lifted}} = V_{\text{variac}} I_{\text{variac}}$$  \hspace{1cm} (4)

where $V_{\text{variac}}$ and $I_{\text{variac}}$ are the voltage and current, respectively, delivered to the heater at the “acceptor head”. Of course, this assumes that $\dot{Q}_{\text{lost}}$ is zero.

$\dot{Q}_{\text{rejected}}$ is the energy per unit time removed from the “heat rejector.” Heat is rejected (or removed from the rejector head) by passing cold tap water through the heat rejector. To determine $\dot{Q}_{\text{rejected}}$,
the water flow rate, and the inlet and outlet water temperatures to the heat rejector must be measured as shown in Equation 5.

\[
\dot{Q}_{\text{rejected}} = \dot{m} c_p (T_{\text{outlet}} - T_{\text{inlet}})
\]  

(5)

where \(\dot{m}\) is the mass flow rate of the water, \(c_p\) is the specific heat of water (4181 J/kg-K @ 22ºC), and \(T_{\text{outlet}}\) and \(T_{\text{inlet}}\) are the outlet and inlet temperatures to the heat rejector, respectively. The mass flow rate is determined by Equation 6.

\[
\dot{m} = \rho_{\text{water}} \dot{V}
\]  

(6)

where \(\rho_{\text{water}}\) is the mass density of water (998 kg/m³ @ 22ºC), and \(\dot{V}\) is the volumetric flow rate of the water. The water’s flow rate is approximated and controlled with the Cole-Parmer flow meter. The flow rate is determined by timing how long it takes to fill a 100 ml graduated cylinder. An average of three trials should be sufficient.

**Heat Lost (or Gained)**

Equation 4 should ideally balance due to the conservation of energy. However, the determined values of \(\dot{Q}_{\text{rejected}}\) and \(\dot{Q}_{\text{Lifted}}\), maybe affected by both heat lost or gained by the system and uncertainties in measurement. Of course, the uncertainties of \(\dot{Q}_{\text{rejected}}\) and \(\dot{Q}_{\text{Lifted}}\) must be calculated and discussed with respect to any unbalance in Equation 4 (check if the uncertainty explains the unbalance and discuss). But more likely, there is actual heat gained or lost.

For your steady state analysis, you are asked to find Stirling input power two ways – one (\(\dot{W}_{\text{stirling}}\)) from the power meter (which is accurate), and the other is from the difference in heat rejected and heat lifted, as shown in Equation 3. You were asked to assume there is no heat lost (or gained).

However, you will likely find a difference between your measured \(\dot{W}_{\text{stirling}}\) and calculated \(\dot{W}_{\text{stirling}}\). This must be \(\dot{Q}_{\text{lost}}\) (or gained), as shown in Equation 7.

\[
\dot{P}_{\text{stirling}} = \dot{Q}_{\text{rejected}} + \dot{Q}_{\text{lost}} - \dot{Q}_{\text{Lifted}}
\]  

(7)

One way to account for this in the effect on COP is to do as the lab guidelines say in using Equation 8 (shown next) – **BUT TAKE NOTE – EQ 8 IS ONLY VALID IF YOU FIND THAT \(Q_{\text{lost}}\) IS NEGATIVE** (heat being absorbed from the room). All of you found either little heat lost or your heat lost was positive.

\[
\dot{Q}_{\text{Lifted}} = V_{\text{varia}} I_{\text{varia}} - \dot{Q}_{\text{lost}}
\]  

(8)
If heat loss is positive, then Equation 8 IS NOT VALID. Positive heat loss (heat transferred from the Stirling cooler to the surrounds) must occur at the heat rejection surface and cannot occur at the heat acceptor (because it is colder than the room). In this case, the only effect is on heat rejected (which does not affect COP) and is given in Equation 9 as

\[ Q_{\text{Rejected}} = m c_p (T_{\text{outlet}} - T_{\text{inlet}}) + Q_{\text{lost}} \]  

(9)

**Comparison to Global Cooling COPs**

Global Cooling has already developed relationships between \( \dot{Q}_{\text{Lifted}} \) and \( \dot{W}_{\text{Stirling}} \) as shown in Figure 6 for a Global Cooling Stirling Cooler. Using the data in Figure 6, you can determine heat lifted for your input powers and determine the \( COP_R \) from Global Cooling testing.

![Figure 6: Representative data for heat lifted versus power input (from Global Cooling).](image)

**Carnot COP**

A Stirling cooler has an excellent coefficient of performance compared to other cooling devices, due in part to the fact that there is very little friction within its mechanical parts. This means that very little energy is lost due to friction and vibration. In order to compare the Stirling cycle with that of the ideal Carnot cycle, the Carnot efficiency can be solved for the conditions of this experiment using Equation 10.
\[ \text{COP}_{R,Carnot} = \frac{1}{\frac{T_{\text{outlet}}}{T_{\text{Lifted}}} - 1} \]  

(10)

where \( \text{COP}_{R,Carnot} \) is the coefficient of performance for a comparable Carnot system, \( T_{\text{outlet}} \) is the absolute temperature of the water flowing out of the Stirling cooler, and \( T_{\text{Lifted}} \) is the absolute temperature of the heat acceptor.

**Uncertainties**

You are to estimate reasonable uncertainty values for the measurements that you are making to calculate \( \dot{Q}_{\text{rejected}} \), \( \dot{Q}_{\text{Lifted}} \), and \( \text{COP}_R \). Uncertainties of \( \dot{Q}_{\text{rejected}} \) and \( \dot{Q}_{\text{Lifted}} \) must justify the energy balance, which in turn will provide credibility to your \( \text{COP}_R \) and uncertainty in \( \text{COP}_R \).

**Procedure**

1. Using cold tap water, set the flow so the steel ball indicates a reading between 15 and 20. **Never operate the Stirling cooler without the cooling water flow.**
2. Determine the flow rate by timing how long it takes to fill a 100 ml graduated cylinder. Take 3 readings and calculate an average. Note your uncertainties in time and volume.
3. Set the sensitivity of the thermocouple displays to 0.1˚C. Note that just because you are measuring temperature to one decimal place does not necessarily mean than this is your uncertainty.
4. Make sure that heat acceptor temperature is approximately 20°C at the start of the test.
5. Connect the acceptor head thermocouple to the channel 1 of the USB data acquisition module and configure the software per the instructions (see other handout).
6. When the experiment is ready to go, start the data acquisition system. Slowly increase voltage of Stirling cooler to approximately 80V (do not exceed 85V). If a rapping sound is heard, quickly back off the voltage until the sound stops. This is the result of the piston over-stroking. Continuous over-stroking may cause damage to the cooler.
7. Record the power to the Stirling Cooler when the acceptor head is at 5, 0, -10 and -23C.
8. When the acceptor head reaches about -25C, activate the heater and slowly adjust its Variac (probably to about 20) such that an equilibrium acceptor head temperature of -20C is maintained for several minutes. -20C is only a guideline. Take whatever steady temperature you get!
9. When the equilibrium temperature is reached, record the power to the Stirling cooler, the inlet and outlet temperature of the water across the heat rejector, and the voltage and current being supplied to the heater. These data will be used for the energy balance.
10. Never allow the heater to increase the acceptor head temperature to above 20 C as it will cause damage to the cooler; however the temperature controller should prevent this from occurring.
11. Once the experiment is completed, turn the heater off first, and then the Stirling cooler. Let the water flow through the system for a few minutes to allow the Stirling cooler to dissipate the heat. Also, make sure you have turned off the data acquisition.
Requirements for analysis (and lab report)

1. Convert all data to SI units.
2. Input your data into suitable analysis software (i.e., Excel, Matlab, etc.) and plot your data as absolute temperature as a function of time (seconds).
3. Use Equations 1 and 2 to determine $COP_R$ at 5°C, 0°C, -10°C and -23°C. Use an appropriate technique to determine $dT/dt$ at these temperatures of interest. Also determine $COP_{Carnot}$ and $COP_{GlobalCooling}$ (using Figure 6, other data and equations) at each temperature. Note: This means you will have to record the power input to the Stirling cooler and the outlet water temperature from the heat rejection head.
4. For the equilibrium test data, calculate the power input into heater, the heat rejected to the cooling water, $COP_R$, and perform the energy balance of Equations 7-9 to find heat lost.
5. For the equilibrium test, compare $COP_R$ for the Stirling cooler to the $COP_R$ of a Carnot system operating at the same conditions.
6. Perform an uncertainty analysis on $\dot{Q}_{rejected}$ and $\dot{Q}_{Lifted}$ and the $COP_R$ of the Stirling cooler. Use the uncertainty analyses of $\dot{Q}_{rejected}$ and $\dot{Q}_{Lifted}$ to justify your energy balance. The uncertainty of the $COP_R$ is based on your estimated uncertainties for the measuring devices; however you must also state the variability in your temperature measurements.

References

1. Walker, G. Stirling Engines
2. Urieli and Berchowitz, Stirling Cycle Engine Analysis
3. www.ent.ohiou.edu/~urieli/Stirling/Stirling.html
4. www.globalcooling.com
Computer Data Acquisition Instructions for Stirling Cooler Lab

1. Open the program “PDAQView” on the computer desktop and you should have the following two windows.

2. Configure the first channel (PD1_A01) as follows to take the acceptor head temperature for the experiment. Click on the item you want to change (i.e., “User Label”, “On” and “Range”) and then change the entry in the menu that becomes visible above the tabs. In the figure above and below, “Range” has been highlighted and the menu indicates “Select input range” and this is a drop-down menu. Configure channel 1 as a “Type K” thermocouple, which is the temperature measurement device on the acceptor head. Note that cold junction compensation is automatically performed internally by this unit.

3. Set the sampling rate for the data acquisition. Click the icon and input “1” for the scan rate in the following menu. Thus, temperature for channel 1 will be recorded at a sampling
rate of 1.0 Hz or one sample every second.

4. To tell the computer where to send the data file click on the icon. The following window will be visible. Input the destination (under “Folder”) to where you want the file to be generated and the name (under “File”) you want to assign to the data file.

Click on the “Conversion Formats” icon to specify the type of data file as an ASCII file as follows. Click as shown in the following window. This will allow you to import your data into MS Excel or other data analysis software. Make sure only “ASCII Text” is checked.
5. Once you have configured the channels, sampling rate and data file, clicking on ![play](image) will begin data recording. Every time a data point is recorded, the number of scans (which is indicated in the figure below) will update to the next number.

However, before you start data recording, place and activate a meter on the desktop (see below) so you can view the data that is being generated during the test. This will allow you to monitor the temperature during the test as is required by the lab. **Just before you turn the power on to the Stirling cooler with the variac, start recording data so no data are lost.** Make sure to activate the heater when the head temperature is about -25°C.

6. Click on the ![play](image) icon to open and configure the meter window. The following window will open and can be dragged to a suitable location on your desktop. You can set the decimal places by “right-clicking” on the values and then configuring the “properties”. Set channel 1 (the K-type thermocouple) to 1 decimal place. The number of channels displayed is set with the ![drop-down](image) drop-down window. Clicking on the ![play](image) icon will commence the thermocouple temperature being displayed on the meter only. **This icon only activates the data display on the computer screen. You must click on the appropriate icon per step 5 to begin data acquisition (recording to file).**