Stirling cooler

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1 Purpose

The purpose of this experiment was to measure the Coefficient of performance (COP), heat rejected, and heat lifted of model 100B Stirling cooler. Stirling coolers are significant due to their high efficiency at relatively low temperatures. Cooler using the free piston design also have only 2 moving parts, the piston and displacer, so modes of failure are greatly reduced compared to a normal compressor. A schematic of the model 100B can be seen in Figure 1.

![Stirling cooler schematic](image)

Figure 1: Schematic of the Stirling cooler model 100B.

Theoretically a Stirling cooler/engine is the only machine capable of achieving Carnot efficiency. The thermodynamic operation of a Stirling cooler can be seen in Figure 2 where $Q_{\text{rej}}$ is the heat removed from
the system at the heat rejector, $Q_{lift}$ is the heat lifted from the object to be cooled at the heat acceptor, and $Q_R$ is the recovered regenerator heat.

![Diagram](image)

Figure 2: P-V diagram showing ideal Stirling cooling cycle.

The isothermal ($\Delta T = 0$) processes 1-2 and 3-4 follow the idea gas law as seen in Equation 1 where $P$ is pressure, $V$ is volume, $m$ is mass, $T$ is absolute temperature, and $R$ is the ideal gas constant.

$$PV = mRT = \text{const}$$  \hspace{1cm} (1)

From this the work from 1-2, $W_{1-2}$, can be calculated from Equation 2. The same process is followed for $W_{3-4}$. It can clearly be seen from Figure 2 that no work is done between 2-3 and 4-1.

$$W_{1-2} = P \frac{dV}{V} dV = mRT \frac{1}{V} dV = mRT \ln \left( \frac{V_2}{V_1} \right)$$  \hspace{1cm} (2)

This is a closed system so it is described by the First Law of Thermodynamics in Equation 3 where $Q$ is the heat added to the system, $W$ is work done by the system, and $\Delta U$ is the internal energy.

$$Q - W = \Delta U$$  \hspace{1cm} (3)

For an isothermal process the internal energy is zero ($\Delta U = mC_v\Delta T = 0$) and the First Law simplifies to Equation 4.
\[ Q = W \] (4)

Thus for the entire cycle the work required can be calculated as seen in Equation 5.

\[ W_{in} = W_{3-4} - W_{1-2} = Q_{4-3} - Q_{1-2} = Q_{rej} - Q_{lift} \] (5)

To quantify how efficient the cooler is the coefficient of performance, \( COP_R \), is used. \( COP_R \) is defined in Equation 6, using power which is more appropriate for measuring on a running cooler instead of energy as used above.

\[ COP_R \triangleq \frac{\dot{Q}_{lift}}{W_{in}} \] (6)

The \( COP_R \) can be compared to an ideal reversible cooler operating under the same temperature conditions with \( COP_{R,carnot} \) as seen in Equation 7.

\[ COP_{R,carnot} = \frac{Q_{lift}}{W_{in}} = \frac{Q_{lift}}{Q_{rej} - Q_{lift}} = \frac{1}{\frac{Q_{rej}}{Q_{lift}} - 1} \] (7)

And only because a reversible system is being considered the substitution \( \frac{T_{out}}{T_{lift}} = \frac{Q_{rej}}{Q_{lift}} \) can be made leading to Equation 8.

\[ COP_{R,carnot} = \frac{1}{\frac{T_{rej}}{T_{lift}} - 1} \] (8)

where \( T_{rej} \) comes from the water flowing out the Stirling cooler and \( T_{lift} \) is the temperature at the heat accepter in absolute units.

Over varying input power values the heat lifted changes, changing the COP. This trend can be seen in Figure 3 provided by the manufacturer of the model 100B cooler.
Figure 3: Recreated $Q_{lift}$ vs $W_{in}$ graph from Global cooling. Note this has “reject: 30°C” written in the corner and the experiment only have values from 24-27°C.

2 Methods

The experiment consisted of two operations. First a dynamic test with the cooler turned on to approximately 12W under no load. Second an approximately 1W load was placed on the cooler by means of a resistive heater. The system was let run until an equilibrium of about .25°C/min was established. For both operations heat was expelled to water flowing around the rejector at a flow rate, $\dot{V}$, of roughly 100ml/2.5min. This was measured by averaging how long it took to fill a 100ml graduated cylinder 3 times. Figure 4 shows a schematic of all components needed for both static and dynamic tests. Both tests have to follow Equation 9.

$$\dot{W}_{in} = \dot{Q}_{rej} - \dot{Q}_{lift} + \dot{Q}_{lost}$$  \hspace{1cm} (9)

It was assumed that $\dot{Q}_{lost}$ to the environment was negligible given the insulated heat rejector. This assumption was tested later.
Figure 4: Schematic of Stirling cooler testing system. [very nice drawing copied from lab procedure]

2.1 Dynamic test

Starting with the cooler at room temperature (21.3°C) and electronic data collection running the cooler was turned on with the variac. It was set to 57.1V. With no load the head temperature started to decrease quickly. For head temperatures of 5, 0, -10, and -25°C\(^1\) the input power (volts and amps) from the digital power meter was noted. Temperature was allowed to go to -30°C to have slightly more data past the last point to graph later.

For this test $Q_{lift}$ was calculated 1st by considering a control volume around the copper head. The First Law in this case is

$$Q_{lift} = m_{Cu} C_p (C_u - C_r) \Delta T$$

To calculate it this was would require waiting on equilibrium to measure each $\Delta T$. Because of this the First Law as considered as a rate equation instead

\(^1\)Yes I thought I knew better than the lab procedure and used -25.
\[
\dot{Q}_{lift} = m_{Cu} C_{p-Cu} \frac{dT}{dt}
\]  

where \( m_{Cu} \) is the mass of the copper heat acceptor (270g) and \( C_{p-Cu} \) is the specific heat of copper, noting it varies with respect to temperature according to Equation 11 (\( T \) is in °C and the answer in \( \frac{J}{kg°C} \)).

\[
C_{p-Cu} = -8.5 \times 10^{-4}T + 0.25T + 379
\]  

The rate of temperature change, \( \frac{dT}{dt} \), had to be determined for each temperature the input power was measured at after an equation for the temperature vs. time was calculated with the experimental data.

### 2.2 Static test

The cooler voltage was set at 47V. The heater was set at 94.1V. These numbers were somewhat random due to running the dynamic several times previously. This combination settled to within .25°C/min to a temperature of -22°C. At that point \( T_{in}, T_{out} \), power to the cooler, and power to the heater were recorded.

For this test \( Q_{rej} \) and \( Q_{lift} \) were determined based on control volumes around the heat rejecter and copper head with heating element respectively. \( Q_{rej} \) was calculated as seen in Equation 12 and \( Q_{lift} \) as seen in Equation 13.

\[
\dot{Q}_{rej} = \dot{m} C_{water} (T_{rej} - T_{in})
\]

where \( \dot{m} = \rho \dot{V} \) (\( \rho = 998 \frac{kg}{m^3} \) @ 21°C) and \( C_{water} = 4.18 \frac{kJ}{kgK} \).

\[
\dot{Q}_{lift} = I_h V_h
\]

where \( I_h \) and \( V_h \) are the heater current and voltage respectively.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (( \dot{m} ) calculation)</td>
<td>±1s</td>
</tr>
<tr>
<td>Volume (( \dot{m} ) calculation)</td>
<td>±10⁻⁶ m³</td>
</tr>
<tr>
<td>Stirling voltage &amp; current</td>
<td>±1V &amp; ±10mA</td>
</tr>
<tr>
<td>Heater voltage &amp; current</td>
<td>±0.1V &amp; ±1mA</td>
</tr>
<tr>
<td>( T_{rej} ) &amp; ( T_{in} )</td>
<td>±1°C</td>
</tr>
</tbody>
</table>
3 Results

Average flow rate, \( \dot{V} \), was 6.30 \( \times 10^{-7} \text{m}^3/\text{s} \) based on Table 2. That makes the mass flow rate, \( \dot{m} \), 6.29 \( \times 10^{-4} \text{kg}/\text{s} \).

<table>
<thead>
<tr>
<th>Run</th>
<th>Time (min:sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2:41</td>
</tr>
<tr>
<td>2</td>
<td>2:36</td>
</tr>
<tr>
<td>3</td>
<td>2:39</td>
</tr>
</tbody>
</table>

Table 2: Time to fill a 100ml graduated cylinder.

3.1 Dynamic test

Figure 5 shows the head temperature vs time data from the dynamic test.

\[
\hat{T}(t) = 225.2 + 57.64e^{-0.004573 t}
\]

\[ R^2 = .9998 \]

\[
T'(t) = -2636e^{-0.004573 t}
\]

Figure 5: Experimental data shifted 50s with best fit and instantaneous slope at measured temperatures.

Table 3 summarizes collected data and calculated results from the dynamic test.
Table 3: Recorded data and calculated results. $C_{p-Cu}$ based on Equation 11. $T'(t)$ from Figure 5. $\dot{Q}_{lift}$ based on equation 10. $COP_R$ based on Equation 6.

<table>
<thead>
<tr>
<th>Temp. ($^\circ$C)</th>
<th>$I_{cooler}$ (A)</th>
<th>$V_{cooler}$ (V)</th>
<th>$W_{cooler}$ (W)</th>
<th>$C_{p-Cu}$</th>
<th>$T'(t)$</th>
<th>$\dot{Q}_{lift}$</th>
<th>$\dot{Q}_{lift}(\ast)$</th>
<th>$COP_R$</th>
<th>$COP_R(\ast)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>.231</td>
<td>57.1</td>
<td>13.2</td>
<td>380</td>
<td>-0.242</td>
<td>24.8</td>
<td>44.9</td>
<td>1.88</td>
<td>3.40</td>
</tr>
<tr>
<td>0</td>
<td>.226</td>
<td>57.1</td>
<td>12.9</td>
<td>379</td>
<td>-0.219</td>
<td>22.4</td>
<td>39.1</td>
<td>1.74</td>
<td>3.03</td>
</tr>
<tr>
<td>-10</td>
<td>.217</td>
<td>57.1</td>
<td>12.4</td>
<td>377</td>
<td>-0.173</td>
<td>17.6</td>
<td>31.5</td>
<td>1.42</td>
<td>2.54</td>
</tr>
<tr>
<td>-25</td>
<td>.210</td>
<td>57.1</td>
<td>12.0</td>
<td>373</td>
<td>-0.105</td>
<td>10.6</td>
<td>21.9</td>
<td>0.880</td>
<td>1.82</td>
</tr>
</tbody>
</table>

* Global cooling values based on Figure 3 at the same $W_{cooler}$ as used in the experiment.

3.2 Static test

Readings were taken once the head temperature stabilized to 0.25°C/min. This happen at -22.0°C. At that time $T_{in}$ was 21.1°C and $T_{rej}$ was 26°C. The heater and Stirling cooler were using power as seen in Table 4. Using these temperatures and the mass flow rate, $\dot{Q}_{rej}$ from Equation 12 was 12.9W.

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>Current (A)</th>
<th>Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stirling cooler</td>
<td>47</td>
<td>.170</td>
</tr>
<tr>
<td>Heater</td>
<td>9.41</td>
<td>.96</td>
</tr>
</tbody>
</table>

Then we take Equation 9 with the $\dot{Q}_{lost}$ is negligible (0) assumption, $\dot{Q}_{rej}$ from above, and $\dot{Q}_{lift}$ from Table 4 and get

$$\dot{W}_in = \dot{Q}_{rej} - \dot{Q}_{lift} = 12.9 - 9.02 = 3.88$$ (14)

If $\dot{Q}_{lost}$ really was 0, this would be the same as $P_{in}$ from Table 4. As we can see it’s not. Using the following we can find the heat lost.

$$P_{in} = \dot{Q}_{rej} + \dot{Q}_{lost} - \dot{Q}_{lift} \implies \dot{Q}_{lost} = P_{in} + \dot{Q}_{lift} - \dot{Q}_{rej} = 7.99 + 9.02 - 12.9 = 4.11W$$ (15)

For this system as observed in the experiment $COP_R$ was

$$COP_R = \frac{9.02}{3.88} = 2.33$$

A reversible Carnot cooler operated under these same temperates would have a COP of
\[ COP_{R,carnot} = \frac{1}{\frac{26+273.15}{-22+273.15} - 1} = 5.23 \]

At this point it’s worth noting there was air observed entering the water lines at the location noted in Figure 4.

4 Conclusions

The relationship between input power, heat lifted, and COP of a Stirling cooler was demonstrated in 2 tests. In the 1st COP was shown to decrease with lower temperatures. Experimental results were an average of 46% lower than Global Cooling’s COP values at the same input power.

In the 2nd test a COP value of 2.33 was calculated (45% of Carnot under the same temperatures). That’s not realistic given a Stirling cooling should only at best operate around 30% Carnot. Given the heat loss of 4.11W, or 51% of the input power to the cooler or 46% of the load this is not negligible. Not accounting for this in the COP clearly lead to unrealistic COP values.

5 Recommendations

The original “Representative data for heat lifted versus power input (from Global Cooling)” (Figure 6) from the lab procedure looks like a scan of a copy of a copy. Also the range of interest on the graph (0-20W input power) is tiny compared to the whole graph. Trying to read \( Q_{lift} \) values off that graph should realistically be at least ±4W. That graph has always been horrible so I made a better one. It will be emailed.

6 Appendix

Data spread sheet and copy of Global cooling graph. An electronic copy will be provided.