1. Find derivative of each function:

   (a) (4 points) \( f(x) = \cot x - \frac{\sin x}{x-1} \).

   \textbf{SOLUTION:} \( f'(x) = -\csc^2 x - \frac{(\cos x)(x-1) - (\sin x)(1)}{(x-1)^2} \).

   (b) (4 points) \( f(x) = \tan(x^2 + 2x - 1) + e^{x^2+3x} \).

   \textbf{SOLUTION:} \( f'(x) = [\sec^2(x^2 + 2x - 1)][2x + 2] + e^{x^2+3x}(2x + 3) \).

   Hence \( f'(x) = (2x + 2)\sec^2(x^2 + 2x - 1) + (2x + 3)e^{x^2+3x} \).

   (c) (4 points) \( f(x) = \csc x + \sec x - e^x \sin x \).

   \textbf{SOLUTION:} \( f'(x) = -\csc x \cot x + \sec x \tan x - [e^x \sin x + e^x \cos x] \).

   (d) (4 points) \( f(x) = \cot^{-1} x + 3\sec^{-1}(2x) \).

   \textbf{SOLUTION:} \( f'(x) = -\frac{1}{1+x^2} + (3)\frac{1}{2x\sqrt{4x^2-1}}(2) = -\frac{1}{1+x^2} + \frac{3}{x\sqrt{4x^2-1}} \).

   (e) (4 points) \( f(x) = \sin(\cos(\tan(x))) \).

   \textbf{SOLUTION:} \( f'(x) = \cos(\cos(\tan(x))).(-\sin(\tan(x))).(\sec^2 x) \).

   (f) (4 points) \( f(x) = \ln(\sin(x^2 + 2x - 1)) \).

   \textbf{SOLUTION:} \( f'(x) = \frac{1}{\sin(x^2 + 2x - 1)}.\cos(x^2 + 2x - 1)(2x + 2) \).

2. Find \( \frac{dy}{dx} \) from each equation below.

   (a) (4 points) \( xy = \sin(xy) + xy^2 \).

   \textbf{SOLUTION:} \( y + x \frac{dy}{dx} = \cos(xy).[y + x \frac{dy}{dx}] + (y^2 + 2xy \frac{dy}{dx}) \Rightarrow y + x \frac{dy}{dx} = y\cos(xy) + x\cos(xy) \frac{dy}{dx} + y^2 + 2xy \frac{dy}{dx} \).

   Hence \( [x - x\cos(xy) - 2xy] \frac{dy}{dx} = y^2 - y + y\cos(xy) \).

   Therefore,

   \[
   \frac{dy}{dx} = \frac{y^2 - y + y\cos(xy)}{x - x\cos(xy) - 2xy}.
   \]

   (b) (4 points) \( xy^3 = 4x + y^2 - xy \).

   \textbf{SOLUTION:} \( y^3 + 3xy^2 \frac{dy}{dx} = 4 + 2y \frac{dy}{dx} - (y + x \frac{dy}{dx}) \Rightarrow (3xy^2 - 2y + x) \frac{dy}{dx} = 4 - y - y^3 \).

   Hence

   \[
   \frac{dy}{dx} = \frac{4 - y - y^3}{3xy^2 - 2y + x}.
   \]
(c) (4 points) \( \cos(siny) = x\cot y \).

**SOLUTION:** 

\[ -\sin(siny)\cos y \cdot \frac{dy}{dx} = \cot y + x(-\csc^2 y) \cdot \frac{dy}{dx} \Rightarrow \]

\[ \frac{dy}{dx} = \frac{\cot y}{xcsc^2 y - \sin(siny)\cos y} \]

3. (6 points) The altitude of a triangle is increasing at a rate of 2 ft/min while its area is increasing at a rate of 5 \( ft^2/min \). At what rate does the base of the triangle changing when the altitude is 12 ft and the area is 120 \( ft^2 \)?

**SOLUTION:** Let’s denote by \( h \) the altitude, and \( x \) the base of the triangle.

Then the area \( A \) is

\[ A = \frac{1}{2}hx. \]

We know that \( A, h \) and \( x \) are functions of the time \( t \). Hence

\[ \frac{dA}{dt} = \frac{1}{2}x \frac{dh}{dt} + \frac{1}{2}h \frac{dx}{dt}. \]

Now for \( h = 12 \), then \( A = 120 \). Hence from the above formula we get \( x = \frac{(120)(2)}{12} = 20 \).

Moreover at this situation, \( \frac{dA}{dt} = 5, \frac{dh}{dt} = 2 \). Thus

\[ \frac{dx}{dt} = \frac{(5)(2) - (20)(2)}{12} = \frac{-30}{12} = -2.5. \]

The rate of change of the base at this situation is \(-2.5 \) feet per minute.

4. Find the absolute maximum and absolute minimum each function on a given interval.

(a) (4 points) \( f(x) = x^3 - 6x^2 + 9x + 4 \) on \([0, 4]\).

**SOLUTION:** \( f'(x) = 3x^2 - 12x + 9 \), hence the critical numbers are \( x = 1, x = 3 \). These are in \([0, 4]\). Now we evaluate \( f(x) \) at \( x = 0, 1, 3, 4 \) to get the answers: \( f(0) = 4, f(1) = 8, f(3) = 4, f(4) = 8 \). Answer: Absolute maximum is 8, and absolute minimum is 4 (on the interval \([0, 4]\)).

(b) (4 points) \( f(x) = \frac{3x}{x^2 + 1} \) on \([0, 3]\).

**SOLUTION:** Notice that \( x^2 + 1 \) is never zero, so \( f(x) \) is continuous everywhere on \((-\infty, \infty)\). \( f'(x) = \frac{3(x^2+1)-(3x)(2x)}{(x^2+1)^2} \). Set \( f'(x) = 0 \Rightarrow (3)(x^2 + 1) - (3x)(2x) = 0 \Rightarrow -3x^2 + 3 = 0 \Rightarrow x = 1 \) or \(-1 \). These are critical numbers of \( f(x) \), but \(-1 \) is not in \([0, 3]\), so we need to evaluate \( f(x) \) at 0, 1, and 3 to get the answers: \( f(0) = 0, f(1) = \frac{3}{2}, f(3) = \frac{9}{10} \). Hence on \([0, 3]\) absolute minimum of \( f(x) \) is 0, absolute maximum is 1.5.