MATH 163A, Winter Quarter 2001, MIDTERM 2

Student’s Name (in capital letters):
S.S. Number:

Show all your work to get full credit. No work will amount to no
credit. Circle your final answers.

1. Evaluation each limit:

(a) (2 points) $\lim_{x \to 2} \frac{x^2 - 4x + 5}{2x - 1} = \frac{4 - 8 + 5}{4 - 1} = \frac{1}{3}$

(b) (2 points) $\lim_{x \to 5} \sqrt{x^2 - 4x + 3} = \sqrt{25 - 20 + 3} = \sqrt{8} = 2\sqrt{2}$

(c) (3 points) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2} = \frac{(2-2)(2+2)}{2-2} = \frac{0}{0}$

(d) (3 points) $\lim_{x \to 1} \frac{x-1}{1-\sqrt{x}} = \lim_{x \to 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(\sqrt{x}-1)} = -2$

(e) (3 points) $\lim_{x \to 3} \frac{x^2 + x - 12}{x - 3} = \lim_{x \to 3} \frac{(x-3)(x+4)}{x-3} = 7$
2. Let \( s(t) = t^2 + 3t + 5 \) be a position function of a moving object.

(a) (3 points) Find the average rate of change of \( s(t) \) (= average velocity) when \( t \) changes from 1 to 4.

\[
\frac{s(4) - s(1)}{4 - 1} = \frac{(4^2 + 3(4) + 5) - (1 + 3 + 5)}{3} = 8
\]

(b) (4 points) Find the instantaneous velocity \( v(t) \) at \( t = 1 \) by evaluating the limit:

\[
v(1) = s'(1) = \lim_{h \to 0} \frac{s(1+h) - s(1)}{h} = \lim_{h \to 0} \frac{(1+h)^2 + 3(1+h) + 5 - 9}{h} = \lim_{h \to 0} \frac{1 + 2h + h^2 + 3 + 3h + 5 - 9}{h} = \lim_{h \to 0} \frac{h(3h + 5)}{h} = \lim_{h \to 0} (3h + 5) = 5
\]

4. Let \( f(x) = x^3 - 3x^2 + 4x - 5 \).

(a) (2 points) Using the the formula of derivative to find \( f'(x) \).

\[
f'(x) = 3x^2 - 6x + 4
\]

(b) (4 points) Find an equation of the tangent line to the graph of \( f(x) \) at \( x = 2 \).

Point \((2, f(2)) = (2, -1)\), slope \( = f'(2) = 4 \)

Equation of the tangent line:

\[
y + 1 = 4(x - 2) \Rightarrow y = 4x - 9
\]

(c) (4 points) Find all values of \( x \) at which the tangent line has the slope 4.

Set \( f'(x) = 4 \) solve for \( x \).

\[
3x^2 - 6x + 4 = 4 \Rightarrow 3x^2 - 6x = 0 \Rightarrow 3x(x - 2) \Rightarrow 3x = 0 \Rightarrow x = 0 \text{ or } x - 2 = 0 \Rightarrow x = 2
\]
5. Find the derivative of each function.

(a) (3 points) \( f(x) = x^5 - 3x + 1 \)
\[
\frac{df}{dx} = 5x^4 - 3
\]

(b) (3 points) \( g(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}} \)
\[
g'(x) = -\frac{1}{2} x^{-\frac{3}{2}}
\]

(c) (4 points) \( h(x) = 2x^2 - 5x^\frac{2}{3} - x^{-1} + 7 \)
\[
h'(x) = 4x - 5 \left( \frac{2}{3} \right)x^{\frac{2}{3} - 1} - (-1)x^{-2}
\]
\[
h'(x) = 4x - \frac{10}{3}x^{-\frac{1}{3}} + x^{-2}
\]