MATH 163A, Fall Quarter 2001, MIDTERM 1

Student's Name (in capital letters): Solutions

Show all your work to get full credit. No work will amount to no credit. Circle your final answers.

1. Find an equation of each line L described in (a), (b), (c).

   (a) (2 points) L passes through the points (1,3) and (5,11).
   
   \[
   \text{Slope} = \frac{11 - 3}{5 - 1} = \frac{8}{4} = 2, \quad \text{Eqn: } y - 3 = 2(x - 1) \\
   \Rightarrow y = 2x + 1
   \]

   (b) (2 points) L has slope 3 and y-intercept 4.
   
   By slope-intercept form: \( y = mx + b \)  
   
   \[
   y = \frac{m}{x} + b \quad \rightarrow \quad \text{answer: } y = 3x + 4
   \]

   (c) (3 points) L passes through the point (3,1) and perpendicular to the line with equation \( x - 2y = 6 \).
   
   \[
   x - 2y = 6 \Rightarrow y = \frac{1}{2}x - 3. \quad \text{Hence, slope of } L \text{ is } -2
   \]
   
   Eqn \( x = i \). \quad y = -2(x - 3) \Rightarrow \quad 2x + y = 7

2. (4 points) Mark to each statement below if it is true (T) or false (F):

   \( \checkmark \) (a) To find the x-intercepts of the graph of a function \( f(x) \), we solve for \( x \) from the equation \( f(x) = 0 \), and to find y-intercept we evaluate \( f(0) \).

   \( \times \) (b) The graph of \( f(x) = 5 \) is a vertical line.

   \( \checkmark \) (c) The slope of a linear function is well-defined.

   \( \checkmark \) (d) The graph of \( f(x) = ax \) is a straight line that passes through the origin.
3. (4 points) Find the linear cost function $C(x)$, if the fixed cost is $100$, and if 50 items are made, it will cost $1600$.

$$C(x) = mx + \text{fixed cost}.$$  
$$1600 = C(50) = m \cdot 50 + 100 \Rightarrow m50 = 1600 - 100 = 1500$$  
$$\Rightarrow m = 30,$$  
Therefore  
$$C(x) = 30x + 100$$

4. Find the domain of each function:

(a) (2 points) $f(x) = x^3 - 2x^2 + 4x - 11$.  
\text{Domain} = (-\infty, \infty)

(b) (3 points) $f(x) = \frac{x+1}{x^2 - 4}$  
\text{Domain} = all real numbers except $\pm 2$  
or in symbol: $(\infty, -2) \cup (-2, 2) \cup (2, \infty)$

5. (a) (5 points) Sketch the graph of the parabola $f(x) = x^2 - 2x - 3$, by finding first the $y$-intercept, $x$-intercepts, the vertex, and the symmetric axis. (The use of a graphic calculator is not allowed for this question.)

$y$-intercept: $-3$  
$x$-intercepts: $-1$, $3$  

vertex $(1, -4)$  

symmetric axis $x = 1$
(b) (2 points) Use the graph in part (a) to give the domain of the function \[ g(x) = \frac{1}{x^2 - 2x - 3}. \]

\[ \text{Domain: all real numbers except } -1 \text{ and } 3 \]

\[ \text{or in symbol: } (-\infty, -1) \cup (-1, 3) \cup (3, \infty) \]

(c) (2.5 points) Use the graph in part (a) to give the domain of the function \[ h(x) = \sqrt{x^2 - 2x - 3}. \]

\[ \text{Domain: } -\infty < x \leq -1 \text{ or } 3 \leq x < \infty \]

\[ \text{or in symbol: } (-\infty, -1] \cup [3, \infty) \]

(d) (2.5 points) Use the graph in part (a) to give the domain of the function \[ k(x) = \frac{1}{\sqrt{x^2 - 2x - 3}}. \]

\[ \text{Domain: } -\infty < x < -1 \text{ or } 3 < x < \infty \]

\[ \text{in symbol: } (-\infty, -1) \cup (3, \infty) \]

6. Evaluate each limit below:

(a) (2 points) \[ \lim_{x \to 1} (x^4 - x^3 - 3x^2 + 4) = 1 - 1 - 3 + 4 = 1 \]

(b) (3 points) \[ \lim_{x \to 1} \frac{x^2 - 4x + 3}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x-3)}{(x-1)} = \lim_{x \to 1} (x-3) = -2 \]

(c) (3 points) \[ \lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)} = \lim_{x \to 2} (x+2) = 4 \]