**SH-Wave Intromission Concept**

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Abstract—The existence of an SH-wave incidence angle for which the reflected amplitude is zero (SH-wave intromission angle) is established for the case of plane-wave scattering by a planar interface joining two homogeneous, isotropic, and linearly elastic half-spaces. Such an incidence angle is numerically shown to exist for two combinations of bimaterial interface properties. The SH-wave intromission angle is roughly parallel to the electromagnetic Brewster angle and the acoustic P-wave intromission angle, and the concept should find new applications for non-intrusive characterization of interfaces. © 2004 MAIK “Nauka/Interperiodica”.

**Introduction.** Three types of elastic body waves composed of longitudinal primary (P) and transverse secondary (SV and SH) waves can exist in homogeneous, isotropic, and linearly elastic media. P-waves are polarized in the propagation direction and within the incidence plane, SV-waves are polarized orthogonal to the propagation direction and within the incidence plane, and SH-waves are polarized orthogonal to the propagation direction and within a plane that is orthogonal to the incidence plane. When elastic waves are incident on an interface joining media with different impedances, scattering (reflection and refraction/transmission) occurs. Amplitude and energy partitioning between scattered components is dependent on the incident wave type, the incidence angle, and the media impedances.

Amplitudes and energies of planar interface-scattered components from incident P- and SV-waves depend on media compressional velocities, shear velocities, and densities, whereas partitioning from incident SH-waves depends only on media shear velocities and densities [1]. For incident P- or SV-waves, mode-conversion can occur at the point of oblique incidence on a welded planar interface, with four possible wave types (reflected P, reflected SV, refracted P, and refracted SV) being generated. From an interface parallel to incident SH-wave polarization however, only scattered SH-waves are generated, regardless of the incidence angle. We have derived equations from the plane SH-wave reflection coefficient, which predict an angle of SH-wave incidence at which the reflected amplitude is zero (SH-wave intromission angle).

**Theory.** For plane SH-waves in homogenous, isotropic, and linearly elastic media, the reflection coefficient [1] for any angle of incidence can be written as

\[
SH_1,SH_1 = (Zs_1 \cos \theta_{SH,SH_1} - Zs_2 \cos \theta_{SH,SH_1})/(Zs_1 \cos \theta_{SH,SH_1} + Zs_2 \cos \theta_{SH,SH_1}),
\]

where \(SH_1,SH_1\) is the reflection displacement amplitude coefficient, \(\theta_{SH,SH_1}\) is the incidence and reflection angle, and \(Zs_1\) and \(Zs_2\) are the incident (medium 1) and refracted (medium 2) media shear impedances. Shear impedance is the product of medium shear velocity (\(V_s_1\) or \(V_s_2\)) and density (\(\rho_1\) or \(\rho_2\)).

The reflection coefficient is zero (only the refracted wave will remain) when the incidence angle is equal to the SH-wave intromission angle (\(\theta_i\)), and this occurs when

\[
Zs_1 \cos \theta_{i} = Zs_2 \cos \theta_{SH,SH_1}.
\]

To solve for \(\theta_i\),

\[
Zs_1/Zs_2 = \cos \theta_{SH,SH_1}/\cos \theta_{i}.
\]

Using the relation \(\sin^2 x + \cos^2 x = 1\) and Snell’s law [1] to eliminate \(\theta_{SH,SH_1}\),

\[
Zs_1/Zs_2 = (1 - (V_s_2/V_s_1)^2 \sin^2 \theta_i)^{1/2} / (1 - \sin^2 \theta_i)^{1/2}
\]

yields a solution for \(\theta_i\) in terms of media shear velocities and shear impedances,

\[
\theta_i = \sin^{-1} \left(1 - \left(Zs_1/Zs_2\right)^2/(V_s_2/V_s_1)^2 - (Zs_1/Zs_2)^2\right)^{1/2}
\]

or a solution for \(\theta_i\) in terms of media shear velocities

\[
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Numerical examples. To illustrate \(SH\)-wave intromission angle existence, solutions to equations (obtained using the PSHSV computer program [2]) describing plane \(SH\)-wave scattering from a planar interface joining two homogeneous, isotropic, and linearly elastic media were obtained using the PSHSV computer program [2]. \(\theta_0\) is the ratio of a given scattered wave maximum amplitude particle displacement to that of the incident wave; \(E/E_0\) is the ratio of a given scattered wave energy to that of the incident wave. \(V_s^1\) and \(V_s^2\) are the incident (medium 1) and refracted (medium 2) media shear velocities; \(\rho_1\) and \(\rho_2\) are the incident and refracted media densities; and \(Z_s^1\) and \(Z_s^2\) are the incident and refracted media shear impedances. A critical angle \(\theta_c\) occurs for these media conditions. See text for discussion of this figure.

and densities,

\[
\theta_1 = \sin^{-1}\left(\frac{(V_s^2/V_s^1)^2}{(\rho_1/\rho_2)^2/(V_s^2/V_s^1)^4 - (\rho_1/\rho_2)^2}\right)^{1/2},
\]

where \(\theta_1\) is a particular real angle of incidence between zero and 90°. To obtain a real solution to Eq. (5) or (6), the numerator and denominator in the respective equation must have the same sign, and the absolute value of the numerator must be equivalent to or less than the absolute value of the denominator in the respective equation. In terms of media shear velocities and shear impedances, Eq. (5) shows that these conditions will be met if \((V_s^2/V_s^1) \geq 1 \geq (Z_s^1/Z_s^2)\), or if \((Z_s^1/Z_s^2) \geq 1 \geq (V_s^2/V_s^1)\). When both \(V_s^1 = V_s^2\) and \(\rho_1 = \rho_2\), the \(SH\)-wave reflection coefficient is zero for all incidence angles.

Fig. 1. \(SH\)-wave intromission angle \(\theta_1\) occurrence for the case of an \(SH\)-wave incident on an interface with increasing shear velocity and shear impedance; \((V_s^2/V_s^1) > 1 > (Z_s^1/Z_s^2)\). Plots of amplitude coefficients, square root energy coefficients, energy coefficients, and phase changes as a function of incident angle for a plane \(SH\)-wave incident on a planar interface joining homogeneous, isotropic, and linearly elastic media were obtained using the PSHSV computer program [2]. \(\theta_0\) is the ratio of a given scattered wave maximum amplitude particle displacement to that of the incident wave; \(E/E_0\) is the ratio of a given scattered wave energy to that of the incident wave. \(V_s^1\) and \(V_s^2\) are the incident (medium 1) and refracted (medium 2) media shear velocities; \(\rho_1\) and \(\rho_2\) are the incident and refracted media densities; and \(Z_s^1\) and \(Z_s^2\) are the incident and refracted media shear impedances. A critical angle \(\theta_c\) occurs for these media conditions. See text for discussion of this figure.
where \( u \) is the slowness, \( pp \) is the horizontal slowness, \( \eta \) is the vertical slowness, and \( \Theta \) is the reflected or refracted ray angle measured from the normal. In terms of \( Vs_1 \) and \( Vs_2 \), the reflection and refraction angle cosines are

\[
\cos \theta_{sh1,sh1} = Vs_1((1/Vs_1^2) - pp^2)^{1/2}, \tag{10}
\]

\[
\cos \theta_{sh1,sh2} = Vs_2((1/Vs_2^2) - pp^2)^{1/2}. \tag{11}
\]

The square root energy and energy coefficients plotted in Figs. 1 and 2 were calculated as

\[
ESH_{sh1}SH_1 = SH_{1,sh1}, \tag{12}
\]

\[
ESH_{sh1}SH_2 = (SH_{1,sh2})(Zs_2 \cos \theta_{sh1,sh2},)(Zs_1 \cos \theta_{sh1,sh1},)^{1/2}, \tag{13}
\]

\[
ESH_{1,sh1} = (ENSH_{1,sh1})^2, \tag{14}
\]

\[
ESH_{1,sh2} = (ENSH_{1,sh2})^2, \tag{15}
\]

where \( ENSH_{1,sh1} \) and \( ENSH_{1,sh2} \) are the normalized reflected and refracted square root energy coefficients and \(ESH_{1,sh1}\) and \(ESH_{1,sh2}\) are the reflected and refracted energy coefficients. Whereas reflected and refracted amplitudes do not necessarily sum to unity for a given angle of incidence, the energy coefficients (representing what fraction of an incident wave energy flux is reflected or refracted normal to the interface) must sum to unity for all angles of incidence.

The phase angles plotted in Figs. 1 and 2 were calculated using the following equation,

\[
\varphi = \tan^{-1}(b/a), \tag{16}
\]

where \( \varphi \) is the phase angle (between +180° and –180°, with 0° specified between quadrants 1 and 4) and \( a \) and \( b \) are the real and imaginary parts of a given displacement amplitude coefficient.

The Fig. 1 solutions were obtained for a case where a bimaterial interface physical properties were such that \( (Vs_2/Vs_1) > 1 \sim (Zs_1/Zs_2) \). Both an intromission angle \( \theta_i \) and a critical angle \( \theta_c \) are observed in the Fig. 1 subplots, and these results are consistent with Eq. (5) predictions. The critical angle occurs, as the incident medium velocity is less than that of the refracted medium; it is defined as

\[
\theta_c = \sin^{-1}(Vs_1/Vs_2). \tag{17}
\]

A 180° phase shift for the reflected \( SH \)-pulse occurs at \( \theta_i \) (approximately 42°), and the phase is then constant until \( \theta_c \) (approximately 64°) is reached, beyond which the reflected pulse phase is variable. From the real and imaginary parts of the reflection amplitude coefficient in Fig. 1, it is seen that \( \theta_i \) occurs when the real portion of the reflected signal goes from negative to positive prior to \( \theta_c \). As the real portion of the reflected signal changes sign prior to all of the energy being reflected from the interface at \( \theta_c \), the reflected energy goes to zero at \( \theta_c \). Not all cases involving an incident \( SH \)-wave going from relatively low to relatively high shear velocity would result in \( \theta_i \) occurrence. For instance, for a
case where the physical properties were such that
\((V_{S2}/V_{S1}) = (Z_{S1}/Z_{S2}) > 1\), a \(\theta_C\) would occur, but Eq. (5) predicts that \(\theta_I\) would not occur.

The Fig. 2 solutions were obtained for a case where bimaterial interface physical properties were such that
\((Z_{S1}/Z_{S2}) > 1 > (V_{S2}/V_{S1})\). A \(\theta_I\) is observed, but no \(\theta_C\) is observed, in the subplots of Fig. 2, and these results are consistent with Eq. (5) predictions. The reflected \(SH\)-pulse phase is constant until the \(\theta_I\) (approximately 46°), at which the reflection amplitude goes to zero and beyond which a 180° phase shift of the reflected pulse occurs. From the real and imaginary parts of the reflection amplitude coefficient in Fig. 2, it is seen that \(\theta_I\) occurs when the real portion of the reflected signal goes from positive to negative. Not all cases involving an incident \(SH\)-wave going from relatively high to relatively low shear velocity would result in \(\theta_I\) occurrence. For instance, for a case where the physical properties were such that \(1 > (V_{S2}/V_{S1}) = (Z_{S1}/Z_{S2})\), Eq. (5) predicts that a \(\theta_I\) would not occur.

**Conclusions.** An \(SH\)-wave intromission angle occurs for a given incidence angle when the interface is characterized by increasing velocity and impedance and \((V_{S2}/V_{S1}) \geq 1 \geq (Z_{S1}/Z_{S2})\) or decreasing velocity and impedance and \((Z_{S1}/Z_{S2}) \geq 1 \geq (V_{S2}/V_{S1})\). Recognition and utilization of the \(SH\)-wave intromission concept should lead to new non-intrusive interface characterization applications (as in the widely utilized electromagnetic Brewster angle in optics [3–5] and the \(P\)-wave intromission angle in acoustics [6, 7]) and should stimulate further investigations of this effect.

**REFERENCES**