Computer programs for application of equations describing elastic and electromagnetic wave scattering from planar interfaces

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Abstract

MATLAB programs are presented which solve equations describing the scattering of plane elastic and electromagnetic waves from a planar interface separating homogenous, isotropic, and semi-infinite geologic media. The PSHSV program calculates and plots amplitude (reflection and refraction/transmission) coefficients, square root energy ratios, energy coefficients, and phase changes for elastic waves of P-, SH- or SV-type incident on an interface between elastic media. The EHEV program calculates and plots amplitude coefficients, square root energy ratios, energy coefficients, and phase changes for electromagnetic waves of EH- or EV-type incident on an interface between dielectric media. The applicability of the programs is demonstrated through the presentation of solutions (plotted as a function of incidence angle) obtained for geologic environments commonly encountered in seismic and ground penetrating radar applications.

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1. Introduction

Equations describing the amplitude and energy partitioning of elastic (seismic) and electromagnetic waves at planar geologic interfaces are well established, and have a wide range of applications in the earth and physical sciences (Castagna, 1993; Hilterman, 2001; Nobes and Annan, 2000; Radzevicius, 2001). The purpose of this paper is to present and make available the PSHSV and EHEV computer programs, which solve these equations and make rapid analysis of results possible. These programs will be useful for engineers and scientists that are conducting research, performing exploration work, or teaching. The nomenclatures used for equations in program source codes are explained in this paper, the equations are well documented in program source codes (allowing the programs to be easily modified), and references from which equations in the source codes are from, modified from, or derived from are also provided. Source codes for the programs are available by anonymous ftp from the ftp.iamg.org site, or via the internet at www.iamg.org.

1.1. Overview of elastic and electromagnetic wave planar interface scattering

When elastic or electromagnetic waves are incident on a planar interface separating media with an impedance contrast (elastic or electromagnetic impedance depending on the incident wave type), scattered (reflected and/or refracted/transmitted) waves are generated. The
amplitude and energy partitioning between scattered waves is dependent on the incident wave type, the angle of incidence, and the contrast in impedance across the interface. The programs presented in this paper consider amplitude and energy partitioning from elastic waves of P-, SH- or SV-type incident on an interface between elastic media, and from electromagnetic waves of EH- or EV-type incident on an interface between dielectric media.

From an interface between homogenous and isotropic solids that is parallel to the polarization of an incident elastic SH-wave, only scattered SH-waves are generated (Fig. 1B), regardless of the incidence angle. The amplitudes and energies of scattered SH-waves are dependent upon the angle of incidence, and the incident and refracted media (media 1 and 2, respectively) shear-wave velocities ($V_s^1$ and $V_s^2$) and densities ($\rho_1$ and $\rho_2$). SH-wave particle motion is perpendicular to the direction of propagation, and is contained within a plane that is perpendicular to the plane of incidence. For incident elastic P- or SV-waves, mode conversion can occur at the point of oblique incidence on a welded interface, with four possible wave types being generated: (1) a reflected P-wave, (2) a reflected SV-wave, (3) a refracted P-wave, and (4) a refracted SV-wave (Figs. 1A and C). P-wave particle motion is in the direction of propagation and contained within the plane of incidence, whereas SV-wave particle motion is also within the plane of incidence, but perpendicular to the propagation direction. The amplitudes and energies of scattered P- and SV-waves are dependent upon the angle of incidence, and the incident and refracted media compressional-wave velocities ($V_p^1$ and $V_p^2$), shear-wave velocities, and densities.

When electromagnetic waves are incident on an interface between two dielectric media, it is necessary to consider two cases (Figs. 2A and B): (1) the electric field components are polarized transverse to the plane of incidence (EH or E horizontal polarization), and (2) the electric field components are polarized parallel to the plane of incidence (EV or E vertical polarization). Wave propagation in geologic materials (non-metallic) within the ground penetrating radar frequency range is primarily a function of dielectric permittivity and magnetic permeability variations. For non-magnetic materials scattering is primarily a function of dielectric permittivity. Amplitude and energy partitioning for each case (EH and EV) is different, and is primarily a function of the angle of incidence, and the incident and refracted media (media 1 and 2, respectively) permittivities ($\varepsilon_1$ and $\varepsilon_2$) and permeabilities ($\mu_1$ and $\mu_2$). The electromagnetic EH and EV cases are similar to elastic SH-waves in that polarization is perpendicular to the direction of propagation, and no conversion between modes occurs for any incidence angle.

1.2 Previous elastic wave scattering solution errors

Although the planar interface scattering equations for elastic and electromagnetic waves are fairly similar in certain regards (despite being a function of different media physical properties), equations describing elastic wave scattering are complicated by mode conversions (for P- and SV-waves) that must be considered. This complexity has led to numerous publications containing erroneous results or misprints, and many of these mistakes have been reported in the literature (Gutenberg, 1944; Singh et al., 1970; Hales and Roberts, 1974; Young and Braile, 1976; Denham and Palmeira, 1984). A number of additional misprints and errors in the literature related to elastic and electromagnetic planar interface scattering equations that have not previously

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**Fig. 1.** Nomenclature used in PSHSV program for incident and scattered wave types, and for calculated coefficients. $P_1P_2$ for example, signifies a refracted P-wave in medium 2 resulting from an incident P-wave in medium 1.
been reported in the literature were also recognized during the development of the PSHSV and EHEV computer programs (for a discussion of elastic wave scattering solution errors in the literature see Guy (2002)).

2. Description of programs

The PSHSV and EHEV computer programs presented in this paper were written in MATLAB (version 6, release 12). The PSHSV program was developed using equations primarily from, and derived or modified from, those presented by Aki and Richards (1980), Sheriff and Geldart (1982), and Shearer (1999). The signs of calculated elastic wave amplitude coefficients depend upon the specified directions of positive particle displacement in the equations. The direction of wave propagation was specified as the direction of positive particle displacement for P-waves and the direction towards the interface between incident and refracted media was specified as the direction of positive particle displacement for SV-waves. The EHEV program was developed using equations primarily from, and derived or modified from those presented by Jenkins and White (1957) and Balanis (1989).

The nomenclatures used for user-input parameters, variables within the source codes (incident and scattered wave types and ray angles), and calculated coefficients (i.e. ratios) are consistent with those shown in Figs. 1 and 2. The nomenclatures are such that the first capital letter of terms indicates the incident wave type, and the second capital letter indicates the scattered wave type. The subscripts associated with capital letters representing wave types indicate the medium in which the wave is traveling. A subscript of 1 indicates that the wave is traveling in the incident medium, while a subscript of 2 indicates that the wave is traveling in the refracted medium. For example, $P_1P_2$ indicates a refracted P-wave in medium 2 resulting from an incident P-wave in medium 1 (Fig. 1A).

2.1. Execution and example solutions

Upon execution of the PSHSV program the user is prompted to enter elastic media P- and S-wave velocities and densities ($V_p$, $V_s$, $\rho_1$ and $\rho_2$). Upon execution of the EHEV program the user is prompted to enter media relative dielectric permittivities and relative magnetic permeabilities ($\varepsilon_r$, $\mu_r$, $\mu_1$ and $\mu_2$). After user-specified parameters are input for either of the programs, amplitude (reflection and refraction/transmission) coefficients, square root energy ratios, energy coefficients, and phase changes of scattered waves resulting from the possible incident (elastic or electromagnetic) wave types are calculated and plotted as a function of incidence angle.

Shown in Fig. 3 are the three figures that were generated after the PSHSV program was executed and user-specified parameters were input. For the example solutions shown in Fig. 3, the incident medium (medium 1) parameters are representative of a shale unit, and the refracted medium (medium 2) parameters are representative of a coal unit (user-input parameters are plotted on figures, and are similar to quantities in Dey-Sarkar and Svatek (1993)). Fig. 3A contains solutions from an incident P-wave, Fig. 3B contains solutions from an incident SH-wave, and Fig. 3C contains solutions from a shear wave.
Fig. 3. Plots generated using PSHSV program showing amplitude coefficients, square root energy coefficients, energy coefficients, and phase changes as a function of incidence angle for elastic waves incident on a shale (medium 1) and coal (medium 2) interface: (A) P-wave, (B) SH-wave, and (C) SV-wave. Density (units of g/cm$^3$) and velocity (units of m/s) values used are plotted on figures.
solutions from an incident SV-wave. Shown in Fig. 4 are the two figures that were generated after the EHEV program was executed and user-specified parameters were input. For the example solutions shown in Fig. 4, the incident medium parameters are representative of an unsaturated sandy soil, and the refracted medium parameters are representative of a saturated sandy soil below a water table (user-input parameters are plotted on figures). Fig. 4A contains results from an incident EH-wave, and Fig. 4B contains results from an incident EV-wave. The main objective of this paper is to present computer programs and demonstrate their functionality, and therefore detailed discussions regarding solutions plotted for the example geologic cases are not included, but can be found in Radzevicius (2001) or Guy (2002).

Each incident wave type figure generated (Figs. 3 and 4) contains six subplots that show how calculated amplitude coefficients, square root energy ratios, energy coefficients, and phase angles change as a function of incident angle, for each wave type generated at the interface (note: color plots are actually generated upon execution of the programs, however, grayscale plots are presented in this paper). For elastic wave cases (Fig. 3), the amplitude coefficient curves represent the ratio of the maximum amplitude particle displacement of reflected or refracted waves to that of the incident wave (exact Zoeppritz solutions are plotted). For electromagnetic wave cases (Fig. 4), the amplitude coefficient curves represent the ratio of the amplitude of maximum (EH or EV) electric fields of reflected or refracted waves to that of the incident field (Fresnel solutions are plotted).

The amplitudes of the reflection and refraction/transmission coefficients for both elastic and electromagnetic waves are determined by the boundary conditions and their amplitudes do not necessarily sum to unity. This is not the case for energy coefficients that represent what fraction of an incident wave’s energy flux is reflected or transmitted normal to the interface. Because of energy conservation, the energy coefficients for reflected and transmitted modes must sum to unity regardless of the incidence angle. Beyond a critical angle, energy coefficients for reflected and transmitted waves become unity and zero, respectively. The energy coefficients do not describe the partitioning of energy parallel to the interface, and thus transmission coefficients can remain above zero beyond a critical angle. These non-zero transmission coefficients represent evanescent waves that travel along the interface, decay exponentially away from it, and do not transport energy normal to the interface. While energy is directly proportional to the square of the amplitudes, energy only equals amplitude squared for reflected waves that travel in the same medium (and at the same velocity) as the incident waves. When waves are refracted across the interface, the energy coefficients are still directly proportional to the squared amplitudes, but must also
Fig. 4. Plots generated using EHEV program showing amplitude coefficients, square root energy coefficients, energy coefficients, and phase changes as a function of incidence angle for electromagnetic waves incident on an unsaturated sandy soil (medium 1) and a saturated sandy soil below the water table (medium 2) interface: (A) EH polarization, and (B) EV polarization. Relative dielectric permittivity (dimensionless) and relative magnetic permeability (dimensionless) values used are plotted on figures.
be weighted by direction cosines and impedance ratios (elastic or electromagnetic) of the two media.

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References


