Do as many problems as possible. Passing or failing will be determined by your 8 best solutions (no part marks from any remaining partial solutions). If any notation or terminology is unfamiliar, please ask.

1. Let $X = \bigcup \{X_i : i \in \omega\}$ be a compact Hausdorff space with countable Suslin number. Suppose that each $X_i$ is a metrizable dense subspace of $X$. Prove that $X$ is metrizable.

2. Let $Y$ and $Z$ be normal subspaces of a Hausdorff space $X$. Suppose that $Y \times Z$ is normal. Prove that $Y \cap Z$ is normal.

3. Prove that every countable completely metrizable space has a dense set of isolated points.

4. Let $f$ be a continuous mapping of the space $\mathbb{P}$ of irrational numbers with the usual topology onto a pseudocompact space $Y$. Is $Y$ metrizable? Explain.

5. Is the space $\omega_1$ with the order topology homeomorphic to a subspace of $\beta\mathbb{Q}$? Explain.

6. Suppose that $X$ is regular and that $(\mathcal{U}_n)_{n \in \omega}$ is a sequence of open covers such that each finite open cover of $X$ is refined by some $\mathcal{U}_n$. Prove that $X$ is metrizable.

7. Prove that every paracompact space with a uniform base is metrizable.

8. Suppose that $X = Y \cup Z$ and that $Y$ is a dense set of isolated points in $X$ and $Z$ is closed discrete in $X$. Prove that the following are equivalent.
   (a) The points of $Z$ can be separated by a family of disjoint open sets.
   (b) Every open cover of $X$ has a disjoint refinement.
   (c) $X$ is paracompact.

9. Suppose that $X = \{x_\alpha : \alpha \in \omega_1\} \subseteq \mathbb{R}$ is a 1-1 enumeration of a set of real numbers. Consider the topology on $X$ obtained by refining the usual subspace topology by declaring sets of the form $\{x_\alpha : \alpha < \beta\}$ open for each $\beta < \omega_1$. Prove or disprove each of the following:
   (a) $X$ is separable.
   (b) $X$ is Lindelöf.
   (c) $X$ is Hausdorff.
   (d) $X$ is regular.

10. Give an example of a nonmetrizable Tychonoff space whose topology is generated by a symmetric. Make sure to prove it has the relevant properties.