formulations of a (classical or non-classical) logical system. Or you might be looking for fixed-point combinators (in the sense of Haskell Curry and Raymond Smullyan). You could be doing group theory (or Moufang loop theory). In all of these areas (and many more), the successes of OTTER and its cousins are dwelt upon.

What, however, is an appropriate philosophy for the tools explored here? The reviewer is much impressed by that espoused by the authors. They do not consider Automated Reasoning AR to be a branch of Artificial Intelligence AI. They see it as a task of AI to “emulate person-oriented reasoning” (p. 441). There is no reason, they claim, to see this as among the aims of AR, whose goal is after all to draw logical conclusions as flawlessly and as efficiently as possible.

Put otherwise—it is as silly to take human reasoning as our model of good logical reasoning as it is to suppose that the wings of an airplane should flap like a bird’s. Our interest in airplanes is to use them to get from place to place. Just so, our interest in AR logics is to use them, as assistants, to get from premises to conclusions. As consumers of these logics, the details of what is merely implementation need not interest us.

ROBERT K. MEYER

MI., LLL, Australian National University, Canberra, Australia. bob.meyer@anu.edu.au.


According to the author, “[t]his book has a double purpose. First, to trace the historical development of the concepts of the continuous and the infinitesimal; and second, to describe the ways in which these two concepts are treated in contemporary mathematics” (2005, p. 11). Although the book falls short of realizing either of these lofty goals, it does provide synopses of some of the important episodes in the historical development of the concepts of the continuous and the infinitesimal as well as overviews of some of the important ways in which these two concepts are treated in contemporary mathematics. In so doing, the book weaves together selected fruit from a large body of research and expository work carried out by a wide array of authors during the twentieth century, the selected fruit (together with the presentation thereof) often being shaped and colored by the research interests and philosophical predilections of the book’s distinguished author.

The historico-philosophical part of the book consists of five chapters: Chapter 1 covers the continuous and the discrete in Ancient Greece, the Orient and the European Middle Ages; Chapter 2 treats the founding of the infinitesimal calculus in the 16th and 17th centuries; and Chapter 3 deals with the philosophies of the continuum and/or the infinitesimal of a host of 18th- and early 19th-century mathematicians and philosophers from Euler to Hegel. Chapter 4 begins with Bolzano’s and Cauchy’s clarifications of the concepts of continuous function and limit, moves on to Riemann’s work in differential geometry, and then to the arithmetization of analysis and accompanying banishment of infinitesimals therefrom via the work of Cantor, Dedekind and Weierstrass. Bertrand Russell’s rejection of infinitesimals is also discussed as is the rejection of infinitesimals in analysis by E. W. Hobson, the latter being based more on matters of convenience than on a claim of inconsistency, as was Russell’s following in the footsteps of Cantor. Finally, Chapter 5 foreshadows the non-classical theories of continua that are discussed in second part of the book with sketches of the views of two groups of philosophers and mathematicians who did not fully embrace the Cantor-Dedekind philosophy of the continuum—du Bois-Reymond, Veronese, and Peirce, who pursued infinitesimalist conceptions, and Brentano, Poincaré, Brouwer and Weyl, who had phenomenological and/or constructivist predilections.

Given the breadth of material Bell attempts to cover in the five historical chapters, the discussions are inevitably sometimes rushed and incomplete. Still, given Bell’s frequent use of citations and quotations from original sources, readers seeking more on topics that are
covered will often learn where to look, though the references to the important secondary literature are sometimes dated and incomplete. Moreover, Bell's choices of which topics to omit, which to include, and of those included which to give more attention than others are sometimes puzzling, given his stated goals. For example, whereas Hegel's obscure remarks on the continuum and the calculus are given eight pages of discussion—three more than accorded Newton's calculus together with his philosophy thereof—the method of exhaustion and its underlying Eudoxian theory of proportions are treated in less than one page. Similarly, Bell devotes merely two sentences to the discovery of incommensurable magnitudes (p. 21), and never mentions that this and related discoveries, coupled with ancient conception of number as a multitude of units, convinced the ancients that it was impossible to bridge the gap between the discrete domain of number and the continuous domain of geometry. Having not mentioned this critical component of the standard ancient philosophy of the continuum, Bell further fails to mention the revolutionary rethinking of the number concept and its relation to the geometrical continuum that began to emerge when mathematicians such as Simon Stevin (1548–1620) argued that not only is 1 also a number but there is a complete correspondence between (positive) number and continuous magnitude, as well as a parallelism between certain geometrical constructions and the now familiar arithmetic operations on numbers. Thus, while there is much that is useful in Bell's sweeping historical account, these and other historical shortcomings would make this author hesitant to recommend it as a comprehensive overview of the development of the concepts of the continuous and the infinitesimal.

The mathematical part of the book is likewise divided into five chapters, the first two of which provide preliminaries for the subsequent three, the vast bulk being for the chapter on Smooth Infinitesimal Analysis/Synthetic Differential Geometry—SIA/SDG. Chapter 6 covers continuity in topological spaces and the concept of a smooth manifold, and Chapter 7 provides introductions to categories, functors and pointless topology as well as overviews of Grothendieck topologies, sheaves, and elementary topoi. Though brief, these chapters cover a good deal of material in a clear and revealing fashion. Chapter 8 covers non-standard analysis—NSA; Chapter 9 treats the continuum from Bishop's constructivist and Brouwer's intuitionist standpoints, and offers an account of Vesley's intuitionistic theory of infinitesimals, and Chapter 10 covers SIA/SDG and provides a brief comparison of SIA and NSA.

Here Bell's apology early on that "it is inevitable that a number of topics have not received the attention they deserve" (p. 11) is an understatement. Bell's five-page discussion of NSA (including non-standard topology) is perfunctory and appears to play no other role than serve as a basis for the aforementioned comparison with SIA. Moreover, while descriptions of the roles played by infinitesimals in NSA and SIA are undoubtedly indispensable to an understanding of some of the important ways infinitesimals are treated in contemporary mathematics, such accounts neither provide a substitute for, nor obviate the need of, descriptions of the in-depth investigations of the various types of systems of finite, infinite and infinitesimal elements undertaken by contemporary model theorists, order-algebraists and geometers alike, investigations that are neither referred to nor referenced by Bell. Nor is there mention of the distinctive infinitesimal conceptions that arise in Alain Connes's noncommutative geometry and in $p$-adic analysis. J. H. Conway's theory of surreal numbers is referred to, but only in a footnote (p. 209). Still, the treatment of SIA is informative and provides, for the most part, a revealing account of its nilpotent-infinitesimalist theory of continua and category-theoretic, intuitionistic-logical framework.

During his discussion of SIA, Bell complements C. S. Peirce for his "awareness, even before Brouwer, of the fact that a faithful account of the truly continuous would involve abandoning the unrestricted applicability of the law of excluded middle . . . " (p. 295: also see pp. 184, 208, and 327). One wonders if this "fact" explains the short shrift given NSA and the absence of any reference to Paolo Giordano's important work in which he develops an alternative
category-theoretic, nilpotent-infinitesimalist approach to analysis and differential geometry that is compatible with classical logic (Nilpotent infinitesimals and synthetic differential geometry in classical logic in Reuniting the antipodes—constructive and nonstandard views of the continuum, Kluwer Acad. Publ., 2001, pp. 75–92; Infinitesimal differential geometry, Acta Mathematica Universitatis Comenianae, New Series, vol. 73 (2004), no. 2, pp. 235–278). Also absent are references to the vast literature on geometries over rings containing nilpotent infinitesimals that has its roots in the classical work of J. Hjelmslev (1898; 1929; 1949), C. Segre (1911) and W. Klingenberg (1954; 1955) (see Geometry over Rings by F. D. Veldkamp in Handbook of Incidence Geometry, edited by F. Bückenhout, Elsevier, 1995 for references) and to the work of F. Bachmann that unifies a remarkable array of geometries including geometries over division rings that do not employ infinitesimals, non-Archimedean geometries over division rings that employ invertible infinitesimals and geometries over rings that employ nilpotent infinitesimals (Ebene Spiegelungsgeometrie: eine Vorlesung über Hjelmslev-Gruppen, B. I. Wissenschaftsverlag, 1989). It is perhaps worth noting that these geometrical works, which are grounded in classical logic, along with the work of Giordano indicate how misleading Bell's claim is that non-zero nilpotent infinitesimals are “possible” in SIA, unlike in NSA, because whereas “[t]he logic of SIA is intuitionistic . . . the logic of NSA is classical” (p. 308).

Much as Bell fails to discuss many of the important ways that infinitesimals are employed in contemporary mathematics, there is a host of alternative contemporary mathematical theories of continua Bell likewise ignores. Included among these are the predicative theory of Feferman, the Russian constructivist theory of Markov, and the fuzzy theory of Zadeh, to name only a few.

As some of our earlier remarks suggest, the references to these shortcomings are not intended to imply that the book is without merit. Indeed, there is much that is interesting and stimulating in the book. Still, at least to this reviewer, the book appears to misrepresent what it is. It is not that the book does not accomplish the ambitious historical and expository goals it sets for itself—that is understandable—but it all too often reads like a celebration of SIA. In any case, when viewed from this perspective, it is a welcome addition to the literature that can be profitably mined for an array of interesting ideas including the most comprehensive treatment to date of John Bell's own interesting and provocative thinking on the continuous and the infinitesimal in mathematics and philosophy.

Philip Ehrlich

Department of Philosophy, Ohio University, Athens, OH 45701, USA. ehrlich@ohio.edu.


This book contains a few semesters worth of mathematical logic at the graduate level, from introductory material on propositional and first-order logic (assuming no prior training in logic), up through Gödel’s incompleteness theorems and topics in set theory, model theory, and recursion theory. Most sections include several exercises. It can be used quite well as a textbook in an introductory graduate logic course, depending on the student population. Students who are casually interested in taking just one course in graduate logic or students who would be intimidated by the level of abstraction may be better served by another textbook. However, students more inclined toward the subject and students who will appreciate a more thorough treatment should be pleased with Hinman’s text.

Chapter 1, on propositional logic, includes a section on intuitive notions of decidability and effective enumerability, a proof of the compactness theorem, and four extra proofs of the compactness theorem, including proofs using ultrafilters and Boolean algebras. (The reader or teacher will probably skip three or four of the extra proofs the first time through, and come back to them as desired.) The reason for studying all these topics in a chapter on