1 Model with Capital Stock in Domestically-owned Firms

The equations and conditions of the foreign investors remain the same (see main text), so we focus on the blocks of the model that are modified with the introduction of capital in the production function of the domestically-owned firms.

1.1 Domestic Households

Households new budget constraint:

\[ P_tC_t + T_t + (1 + r_{t-1}) D_{ht-1} + (1 + r_d) S_t D_{ht-1}^f + P_{ht} I_{ht} = W_t L_t + \Pi_{ht} + D_{ht}^N + S_t D_{ht}^f + r_t k_t P_{ht} K_{ht-1} \] (1.1)

where

\[ I_{ht} = K_{ht} - (1 - \delta) K_{ht-1} \] (1.2)

Households choose consumption, labor, capital, investment and debt holdings to maximize expected utility subject to their budget constraint 1.1, 1.2, initial conditions \( D_{ht}^N \) and \( D_{ht}^I \), and suitable transversality conditions. Optimality entails that

\[ E_t [v_0 C_{t}^{\lambda} L_{t}^{\nu-1}] = \frac{W_t}{P_t} \] (1.3)

\[ E_t \frac{\Lambda_{ht+1}}{\Lambda_{ht}} [1 + r_t - (1 + r_d) \frac{S_{t+1}}{S_t}] = 0 \] (1.4)

\[ E_t \frac{\Lambda_{ht+1}}{\Lambda_{ht}} \frac{P_{ht+1}}{P_{ht}} [r_t - (1 - \delta)] = 1 \] (1.5)

where \( \Lambda_{ht} \equiv \beta^t U_r(C_t, L_t)/P_t \) is the Lagrange multiplier.
1.2 Domestically-owned Firms

Domestic production is carried out by a continuum of monopolistic competitors. Firm $j$, with $j \in [0, 1]$, employs the following technology

$$Y_{ht}(j) = A_{ht} (K_{ht-1}(j))^\alpha_h (L_{ht}(j))^{1-\alpha_h}$$

(1.6)

where $Y_h(j)$ stands for home output of variety $j$ and $A_h$ is a productivity term. The domestically-owned firms’ dividends are then given by:

$$\Pi_{ht}(j) = P_{ht}(j)Y_{ht}(j) - W_tL_{ht}(j) - r_t^k P_{ht}K_{ht-1}$$

Let $mc_t$ denote the (real) marginal cost of the firms. Cost minimization implies that:

$$\frac{W_t}{P_{ht}(j)} = mc_t(j) \frac{(1 - \alpha_h)Y_{ht}(j)}{L_{ht}(j)}$$

(1.7)

$$r_t^k = mc_t(j) \frac{\alpha_h Y_{ht}(j)}{K_{ht}(j)}$$

(1.8)

Combining 1.7 and 1.8 we obtain

$$mc_t(j) = \frac{1}{A_h} \left( \frac{r_t^k}{\alpha_h} \right)^{\alpha_h} \left( \frac{W_t/P_{ht}}{1 - \alpha_h} \right)^{(1-\alpha_h)}$$

(1.9)

1.3 Market Clearing Conditions

Market clearing then implies that:

$$D^f_{ft} = D^f_{ht}$$

(1.10)

$$D^h_{ht} = 0$$

(1.11)

$$Y_{ht} = C_{ht} + I_{ht} + G_h + X_{ht}$$

(1.12)

$$L_t = \int_0^1 L_{ht}(j) dj + L_{ft}$$

(1.13)

Using the household’s budget constraint (equation 1.1), the dividends equation for domestically-owned firms, the market clearing conditions (1.10 through 1.13), the government’s budget constraint, and
intratemporal conditions, we can derive the balance of payments of the economy expressed in terms of home goods:

\[ 0 = s_t(D_{ft} - D_{f t-1}) - r_d s_t D_{ft-1}^f + (w_t L_{ft} + X_{ht} - s_t C_{ft}) \]

where the first parenthesis contains the change in net debt, the next component represents net factor payments from abroad, and the last one is the trade balance of goods and services (services provided by residents to nonresidents, plus exports of the home good minus imports of the foreign good).

1.4 Equilibrium

Given our initial conditions, international prices \((p_t^f, p_t^h, r_t, r_d)\), fiscal policy \(\{T_t, G_t\}_{t=0}^{\infty}\) and monetary policy \(\{r_t\}_{t=0}^{\infty}\), the symmetric equilibrium is defined as the sequences of prices \(\Delta_p \equiv \{r_{ft}, w_t, r_k^f, s_t\}_{t=0}^{\infty}\) and allocations \(\Delta_a \equiv \{C_{ht}, C_{ft}, C_t, C_{t*}, L_{ht}, L_{ft}, B_{ft}, B_{wt}, \Omega_{ft}, K_{ft}, D_{ft}, D_{ht}, D_{hf}\}_{t=0}^{\infty}\), such that: (i) foreign investors maximize utility and their parent companies (the foreign-owned firms) maximize dividends subject to their budget constraints; (ii) domestically-owned firms maximize dividends and set goods prices optimally; (iii) households maximize their utility subject to their budget constraints; (iv) the monetary-fiscal authority balances its budget and sets (optimally) the monetary policy rate; and (v) assets, goods and labor markets clear.

1.5 Steady-State Equilibrium

In the symmetric steady-state equilibrium, the model owns basically 29 endogenous variables denoted by \(C_t^f, C_h, C_f, C, R_f, \Pi_f, \Omega_f, B_{ft}, B_{wt}, L, L_h, L_{ft}, K_f, K_h, I_h, Y_f, Y_h, \Pi_h, D_{ft}, D_{ht}, D_{hf}, X_h, mc, T/P_h, P/P_h, w, r, r_k, \) and \(s\). We can characterize the zero-inflation-rate equilibrium by the following equations:

\[
R_f = 1 + \frac{\Pi_f}{B_f} \tag{1.14}
\]

\[
\Omega_f = p_t^f C_t^f + B_{wt} + B_f \tag{1.15}
\]

\[
\Omega_f = R_w B_w + R_f B_f - (\psi_w/2) (B_w - \bar{B}_w)^2 - (\psi_f/2) (B_f - \bar{B}_f)^2 \tag{1.16}
\]

\[
1 = \beta [R_w - \psi_w (B_w - \bar{B}_w)] \tag{1.17}
\]

\[
1 = \beta [R_h - \psi_f (B_f - \bar{B}_f)] \tag{1.18}
\]

\[
B_f = K_f + D_f^f \tag{1.19}
\]

\[
Y_f = A_f K_f^{\alpha_f} L_f^{\alpha_l} \tag{1.20}
\]

\[
\Pi_f = QA_f K_f^{\alpha_k} L_f^{\alpha_l} - \delta K_f + r_d D_f^f \tag{1.21}
\]

\[
\alpha_q QA_f K_f^{\alpha_k - 1} L_f^{\alpha_l - 1} = r_d + \delta \tag{1.22}
\]

\[
\alpha_d QA_f K_f^{\alpha_k - 1} L_f^{\alpha_l - 1} = \frac{w}{s} \tag{1.23}
\]
\[
\begin{align*}
\frac{P}{P_h} &= s^{1-\gamma} \\
\frac{PC + T}{P_h} + r_d s D_h^f &= wL + \Pi_h - \left( r^k - \delta \right) K_h \\
\nu_0 C_s L^{-1} &= \frac{w}{s^{1-\gamma}} \\
1 + r &= \beta^{-1} \\
C_h &= \gamma s^{1-\gamma} C \\
C_f &= (1 - \gamma) s^{-\gamma} C \\
Y_h &= A_h K_h L_h^{1-\alpha} \\
\frac{\Pi_h}{P_h} &= Y_h - w L_h - r^k K_h \\
m_c &= \frac{1}{A_h} \left( \frac{r^k}{\alpha} \right) \left( \frac{w}{1 - \alpha} \right)^{1-\alpha} \\
m_c &= \frac{\vartheta - 1}{\vartheta} \\
X_h &= s\bar{X} \\
T / P_h &= G_h \\
D_f^l = D_h^l \\
D_h &= 0 \\
Y_h &= C_h + I_h + G_h + X_h \\
L &= L_h + L_f \\
r^k &= \beta^{-1} - (1 - \delta) \\
I_h &= \delta K_h \\
r^k &= mc \frac{\alpha \vartheta Y_k}{K_h}
\end{align*}
\]

1.6 Loglinearization

The loglinearized model can be reduced to the following system:

\[
\begin{align*}
\eta_{bf} \tilde{B}_{ft} + \eta_{bw} \tilde{B}_{wt} &= \eta_{rh} \tilde{B}_{ft-1} + \eta_{rw} \tilde{B}_{wt-1} - \eta_{cf} \tilde{C}_{ft} + \left( \eta_{bf} R_f \right) \tilde{R}_{ft} \\
\tilde{R}_{ft+1} &= \left( \frac{R_d B_f - \Pi_f}{R_f B_f} \right) \tilde{B}_{ft-1} - R_f^{-1} \tilde{B}_{ft} - \frac{\alpha_l Q Y_f}{R_f B_f} (\tilde{w}_t - \tilde{s}_t) + \frac{Q Y_f}{R_f B_f} \tilde{A}_{ft} \\
\tilde{B}_{ft} &= \frac{K_f}{B_f} \tilde{K}_f + \frac{D_f}{B_f} \tilde{D}_{ft}
\end{align*}
\]
\[ \eta_{hb} \hat{D}_{ft} = (\eta_{ht} R_d) \hat{D}_{ft-1} + r_d [\eta_{hb} + (1 - \gamma)\eta_c] \hat{s}_t - r_d \left[ \hat{Y}_{ht} + \eta_f (\hat{w}_t + \hat{L}_{ft}) - \eta_c \hat{C}_t - \eta_{ht} \hat{I}_{ht} \right] \] (1.46)

\[ \hat{Y}_{ht} = \frac{C_h}{Y_h} \hat{C}_t + \left[ \frac{(1 - \gamma)C_h + X_h}{Y_h} \right] \hat{s}_t + \frac{I_h}{Y_h} \hat{I}_{ht} + \frac{X_h}{Y_h} \hat{X}_t \] (1.47)

\[ \pi_t = \pi_{ht} + (1 - \gamma) \Delta \hat{s}_t \] (1.48)

\[ \alpha_k \hat{K}_{ft-1} + (\alpha_t - 1) \hat{L}_{ft} + \hat{A}_{ft} = (\hat{w}_t - \hat{s}_t) \] (1.49)

\[ (v - 1) \left[ \frac{L_h}{L} \left( \frac{\hat{Y}_{ht} - \hat{A}_{ht}}{1 - \alpha_h} - \frac{\alpha_h}{1 - \alpha_h} \hat{K}_{ht-1} \right) + \frac{L_f}{L} \hat{L}_{ft} \right] + \chi_h \hat{C}_t = \hat{w}_t - (1 - \gamma) \hat{s}_t \] (1.50)

\[ \hat{X}_t = \rho_x \hat{X}_{t-1} + \xi_{xt} \] (1.51)

\[ \hat{A}_{ft} = \rho_{af} \hat{A}_{ft-1} + \xi_{ft} \] (1.52)

\[ \hat{A}_{ht} = \rho_{ah} \hat{A}_{ht-1} + \xi_{ht} \] (1.53)

\[ \hat{s}_t = E_t \hat{s}_{t+1} - \hat{R}_t + E_t \pi_{ht+1} \] (1.54)

\[ \pi_{ht} = \beta E_t \pi_{ht+1} + \psi_{mc} \tilde{m} c_t \] (1.55)

\[ \chi_f \hat{C}_{ft} = \chi_f E_t \hat{C}_{ft+1} - (\beta \psi_w B_w) \hat{B}_{wt} \] (1.56)

\[ \alpha_t E_t \hat{L}_{ft+1} - E_t \hat{A}_{ft+1} = (1 - \alpha_k) \hat{K}_{ft} \] (1.57)

\[ E_t \hat{R}_{ft+1} = \left( \frac{\psi_f B_f}{R_f} \right) \hat{B}_{ft} - \left( \frac{\psi_w B_w}{R_f} \right) \hat{B}_{wt} \] (1.58)

\[ \hat{K}_{ht} = \delta \hat{I}_{ht} + (1 - \delta) \hat{K}_{ht-1} \] (1.59)

\[ \tilde{m} c_t = \hat{w}_t + \frac{\alpha_h}{1 - \alpha_h} (\hat{Y}_{ht} - \hat{K}_{ht}) - \frac{1}{1 - \alpha_h} \hat{A}_{ht} \] (1.60)

\[ (1 - \alpha_h) \tilde{v}_t = (1 - \alpha_h) \hat{w}_t + \hat{Y}_{ht} - \hat{K}_{ht} - \hat{A}_{ht} \] (1.61)

\[ E_t \tilde{v}_{t+1} = \frac{1}{\beta_{vk}} \left( \hat{R}_t - E_t \tilde{v}_{ht+1} \right) \] (1.62)

where \( \eta_{bf} \equiv \frac{B_f}{R_f} \), \( \eta_{bw} \equiv \frac{B_w}{R_w} \), \( \eta_{rf} \equiv \left[ R_f - \psi_f (B_f - \hat{B}_f) \right] \eta_{bf} \), \( \eta_{rw} \equiv \left[ R_w - \psi_w (B_w - \hat{B}_w) \right] \eta_{bw} \).
The loglinearized version of the model can be reduced to 18 endogenous variables $B_{ut}, \hat{B}_{ft}, \hat{K}_{ft}, \hat{K}_{ht}, \hat{D}_{ft}, \hat{Y}_{ht}, \pi_t, \hat{\pi}_t, \hat{\pi}_{ht}, C_{ft}, \hat{L}_{ft}, \hat{R}_{ft}, \hat{r}_f^h, \hat{m}_c, \hat{I}_{ht},$ and $\hat{R}_t$. In addition, there are three exogenous stochastic processes for $\hat{X}_t, \hat{A}_{ft},$ and $\hat{A}_{ht}$. The rest of endogenous variables ($\hat{D}_{ft}, \hat{C}_{ft}, \hat{L}_{ft}, \hat{R}_{ht}, \hat{X}_{ht}$, among others) can then be expressed as functions of the initial 18 variables and the exogenous shocks.

Let $\vec{x}_{1t}$ and $\vec{x}_{2t}$ denote the vectors of backward- and forward-looking variables, respectively. It is convenient then to rewrite the loglinearized model in its state-space form:

$$H_x\begin{bmatrix} \vec{x}_{1t+1} \\ E_t \vec{x}_{2t+1} \end{bmatrix} = A_x \begin{bmatrix} \vec{x}_{1t} \\ \vec{x}_{2t} \end{bmatrix} + B_R \vec{R}_t + \begin{bmatrix} \xi_{t+1} \\ 0 \end{bmatrix}$$

(1.63)

where $\vec{x}_{1t} = (B_{ut-1}, \hat{B}_{ft-1}, \hat{K}_{ft-1}, \hat{D}_{ft-1}, \hat{Y}_{ht-1}, \pi_{t-1}, \hat{\pi}_{t-1}, \hat{\pi}_{ht-1}, C_{ft}, \hat{L}_{ft-1}, \hat{R}_{ft-1}, \hat{r}_f^h, \hat{m}_c, \hat{I}_{ht-1}, \hat{R}_t, \hat{X}_{ht})'$, and $\vec{x}_{2t} = (\hat{s}_t, \pi_{ht}, \hat{C}_{ft}, \hat{L}_{ft}, \hat{R}_{ft}, \hat{r}_f^h)'$. For the rest of details, please see main text.

### 2 Sensitivity Analysis: Other Loss Functions

We use the loss function derived by Galí and Monacelli (2005) in a similar model. Using our notation, their loss function without terms independent of policy would be approximately

$$(1/2)E_t \sum_{\tau=0}^{\infty} \beta^\tau \left[ \frac{\vartheta}{\psi_{mc}} \pi_{ht+\tau}^2 + \nu \hat{Y}_{gt+\tau}^2 \right]$$

(2.64)

where $\psi_{mc} = \frac{(1-\theta)(1-\beta)}{\theta}$ and now $\hat{Y}_{gt}$ is the output gap, that is, the log-deviation of output from its level under flexible prices and complete asset markets, where the latter depends on the exogenous external demand and productivity shocks. In this case the sensitivity consisted of changing values of deep parameters of the model such as the degree of price stickiness ($\theta$), the curvature parameter of labor disutility ($\nu$), and the elasticity of substitution across goods varieties ($\vartheta$). The first exercise is reported in the main text (see figure 20). The exercises for $\nu$ and $\vartheta$ are reported below.
Output-Interest Correlation

Figure 24. Sensitivity Analysis: v
Figure 25. Sensitivity Analysis: 9