Online Appendix
A Simple Model to Teach Business Cycle Macroeconomics for Emerging Market and Developing Economies

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The AG Model

The consumers

The consumer’s problem consists of choosing $C, C', N, N'$ and $B'$ to:¹

$$\max \quad \ln \left( C - \frac{N^2}{2} \right) + \beta \ln \left( C' - \frac{N'^2}{2} \right)$$

s.t.:

$$C + B' = wN + \pi$$
$$C' = w'N' + \pi' + (1 + r)B'$$

where the initial stock of net assets ($B$) is given, we assume $B = 0$, and the transversality condition $B'' = 0$.²

The Lagrangean function is

$$\mathcal{L} = \ln \left( C - \frac{N^2}{2} \right) + \beta \ln \left( C' - \frac{N'^2}{2} \right) + \lambda [wN + \pi - C - B'] + \lambda' [w'N' + \pi + (1 + r)B' - C']$$
The first-order conditions (FOCs) are

\[
\frac{\partial L}{\partial C} = \frac{1}{C - \frac{N^2}{2}} - \lambda = 0 \quad (1)
\]

\[
\frac{\partial L}{\partial C'} = \frac{\beta}{C' - \frac{N'^2}{2}} - \lambda' = 0 \quad (2)
\]

\[
\frac{\partial L}{\partial N} = -\frac{N}{C - \frac{N^2}{2}} + \lambda w = 0 \quad (3)
\]

\[
\frac{\partial L}{\partial N'} = -\frac{N'}{C' - \frac{N'^2}{2}} + \lambda' w' = 0 \quad (4)
\]

\[
\frac{\partial L}{\partial B'} = -\lambda + \lambda' (1 + r) = 0 \quad (5)
\]

\[
\frac{\partial L}{\partial \lambda} = wN + \pi - C - B' = 0 \quad (6)
\]

\[
\frac{\partial L}{\partial \lambda'} = w'N' + \pi' + (1 + r)B' - C' = 0 \quad (7)
\]

The firms

Assuming a Cobb-Douglas production function, the firm’s current and future profits \((\pi, \pi')\) are

\[
\pi = zK^\alpha N^{1-\alpha} - wN - [K' - (1 - \delta)K] \quad (8)
\]

\[
\pi' = z'K'^\alpha N'^{1-\alpha} - w'N' + (1 - \delta)K' \quad (9)
\]

where \(0 < \alpha < 1\) is the capital share, \(0 < \delta < 1\) is the depreciation rate, and we impose the transversality condition \(K'' = 0\). Given the initial stock of capital \((K)\), the firm’s problem consists of choosing \(N, N',\) and \(K'\) to maximize its value (the present value of profits, \(V\)). That is,

\[
\max_{N, N', K'} \quad V = \pi + \frac{\pi'}{1 + r}
\]

The FOCs are

\[
\frac{\partial V}{\partial N} = (1 - \alpha)zK^\alpha N^{-\alpha} - w = 0 \quad (10)
\]

\[
\frac{\partial V}{\partial N'} = (1 - \alpha)z'K'^\alpha N'^{-\alpha} - w' = 0 \quad (11)
\]

\[
\frac{\partial V}{\partial K'} = -1 + \frac{\alpha z'K'^{\alpha-1}N'^{1-\alpha} + (1 - \delta)}{1 + r} = 0 \quad (12)
\]
Market-clearing conditions

Markets clear in the present and future period if

\[ N^s = N^d \]  
\[ N^{s'} = N^{d'} \]  
\[ Y^s = Y^d \]  
\[ Y^{s'} = Y^{d'} \]  

Solution

Given that \( r = r^w \), the solution of the model is

\[ N^* = w^* = \left[ (1 - \alpha)zK^\alpha \right]^{\frac{1}{1-\alpha}} \]  
\[ Y^* = zK^\alpha \left[ (1 - \alpha)zK^\alpha \right]^{\frac{1}{1-\alpha}} \]  
\[ K^{t*} = \left( \frac{\alpha z'}{r + \delta} \right)^{\frac{1}{1-\alpha}} \left( 1 - \alpha \right)z' \]  
\[ I^* = K^{t*} - (1 - \delta)K \]  
\[ N^{t*} = w^{t*} = (1 - \alpha)z' \left( \frac{\alpha z'}{r + \delta} \right)^{\frac{1}{1-\alpha}} \]  
\[ Y^{t*} = z'K^{t*\alpha}N^{t_1-\alpha} \]  
\[ C^* = \frac{1}{(1 + \beta)(1 + r)} \left[ \frac{(N^{t*})^2}{2} \right] + \frac{1}{(1 + \beta)(\alpha Y^* + (1 - \delta)K)} + \frac{2 + \beta}{1 + \beta} \left[ \frac{(N^*)^2}{2} \right] \]  
\[ NX^* = Y^* - C^* - I^* \]  
\[ B^{t*} = NX^* \]
\[ C^* = \frac{1 + 2\beta}{1 + \beta} \left[ \frac{(N^*)^2}{2} \right] + \frac{\beta(1 + r)(\alpha Y^* + (1 - \delta)K)}{1 + \beta} + \frac{\beta(1 + r) (N^*)^2}{1 + \beta} \]  

(26)

**Supply and demand functions**

Using equations (1) and (3), we can derive the current labor supply

\[ N^* = w \]  

(27)

Equation (10) yields the current labor demand

\[ N^d = \left[ \frac{(1 - \alpha) zK^\alpha}{w} \right]^\frac{1}{1 - \alpha} \]  

(28)

It is easy to prove that \( \partial N^d / \partial z > 0, \partial N^d / \partial K > 0, \partial N^d / \partial w < 0 \). Recall that the current production function does not depend on the real interest and constitutes the current output supply:

\[ Y^* = zF(K, N) = zK^\alpha N^{1-\alpha} \]  

(29)

where, by assumption, \( \partial Y^* / \partial z > 0, \partial Y^* / \partial K > 0, \) and \( \partial Y^* / \partial N > 0 \). The equations for the current demand for consumption goods, the future capital stock, and the current demand for investment goods (23, 19, 20) yield

\[ C^d = \frac{1}{(1 + \beta)(1 + r)} \left[ \frac{(N^*)^2}{2} \right] + \frac{1}{1 + \beta} \left( \frac{(1 - \alpha) zK^\alpha}{(1 + \beta)(1 + r)} \right) \left(1 + \frac{\delta}{\delta + 1}\right) \]  

\[ I^d = \left( \frac{\alpha z'}{r + \delta} \right) \left( \frac{1}{1 - \alpha} z' - (1 - \delta)K \right) \]  

It is straightforward to prove that \( \partial C^d / \partial z > 0, \partial C^d / \partial z' > 0, \partial C^d / \partial r < 0, \partial C^d / \partial K > 0, \partial I^d / \partial z' > 0, \partial I^d / \partial r < 0, \) and \( \partial I^d / \partial K < 0 \). Putting all of these elements together we have

\[ Y^d = C^d(r, z, z', K) + I^d(r, z', K) + N X \]  

(30)

Finally, rewriting equations (27)-(30) in general form, jointly with the parity of interest rates, we have the equations shown in the second section:³

\[ N^* = w \]

\[ N^d = N^d(w, z, K) \]
\[ Y^* = zF(K, N) \]
\[ Y^d = C^d(r, z, z', K) + I^d(r, z', K) + NX \]
\[ r = r^w \]

Other Applications and Extensions

Next we show other applications and simple extensions of the AG model. We give priority to simplicity and view these applications as complementary explanations to others given in the macroeconomic literature for EMDEs.

The Cyclical Behavior of Prices

Rand and Tarp (2002) find that the cyclical component of the CPI is negatively correlated with (detrended) output in most developing countries. In other words, prices are countercyclical in EMDEs.\(^4\) Let us introduce a money market to the model using a similar approach as in Williamson (2013). For simplicity, the money supply (\(M^s\)) is exogenously controlled by the central bank, and the money demand (\(M^d\)) depends on total income and the interest rate as follows:

\[ M^s = M \]  \hspace{1cm} (31)
\[ M^d = PL(Y, r) \]  \hspace{1cm} (32)

where \(P\) is the domestic price level determined in this market (see Figure 10).\(^5\) In Figure 10, we represent the money supply as a vertical line at the initial level \(M_1\). The money demand is represented by the straight upward-sloping line \(P = M/L(Y, r)\), with slope \(1/L(Y, r)\).
Figure 1: The equilibrium in the money market and the determination of the price level.

As previously shown, a persistent TFP increase raises income and lowers the interest rate. Unambiguously, money demand expands. This can be observed in Figure 11. Given a constant money supply, the subsequent excess demand at the initial price level \( (P_1) \) is only satisfied by a fall in prices \( (P_1 > P_2) \). If the economy is systematically shocked by persistent TFP changes, this result yields to a negative comovement between output and prices.\(^6\)

Figure 2: Countercyclical prices.

Currency Depreciations and Output

Recent empirical evidence reported by Cordella and Gupta (2014) shows that currencies in EMDEs tend to depreciate when GDP growth is low and appreciate when GDP
growth is high. That is, nominal exchange rates—defined as the relative price of a foreign currency in terms of the domestic currency—are countercyclical (currencies are procyclical) in most of the developing world. One possibility is that the nominal exchange rate is indirectly affected by output fluctuations, which in turn are caused by a third factor such as TFP changes. If this mechanism is empirically correct, then how can we explain the negative correlation between currency depreciations and output fluctuations using any of the models? Following Williamson (2013), assume that the purchasing power parity condition holds:

\[ P = eP^* \tag{33} \]

where \( P^* \) is the foreign price level and \( e \) is the nominal exchange rate defined such that an increase represents a depreciation of the domestic currency. If we plug equation (33) in (32), we obtain

\[ M^d = eP^*L(Y, r) \tag{34} \]

Equations (31) and (34) determine the nominal exchange rate in the money market under a flexible exchange rate regime. Figure 12 shows the interaction between supply and demand and the determination of the nominal exchange rate.

![Diagram](image)

Figure 3: The money market and the determination of the nominal exchange rate.

Consider a persistent reduction in TFP that contracts output (in both models) and raises the interest rate (in the NP model). The reduced income leads to a lower demand for money and, as a result, a nominal depreciation of the currency (see Figure 13). Therefore, if the economy is regularly hit by persistent TFP changes, we should expect a negative comovement between the nominal exchange rate and output.
Figure 4: The negative comovement between the nominal exchange rate and output.

Suggested Exercises

1. **A change in the world interest rate.** Show that a reduction in the world interest rate does not produce all of the predictions that we obtain from a persistent TFP fall in the AG model.

2. **The macroeconomic effects of natural disasters.** According to Raddatz (2007), climatic disasters (which include floods, droughts, extreme temperatures, and wind storms) result in declines in real GDP (per person) of 2% in low-income countries. This rate looks modest in absolute value, but the declines are significantly greater than the median growth rates of such countries (0.4%). Analyze the macroeconomic effects of a climatic disaster that destroys part of the current stock of capital using the AG model.

3. **Procyclical prices and monetary policy: The cases of Malaysia and Peru.** In the previous section, we show that if we extend the model with a money market, a persistent TFP increase that raises income and lowers the interest rate will contract money demand, causing a procyclical behavior of the price level. However, there exists evidence that prices are procyclical in certain EMDEs such as Malaysia (Rand and Tarp 2002) and Peru (Castillo, Montoro and Tuesta 2007). In addition, several works conclude that monetary policy tends to be procyclical in EMDEs (see Kaminsky, Reinhart and Végh 2005, Duncan 2014). Show that if we include a procyclical monetary rule that positively links the money supply to output, $M^* = M^*(Y)$, it is possible to observe an increase in the price level and, therefore, procyclical prices.

4. **Does risk matter?** Fernández-Villaverde, Quintana, Ramírez and Uribe (2011) show that changes in risk, measured by the volatility of the real interest rate, can
account for the behavior of output, consumption, investment, and hours worked in Argentina, Brazil, Ecuador, and Venezuela. Suppose we extend the AG model by replacing the output demand equation by

\[ Y^d = C^d(r, \sigma_r, ...) + I^d(r, \sigma_r, ...) + NX \]

where \( \sigma_r \) is the standard deviation of the real interest rate. The intuition behind the negative link is that an increase in \( \sigma_r \) raises precautionary savings and, thus, lowers consumption. In addition, physical capital becomes riskier and, hence, investment falls. Analyze the effects of an increase in \( \sigma_r \) on the main endogenous variables.
References


Notes

1. Strictly speaking, we should include $N^s$ and $N^s'$ in the consumer’s problem and $N^d$ and $N^d'$ in the firm’s problem. To lighten the notation, we just use $N$ and $N'$.

2. GHJ preferences lead to a labor supply that depends only on the real wage but not on consumption, and thus, wealth effects. This type of preference is widely used in the business cycles literature for EMDs. A short list includes Mendoca (1991), Neumeyer and Perri (2005), and Uribe and Yue (2006).

3. Strictly speaking, the arguments shown in the functions below are not the only ones. Preference and technology parameters are omitted. An alternative formulation could be, for example, $N^d = N^d(w, ...)$, to indicate that the function depends on other exogenous, perhaps less important, arguments.


5. To simplify the analysis, we set the inflation rate equal to zero; thus, the nominal equals the real interest rate.

6. In this case, we assume that the monetary policy is acyclical or not used to stabilize prices. One of the suggested exercises in Appendix C addresses the (unusual) case of procyclical prices that might be understood if the central bank performs a (highly) procyclical monetary policy. Procyclical monetary policies are frequently observed in EMDEs (see, e.g., Kaminsky, Reinhart and Végh 2005).

7. There also exists a large literature that tries to explain and verify the negative link by establishing a causal relationship from the nominal exchange rate to output in EMDEs. This is sometimes called the contractionary devaluation hypothesis. See, e.g., Kim and Ying (2007) and the references therein.


9. In this case, $c$ is endogenous. Naturally, the quantity of money ($M$) becomes endogenous under a fixed exchange rate.