Carrier Phase Position Domain Smoothing (CPDS) Algorithm and Flight Test Results for New Dual-Frequency Differential Architecture

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ABSTRACT

Flight test results are presented for a single-frequency, airborne-only carrier phase position domain smoothing (CPDS) algorithm, which is a key component of a new dual-frequency differential architecture for aircraft precision approach and landing. The architecture’s first two major components—a modified dual-frequency Code Noise and Multipath (CNMP) algorithm and new composite protection level equations for horizontal, vertical and radial error—are described elsewhere by the authors. The CPDS algorithm, which is an extension of the divergence-free smoothing method, is the third and final major component.

Conceptually, the CPDS algorithm harnesses the stability of continuously tracked carrier phase measurements, exploiting them to propagate the user’s position with centimeter level accuracy while keeping long-term propagated position solution error close to zero by continual updates from pseudorange measurements. Carrier phase measurements can be used as soon as they are obtained. Ionosphere-free observables are required for high accuracy, high integrity applications over all ionospheric conditions. However, uncompensated code and carrier data may be used in system development and lower-integrity applications and when severe ionospheric anomalies are not present and when the reference station is co-located with the runway threshold.

Using flight test data recorded in an Ohio University King Air aircraft, performance comparison is made between the conventional 100-s LAAS ground-air smoothing technique and the CPDS air-only algorithm. A standalone implementation of the CPDS algorithm, namely one without the benefit of CNMP processing, is used here to show most clearly the benefits of this smoothing approach.

INTRODUCTION

The literature contains a number of suggested modifications to the single-frequency Local Area Augmentation System (LAAS), a differential GPS system specified in [1-2] for precision approach and landing by civil aircraft. Among these suggested modifications are a handful of proposed architectures that use signals from two GPS frequencies to enhance system performance. A recent addition to this group is the Dual-Frequency Differential 1 architecture, or DFD1 for short, proposed by the authors. The first two major components of the new architecture—a modified dual-frequency Code Noise and Multipath (CNMP) algorithm and new composite protection level equations for horizontal, vertical and radial error—are described in [3-4]. The carrier phase position domain smoothing (CPDS) algorithm described in the present effort is the third and final major component of the new architecture.

Before introducing the smoothing algorithm and presenting flight test results, four background areas will briefly be described. These are the baseline LAAS architecture against which performance is measured, key characteristics of the DFD1 architecture, other carrier phase smoothing approaches found in the literature, and a noteworthy dual frequency architecture by Hwang, McGraw and Bader that serves as a point of comparison for the DFD1 architecture.

BACKGROUND

The LAAS baseline architecture

The LAAS reference station architecture, as defined in [1-2], is based on up to four GPS antenna-receiver pairs situated in a low-multipath environment and with a one-way data link from the reference station to the aircraft. Ideally, this installation is co-located with the primary airport it serves in order to maximize differential GPS performance, but this is not a requirement in general.
The corrections from each antenna-receiver pair for each satellite are resolved to a single antenna location and averaged, and then typically smoothed using an alpha filter for a time interval of up to 100 s. The results are transmitted on the data broadcast for use by the airborne system, which measures and smoothes its own satellite measurements according to the same procedure. The differentially corrected, single-frequency GPS position solutions and protection levels are calculated and used in the precision approach operation. As the distance between the aircraft and the reference station decreases, so do errors that arise due to spatial decorrelation between the two, such as residual ionospheric and tropospheric delays. The result is a high-accuracy, high-integrity navigation solution suitable for safety-critical, civil aviation applications.

The standard LAAS configuration is modified slightly in the present effort to match as closely as possible the test configuration of the DFD1 architecture. Only one receiver-antenna pair is used in the reference station, and likewise in the aircraft. A low-multipath pinwheel antenna is used instead of the more typical integrated multipath limiting antenna.

The Dual-Frequency Differential 1 (DFD1) architecture

The physical components of the DFD1 architecture are the same as those used in LAAS, except that dual-frequency antennas and receivers are used. The processing architecture, however, differs in a number of ways. The data broadcast from reference station to aircraft includes not only the differential corrections, but also pseudorange measurements, accumulated Doppler measurements, noise sigmas and bias bounds, all of which enable enhanced processing onboard the aircraft. The architecture incorporates a modified version of the CNMP algorithm, namely one that calculates bias bounds based on real-time observations rather than preset values, and is applied in the aircraft as well as the reference station [3].

Dual-frequency measurements are used only to remove time variations in ionospheric delay—that is, delay that has occurred since CNMP algorithm initialization—from the CMC observable. They are not used to correct code phase and carrier phase measurements for ionospheric delays. Put another way, absolute ionospheric delay is removed from the aircraft solution only as a result of differential processing (which removes several other errors as well), and residual ionospheric error between aircraft and reference station tends to zero only as the aircraft approaches the reference station and “sees” the same ionosphere. Ionosphere-free observables are required to implement high accuracy, high integrity applications, but uncorrected code and carrier measurements are used here as a measurement baseline for the future addition of an ionospheric correction algorithm to the architecture. To minimize ionospheric effects in the meantime while this transitional architecture is used for development and evaluation, flight testing is suspended during severe ionospheric anomalies, and the reference station is located only a short distance (i.e., less than 200 m) from runway threshold.

Only carrier phase data is used from the L2 frequency, along with L1 code and carrier, to remove ionospheric divergence. Code phase data from L2 is discarded. Composite protection levels (PLs), namely those derived by treating bias and noise separately in computing the integrity bound, are another major feature of the DFD1 architecture [4]. Along with the CNMP algorithm and composite PLs, the single-frequency CPDS algorithm completes the major features of the DFD1 architecture.

Previous carrier phase position domain smoothing techniques

Using carrier phase measurements to smooth GPS pseudoranges is a well-known technique first reported in 1982 by Hatch [5]. However, the use of carrier phase data to smooth position or velocity estimates derived from GPS pseudoranges has been comparatively rare. In perhaps the first flight test campaign using this technique, a delta-carrier phase update was applied to unsmoothed pseudorange corrections in differential GPS autoland flight tests in late 1994 and early 1995 [6]. In the 1995 tests, which involved fifty precision approaches with a United Parcel Service Boeing 757 aircraft, a $\mu + 2\sigma$ error performance of 1.1 m vertical and 1.4 m lateral was achieved. The details of this smoothing algorithm were not provided in [6], but an updated version is described in detail here.

At least two additional GPS position domain smoothing algorithms are described in the literature. In the first one, a single GPS receiver with dual frequency capability provides ionosphere-free measurements consisting of undifferenced pseudorange and time-differenced carrier phase for precise point positioning in post-processing [7]. A code-minus-carrier (CMC) observable is used to estimate variance-covariance matrix elements for code noise and multipath, and a variance-covariance matrix for carrier phase terms is also created. A kinematic, sequential least-squares filter, which is a special case of a Kalman filter, uses as innovations the differences between predicted and measured pseudorange as well as the differences between predicted and measured carrier phase updates. This approach, which uses precise orbit products supplied in post-processing, achieves sub-meter-level two-sigma positioning accuracy in post-processed static, aircraft and low-earth orbit satellite data sets.
The second GPS position domain smoothing formulation, likewise a Kalman filter, is described in [8] and [9]. The state vector consists of the error states of current position, current velocity and previous position. The innovations are the single differences between measured and theoretical pseudorange, and between measured and theoretical Doppler, for two different satellites. A double difference is then formed within the Kalman filter by differencing these across the current and previous GPS epochs. This algorithm is shown in [8] to improve both theoretical Doppler, for two different satellites. A double difference is then formed within the Kalman filter by differencing these across the current and previous GPS epochs. This algorithm is shown in [8] to improve both availability and accuracy for land vehicle use, achieving non-differential mode root-mean-square (rms) accuracies of 1-2 m with good geometries and 5-10 m in urban canyons. In [9], the position-domain smoothing is accomplished at both reference station and aircraft where measurements can be made divergence-free or ionosphere-free (terms fully defined in that reference), and only in the aircraft otherwise. Factors listed for consideration when selecting range domain or position domain smoothing included the following:

- Filter memory when transiting an ionospheric front (memory hampers position domain smoothing due to “left over” errors after crossing through the front)
- The possibility of frequent satellite blockages (blockages favor position domain smoothing as long as four satellites are always visible)
- Possible step changes in the position solution due to smoothing if the source of differential corrections changes (i.e., when transitioning to a different reference station)

The position domain smoothing algorithm in the present effort is not a Kalman filter implementation. Instead, an instantaneous integrated velocity estimate of very high quality is created and applied only in the aircraft and not in the reference station. Measurements from only the previous and current GPS epochs are used. An alpha filter smoothes the differential position derived from pseudorange measurements at both the reference station and aircraft. At no time are the pseudorange measurements themselves smoothed using carrier phase information. Pseudorange noise is reduced to a few cm by the CNMP algorithm, and pseudoranges are weighted by 1 / (bias bound) in the least-squares position solution due to the fact that post-CNMP bias dwarfs noise. In this way, the shortcomings of position domain smoothing as described above are resolved. Furthermore, carrier phase measurements from a newly acquired satellite may be used immediately in the position domain smoothing algorithm while incurring jumps of only a few cm or less with a typical GPS constellation, as opposed to 0.5 to 1 m jumps introduced by changes in the code solution. As long as continuous carrier phase tracking is maintained for at least four satellites, or an inertial measurement unit provides data to coast through momentary blockages, the algorithm continues without interruption. The DFD1 architecture is an extension of the divergence-free smoothing algorithm described in [9] and elsewhere. The extensions to the dual-frequency calculation are precise geometry corrections of the type described in [10], and a single-satellite fault exclusion algorithm to remove cycle slip errors.

In attempting to create optimal architectures for future GPS use in the 2025 – 2030 time frame, the GNSS Evolutionary Architecture Study (GEAS) Panel recommended two relative receiver autonomous integrity monitor (RRAIM) architectures [11]. One uses a range domain formulation; the other employs an accumulated carrier phase based projection of the user’s position as its observable in the position domain [12, 13], similar to the present effort. An explicit goal of the recommended second architecture is to reduce the size of PLs, thereby improving system availability and continuity.

**Comparing DFD1 to another dual-frequency civil differential GPS implementation**

A notable set of dual-frequency LAAS architectures was proposed in [14] by Hwang, McGraw and Bader. After defining the techniques of divergence-free and ionosphere-free smoothing for a local-area differential service, the authors describe a reference station that transmits carrier phase measurements and divergence-free smoothed pseudorange measurements. Several options are provided for the airborne system. These include single frequency processing to provide divergence-free, smoothed L1 pseudoranges or dual-frequency processing to provide divergence-free L1 or L2 pseudoranges. This approach creates a useful framework for CMC techniques and clearly illustrates the cost (in added noise) of removing ionosphere delay altogether from code and carrier measurements.

Like the architecture of [14], the DFD1 architecture creates a dual-frequency CMC observable without implementing a Kalman filter, confirms the value of smoothing longer than 100 s to remove multipath oscillations, decouples the smoothing time constants used in reference station and airborne system, and requires continuous carrier phase tracking to realize the architecture’s full benefit. However, the DFD1 architecture smoothes in the position domain rather than in the range domain, and exchanges pseudorange noise and multipath for pseudorange bias instead of filtering the noise and multipath. Using the CNMP algorithm, the DFD1 is able to estimate the bias—which is fixed as long as continuous carrier phase tracking is maintained—over time and remove it, resulting in a net “profit” in the exchange.
CARRIER PHASE POSITION DOMAIN SMOOTHING ALGORITHM

Origin and Benefits

As has been stated, smoothing in the DFD1 processing architecture is accomplished solely in the aircraft instead of using code-carrier techniques specified for ground and air LAAS processing in [1]. A position domain technique first described in [6] is used. Unsmoothed pseudorange corrections (i.e., not smoothed using carrier phase data) and AD data from the reference receiver, along with unsmoothed pseudoranges and AD data from the airborne receiver, are combined to propagate the aircraft’s position from the previous time to the current time with centimeter-level accuracy.

The position domain smoothing approach offers several important benefits. First and perhaps most important, a virtual “rail in the sky” is created to define the aircraft approach path. The smoothness of the rail is provided by the centimeter-level accuracy of the position domain updates, which are derived from carrier phase data. The position solution is thus kept free from abrupt changes that would otherwise occur when satellites enter or leave the visible constellation. This is essential for an aircraft autoland computer, which might execute a dangerous nose-down command on final approach in response to such a step change in GPS position data. The rail is kept centered in its true position by continual updates from the code phase solution.

Integrity of the position solution is enhanced by the carrier phase lock and cycle slip detectors provided by the GPS receivers. The additional cycle slip detection algorithm implemented in software prior to the CNMP algorithm processing stage adds to this integrity. Before actual implementation, a more detailed analysis of these and other integrity components of the DFD1 architecture is needed.

Two additional advantages serve to enhance system continuity. As soon as a satellite’s carrier phase data is available, it may be used to propagate the differentially-corrected aircraft position solution. Transition effects remain a few centimeters or less for a typical constellation while continuous tracking of at least four satellites’ carrier phase data is maintained. Furthermore, the centimeter-level accuracy of the relative position solution from one measurement epoch to the next enables the patching of pseudoranges following a temporary loss of lock. The specification for Category I non-federal LAAS ground facilities already allows such patching to accommodate aircraft overflights by specifying an update rate of 0.5 s, not to exceed 1.0 s (see paragraph 3.2.1.2.8.5.1 of [15]). Previous specification versions described the patching process in more detail, but the most recent document gives more leeway to the manufacturer. The method presented here does, however, require continuous carrier phase tracking of at least four satellites to avoid an algorithm restart. An inertial sensor could be used to patch a temporary loss of carrier phase data.

The position domain algorithm was previously summarized and tested in [6], but no equations were included. The equations are provided below, using Huang’s setup and notation from [16].

Algorithm Description

Let the single difference between simultaneous carrier phase measurements of satellite \( i \) by users at points A and B be defined as follows:

\[
SD_{\text{AB},L,i}(t) = R_{\text{AB},i}(t) + c\Delta t_{\text{AD,AB},L,i}(t) + N_{\text{AB},L,i}\lambda + \epsilon_{\text{P,AB},i}(t) - I_{\text{AB},L,i}(t) + T_{\text{AB},L,i}(t) + mp_{\text{AD,AB},L,i}(t) + \eta_{\text{AD,AB},L,i}(t)
\]

where

\[
R = \text{true geometric range from user to satellite (m)}
\]

\[
c = \text{speed of light defined for GPS (m/s)}
\]

\[
\Delta t = \text{receiver clock offset between identical measurements on the two receivers at points A and B (s)}
\]

\[
N = \text{integer difference in the number of wavelengths between the satellite and users at points A and B}
\]

\[
\lambda = \text{wavelength of the specified GPS frequency (Hz)}
\]

\[
\epsilon = \text{difference in satellite orbit error projections onto line of sight (m)}
\]

\[
I = \text{difference in ionospheric delay (m, with – sign indicating advance)}
\]

\[
T = \text{difference in tropospheric delay (m)}
\]

\[
mp = \text{difference in multipath error (m)}
\]

\[
\eta = \text{difference in noise error (m)}
\]

and with subscripts

\[
\text{AB} \quad \text{indicating point B relative to point A}
\]

\[
\text{AD} \quad \text{indicating accumulated Doppler (also known as deltarange)}
\]

\[
\text{P} \quad \text{indicating geometric projection onto user-to-satellite line of sight}
\]

\[
L = \text{GPS frequency, L1 only for the single differences computed here}
\]

\[
i = \text{index of usable satellites (1, 2, \ldots, n)}
\]

It is assumed that known errors from sources such as antenna phase differences, which depend on azimuth and
elevation, and phase wrap-up have been removed. Given the short baseline assumption, the terms \( I \) and \( \varepsilon_p \) are each on the order of one or two centimeters or less, and may be excluded from consideration in these position calculations. The same is true for the \( mp \) term, which is difficult to model in any event. A small residual ionospheric error \( \sigma \) of 0.02 m is accounted for, however, in the protection level computations in the DFDS architecture.

Differential GPS processing accounts for differential height decorrelation between receivers and between the current and previous epoch. Since this error reduction is accomplished using dual-frequency data in previous processing stages within the DFDS architecture, all position domain smoothing may be accomplished solely using the L1 frequency. Satellite inter-frequency and inter-code biases are also assumed to cancel in the differential corrections. Therefore, the single difference for satellite \( i \) now becomes

\[
SD_{AB,1.1,i}(t) = R_{AB,i}(t) + c\Delta AB_{AD,1.1,i}(t) + N_{AB,1.1,i} + \eta_{AD,AB,1.1,i}(t) + T_{AB,1.1,i}(t)
\]  

(2)

A geometric illustration of the single difference is appropriate at this point. Let the two GPS receivers located at points A and B be separated at time \( t \) by baseline vector \( \vec{b}_{AB}(t) \), as shown in Figure 1 below.

Figure 1: Range difference \( AO \) from users at points A and B to satellite at point S and at time \( t \), with parallel lines of sight

Let the line-of-sight vectors from A and B to satellite \( i \), which is located at point S, be \( \vec{e}_A(t) \) and \( \vec{e}_B(t) \), respectively. Then the difference in length between these two vectors, which is the difference in range from each user to the satellite, may be determined using the inner product:

\[
R_{AB,i}(t) = \vec{e}_{A,i}(t) \cdot \vec{b}_{AB}(t)
\]  

(3)

In terms of line segments, range difference \( AO = AS - OS \). It is apparent that \( OS = BS \).

For mathematical convenience and as Figure 1 illustrates, use of the inner product includes the assumption that vectors \( \vec{e}_A(t) \) and \( \vec{e}_B(t) \) are parallel. In reality, vectors \( \vec{e}_A(t) \) and \( \vec{e}_B(t) \) are not parallel for non-zero baselines between A and B, so the range difference must be corrected:

\[
R_{AB,i}(t) = \vec{e}_{A,i}(t) \cdot \vec{b}_{AB}(t) - corr_{AB}(t)
\]  

(4)

In the true geometry shown in Figure 2, line segment \( \overline{AC} \) is the true range difference. Then segments \( \overline{CS} \) and \( \overline{BS} \) must be equal for \( \overline{AC} \) to be their difference. The nonlinearity correction, \( corr_{AB}(t) \), is represented by line segment \( \overline{OC} \). This length must be subtracted from range difference \( \overline{AO} \), which is calculated by the inner product, to avoid a systematic position error that increases with distance \( \overline{AB} \). The correction will be derived shortly.

Figure 2: True range difference \( AC \) from users at points A and B to satellite at point S, all at time \( t \)

At algorithm initialization, the best relative position estimate available is used for mobile point B. This is the unsmoothed differential solution for the receiver at point B, by definition an estimate of column vector \( \vec{b}_{AB}(t) \). A locally level, east-north-up coordinate frame is used. Since point A is always the origin, this formulation is equally applicable to differential systems, where A is stationary, and relative systems, where A may move as well as B. True user-to-satellite line of sight column vectors \( \vec{e}_A(t) \) and \( \vec{e}_B(t) \) are now exchanged for estimated
line of sight column vectors \( \mathbf{r}_A(t) \) and \( \mathbf{r}_B(t) \). The vector \( \mathbf{r}_B(t) \) is obtained using the estimated position of B for time \( t_1 \), which is the first GPS measurement epoch:

\[
\mathbf{r}_B(t_1) = \mathbf{r}_A(t_1) - \mathbf{b}_{AB}(t_1)
\]

(5)

The nonlinearity correction is calculated for time \( t_1 \) as in [16], and is saved for further use:

\[
corr_{AB}(t_1) = \left[ \mathbf{F}_A(t_1) - \frac{\mathbf{r}_B(t_1) \mathbf{F}_A(t_1)}{\mathbf{r}_B(t_1)} \right]
\]

(6)

The computations above are repeated for each of \( n \) satellites, and the geometry matrix \( \mathbf{H} \) is constructed for point B at time \( t_1 \), with user-to-satellite unit vector elements in the first three columns of each row and the integer value 1 in column 4. The \( i \)-th row of \( \mathbf{H} \) at time \( t_1 \) is then as follows:

\[
\mathbf{H}_{i, \ldots, t_1} = \begin{bmatrix}
    r_{B,1,i,\text{north}}(t_1) & r_{B,1,i,\text{east}}(t_1) & r_{B,1,i,\text{up}}(t_1) & 1
\end{bmatrix}
\]

(7)

At time \( t_2 \), another set of GPS measurements is taken simultaneously by the two receivers at point A and B. In this case, however, position B(\( t_2 \)) is assumed to be different than B(\( t_1 \)), relative to point A. Additionally, the position S(\( t_2 \)) of the \( i \)-th satellite has changed to S(\( t_2 \)). Geometrically, this appears as shown in Figure 3 below.

A new estimated relative position is obtained for B(\( t_2 \)), namely the code phase differential solution for \( \mathbf{b}_{AB}(t_2) \). The quantities \( \mathbf{r}_A(t_2), \mathbf{r}_B(t_2), corr_{AB}(t_2) \) and \( \mathbf{H}(t_2) \) are then computed as above. The position propagation vector \( \Delta \mathbf{b}(t_2 - t_1) \), the desired quantity in this derivation, is obtained as follows.

The single difference defined above for the \( i \)-th satellite between points A and B is here restated in terms of the AD observables in units of meters, at times \( t_1 \) and \( t_2 \):

\[
SD_{AB,i,1,1}(t_1) = AD_{A,i}(t_1) - AD_{B,i}(t_1)
\]

(8)

\[
SD_{AB,i,1,1}(t_2) = AD_{A,i}(t_2) - AD_{B,i}(t_2)
\]

(9)

A double difference quantity needed to obtain the propagation vector \( \Delta \mathbf{b}(t_2 - t_1) \) is then formed by differencing these two. Substituting, this double difference is

\[
DD_{AB,i,1,1}(t_2 - t_1) = \left( AD_{A,i}(t_2) - AD_{B,i}(t_2) \right) - \left( AD_{A,i}(t_1) - AD_{B,i}(t_1) \right)
\]

(10)

For convenience in processing, the double difference may be rearranged as follows:

\[
DD_{AB,i,1,1}(t_2 - t_1) = AD_{A,i}(t_2) - AD_{B,i}(t_2) - \left( AD_{B,i}(t_1) - AD_{A,i}(t_1) \right)
\]

(11)

The double difference is then corrected for tropospheric spatial decorrelation between reference and airborne receivers from time \( t_1 \) to time \( t_2 \):

\[
DD_{AB,i,1,1}(t_2 - t_1) = SD_A(t_2 - t_1) - SD_B(t_2 - t_1) - \left( \mathbf{r}_A(t_2 - t_1) - \mathbf{r}_B(t_2 - t_1) \right)
\]

(12)

Similarly, the double difference is corrected to remove nonlinearities not accounted for in the single difference computations:

\[
DD_{AB,i,1,1}(t_2 - t_1) =\]

(13)

Column vector \( \mathbf{dd} \) is formed with rows \( i = (1, 2, ..., n) \), each of which is a corrected double difference observable.
$DD_i$. Since the calculation is understood as being from A to B and for frequency L1, the remaining subscripts are suppressed. The matrix of partial derivatives in the position domain is calculated using unweighted least squares:

$$G(t_2) = \left( H^T(t_2)H(t_2) \right)^{-1} H^T(t_2)$$

(14)

An additional geometry correction is created using the best unsmoothed estimate of the position of the user at point B at time $t_i$:

$$L(t_2 - t_i) = [H(t_2) - H(t_i)] \left[ \hat{b}_{AB}(t_i) \right]$$

(15)

By applying this correction, the “snapshot” position propagation vector from time $t_i$ to $t_2$ for the user at point B is then obtained as follows:

$$\Delta \hat{b}(t_2 - t_i) = G(t_2) \left( \dd(t_2 - t_i) - L(t_2 - t_i) \right)$$

(16)

The scalar residual of this estimate is formed using the $(n \times n)$ identity matrix $I$ and the vector norm operation, as follows:

$$q(t_2 - t_i) = \left| (I - H(t_2)G(t_2)) \left( \dd(t_2 - t_i) - L(t_2 - t_i) \right) \right|$$

(17)

If the residual $q$ is larger than the empirically derived threshold of 0.2 m and at least six satellites are in the solution, a RAIM operation is performed by excluding the satellites one at a time. If a satisfactory subset of satellites is found, the satellite with excessive error is removed and the position propagation vector $\Delta \hat{b}$ is recalculated. Then at time $t_2$, the propagated position is created:

$$\hat{p}_{AB}(t_2) = \hat{b}_{AB}(t_i) + \Delta \hat{b}(t_2 - t_i)$$

(18)

Otherwise, no smoothed position is returned for time $t_2$, the snapshot differential solution $\hat{b}_{AB}(t_i)$ is used for propagated position $\hat{p}_{AB}(t_2)$ and the algorithm is restarted.

The significance of the expression $\Delta \hat{b}(t_2 - t_i)$ is that it is an instantaneous, very high-quality integrated velocity estimate obtained for the user at point B over the time interval $t_i$ to $t_2$. The same type of geometry corrections that hold errors in average velocity to less than 1 cm/s in [10] have been painstakingly made. Furthermore, this estimate is not derived from a filter, but it may be used as an input to other filters. For example, this estimate can be used to calibrate an inertial navigation system or smooth noisy position states. In the current derivation, it is used to smooth the differential position derived from pseudorange measurements at time $t_2$. The smoothed position is thus created using an alpha filter:

$$\hat{\hat{s}}_{AB}(t_2) = \alpha \hat{b}_{AB}(t_2) + (1 - \alpha)\hat{p}_{AB}(t_2)$$

(19)

Here smoothing weight $\alpha = 0.005$ is used, corresponding to an empirically derived smoothing time constant of 200 s for the 1-s sample rate. The unsmoothed differential code phase solution, which is accurate but also contains step changes when satellites enter and leave the constellation, is smoothed by the low-noise carrier phase update. Put a different way, the longer-term error of the ultra low-noise carrier phase propagation method is kept very close to zero.

In the algorithm’s next iteration, the $t_i$ values just computed become $t_1$ values. The smoothed differential position $\hat{\hat{s}}_{AB}(t_2)$ from the most recent iteration replaces code phase estimate $\hat{b}_{AB}(t_1)$ in the new iteration. This continues as long as continuous carrier phase tracking exists for at least four satellites. If this condition is not met, then the algorithm is restarted.

**Baseline for Comparison: LAAS 100-s Range Domain Smoothing**

The LAAS code phase observable in ground and air is smoothed using carrier phase data, as suggested in [1]. The equation is reproduced here for comparison. The smoothed pseudorange is

$$P_n = \alpha P + (1 - \alpha) \left( P_{n-1} + \frac{\lambda}{2\pi} (\phi_n - \phi_{n-1}) \right)$$

(20)

where

- $P_n$ = smoothed pseudorange (m)
- $P_{n-1}$ = previous smoothed pseudorange (m)
- $P$ = raw pseudorange measurement (m, see [1] for more details)
- $\lambda$ = L1 GPS wavelength (m)
- $\phi_n$ = carrier phase (AD) (rad)
- $\phi_{n-1}$ = previous carrier phase (AD) (rad)
- $\alpha$ = filter weighting function, equal to sample interval divided by $\tau = 100$ s
FLIGHT TEST DESCRIPTION

General Description

Six 150-s straight-in approaches were flown at the Ohio University (OU) airport in Albany, Ohio in the late morning and early afternoon of October 29, 2008. The test period constituted 1 hour, 46 minutes and 18 seconds of data, or 6,378 consecutive GPS epochs. The test aircraft was the OU Avionics Engineering Center’s King Air C90SE, as shown in Figure 4 during a different test event.

Equipment Configuration

The airborne system included two NovAtel OEM-4 L1L2W GPS receivers connected to one dual frequency GPS patch antenna (the S67-1575-14 model by Sensor System Inc.), a laptop computer and a Harris VHF data broadcast (VDB) receiver connected to a wire antenna on the underside of the aircraft. The location of the GPS antenna on the aircraft is indicated in Figure 4. The computation engine was Matlab v2006b in Windows XP. Real-time interfaces used custom Matlab code and serial-to-TCP/IP converters.

NovAtel binary GPS data from the airborne receivers, data received over the VDB and differential positions—smoothed and unsmoothed—computed aboard the aircraft were recorded every second. Of these, however, only the recorded raw GPS data is used in this analysis. No WAAS data is used. The reference station consisted of four NovAtel OEM-4 L1L2 receivers connected to a single NovAtel Pinwheel 600 L1L2 antenna, a desktop computer running Matlab on Windows XP, and a Harris VDB transmitter broadcasting data at 114.450 MHz at 70W effective radiated power with an elliptically polarized dipole antenna. The same Matlab computational engine used in the airborne system was used in the reference station, and six TCP/IP-to-serial converters were used in parallel in real time, controlled from Matlab. NovAtel binary data was recorded at 1 s intervals.

As has been stated, no smoothing is applied to the ground data used in the airborne CPDS algorithm. However, in both air and ground receivers in both LAAS and CPDS cases, a standard value of 20 for the NovAtel CSMOOTH parameter is used. In this case, every third sample provided by the receiver, here at a sampling period of one second, is statistically independent. Thus a preliminary, basic smoothing operation is performed on all data in the receiver. This is considered negligible when comparing the CPDS to LAAS results because it is applied to both methods equally, and the CPDS and LAAS smoothing time constants are significantly longer than that used internally in the receiver.

Flight Tracks and Satellite Geometries

The ground track for the flight test period appears in Figure 5, and the flight profile for each of the six 150-s approaches appears in Figure 6. On each approach, the pilot flew a standard Instrument Landing System (ILS) approach path with a three-degree glideslope, using ILS guidance. Although DGPS data was present in real time in the aircraft, it was not presented to the pilot or used for guidance. Two approximately ten-minute periods, one just after approach #2 and the other just after approach #4, were spent on the ground taxiways for crew change-outs.
Figure 6: King Air precision approach segments for October 29, 2008 flight

The number of satellites, the vertical dilution of precision (VDOP) and the horizontal dilution of precision (HDOP) satellite geometry indicators all appear in Figure 7 for the reference station. The airborne values are nearly the same.

Figure 7: Number of Satellites, VDOP and HDOP for October 29, 2008 flight

Truth Processing

Airborne truth values were obtained using Waypoint Consulting’s GrafNav software, which provides a post-processing kinematic solution from recorded reference station and airborne data. All but 13 of the 6,378 GPS epochs in this data set yielded the desired ambiguity resolved solution. None of those 13 epochs is consecutive, and each provides a “next best” stable float solution. Advertised GrafNav accuracies for ambiguity resolved solutions in properly sited, dual-frequency, kinematic receiver installations similar to this one are 2 cm + 1 mm/km for baselines of 0 to 5 km, and 5 cm + 2 mm/km for baselines of 5 to 35 km.

FLIGHT TEST RESULTS

Results for Entire Flight

Analysis of the flight data shows that the CPDS algorithm yields significant reductions in all measures of navigation system error (NSE) when compared to the baseline LAAS 100-s smoothing algorithm. Over the entire data set, 95% vertical error is reduced from 0.80 m to 0.55 m. Similarly, 95% horizontal error is reduced from 0.41 m to 0.40 m over the same period.

Figure 8 and Figure 9 illustrate these error performance improvements. The time intervals of the six approaches are superimposed over the data sets in the plot using short line segments. The spike in both vertical and horizontal errors at the 4005th iteration is due to a CPDS filter reset (i.e., a reset to the unsmoothed differential solution). This is caused by more than one satellite having a large residual in the simple single fault exclusion check described in Equations (17) and (18) above. This condition is likely due to simultaneous cycle slips on two satellites.

The results for position error are summarized in Table 1 below. The CPDS algorithm provides a significant reduction in error compared to the LAAS 100-s smoothing algorithm, and is free of code-carrier divergence. However, balanced against this benefit is the possibility of error spikes due to simultaneous cycle slips such as those described above. A more sophisticated fault detection and exclusion will greatly reduce the likelihood of these. Although not desirable from a LAAS aircraft equipage point of view, inertial data could also be used to avoid smoothing filter resets.

<table>
<thead>
<tr>
<th>Measure</th>
<th>LAAS (95%), m</th>
<th>CPDS (95%), m</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical NSE</td>
<td>0.80</td>
<td>0.55</td>
<td>31.6%</td>
</tr>
<tr>
<td>Horizontal NSE</td>
<td>0.41</td>
<td>0.40</td>
<td>3.1%</td>
</tr>
</tbody>
</table>
Figure 8: Vertical accuracy and 95% navigation system error performance for LAAS (with 1 reference receiver) and CPDS algorithm, entire King Air flight test.

Figure 9: Horizontal accuracy and 95% navigation system error performance for LAAS (with 1 reference receiver) and CPDS algorithm, entire King Air flight test.
Results for Six Approaches

The improvement in position error performance during the six 150-s approaches is also dramatic, as shown in Figure 10, Table 2 and Table 3 for vertical error; and Figure 11, Table 4 and Table 5 for horizontal error.

Figure 10: Vertical NSE performance for the LAAS architecture (top) and the CPDS algorithm (bottom), six 150-s approaches

Figure 11: Horizontal NSE performance for the LAAS architecture (top) and the CPDS algorithm (bottom), six 150-s approaches

Table 2: LAAS Vertical NSE statistics for six approaches (m)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.02</td>
<td>-0.06</td>
<td>0.74</td>
<td>0.28</td>
<td>0.39</td>
<td>0.10</td>
</tr>
<tr>
<td>Max</td>
<td>0.21</td>
<td>0.32</td>
<td>1.07</td>
<td>0.36</td>
<td>0.45</td>
<td>0.27</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.10</td>
<td>0.16</td>
<td>0.15</td>
<td>0.05</td>
<td>0.03</td>
<td>0.12</td>
</tr>
<tr>
<td>Rms</td>
<td>0.10</td>
<td>0.17</td>
<td>0.76</td>
<td>0.28</td>
<td>0.39</td>
<td>0.16</td>
</tr>
<tr>
<td>95%</td>
<td>0.19</td>
<td>0.30</td>
<td>1.04</td>
<td>0.34</td>
<td>0.44</td>
<td>0.26</td>
</tr>
<tr>
<td>$</td>
<td>\mu + 2\sigma</td>
<td>$</td>
<td>0.21</td>
<td>0.38</td>
<td>1.05</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Table 3: CPDS Vertical NSE statistics for six approaches (m)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.05</td>
<td>-0.09</td>
<td>0.46</td>
<td>0.23</td>
<td>0.32</td>
<td>0.05</td>
</tr>
<tr>
<td>Max</td>
<td>0.13</td>
<td>0.23</td>
<td>0.57</td>
<td>0.30</td>
<td>0.39</td>
<td>0.18</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.06</td>
<td>0.08</td>
<td>0.04</td>
<td>0.05</td>
<td>0.04</td>
<td>0.09</td>
</tr>
<tr>
<td>Rms</td>
<td>0.08</td>
<td>0.12</td>
<td>0.46</td>
<td>0.23</td>
<td>0.32</td>
<td>0.10</td>
</tr>
<tr>
<td>95%</td>
<td>0.12</td>
<td>0.22</td>
<td>0.55</td>
<td>0.28</td>
<td>0.38</td>
<td>0.18</td>
</tr>
<tr>
<td>$</td>
<td>\mu + 2\sigma</td>
<td>$</td>
<td>0.17</td>
<td>0.26</td>
<td>0.54</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 4: LAAS Horizontal NSE statistics for six approaches (m)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.12</td>
<td>0.15</td>
<td>0.30</td>
<td>0.14</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>Max</td>
<td>0.19</td>
<td>0.18</td>
<td>0.45</td>
<td>0.18</td>
<td>0.24</td>
<td>0.28</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.05</td>
<td>0.02</td>
<td>0.05</td>
<td>0.02</td>
<td>0.03</td>
<td>0.07</td>
</tr>
<tr>
<td>Rms</td>
<td>0.13</td>
<td>0.15</td>
<td>0.30</td>
<td>0.15</td>
<td>0.21</td>
<td>0.20</td>
</tr>
<tr>
<td>95%</td>
<td>0.18</td>
<td>0.17</td>
<td>0.40</td>
<td>0.17</td>
<td>0.24</td>
<td>0.28</td>
</tr>
<tr>
<td>$</td>
<td>\mu + 2\sigma</td>
<td>$</td>
<td>0.22</td>
<td>0.19</td>
<td>0.39</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 5: CPDS Horizontal NSE statistics for six approaches (m)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.09</td>
<td>0.07</td>
<td>0.25</td>
<td>0.14</td>
<td>0.18</td>
<td>0.23</td>
</tr>
<tr>
<td>Max</td>
<td>0.16</td>
<td>0.10</td>
<td>0.29</td>
<td>0.16</td>
<td>0.20</td>
<td>0.28</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.01</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>Rms</td>
<td>0.10</td>
<td>0.07</td>
<td>0.25</td>
<td>0.14</td>
<td>0.18</td>
<td>0.23</td>
</tr>
<tr>
<td>95%</td>
<td>0.15</td>
<td>0.10</td>
<td>0.28</td>
<td>0.16</td>
<td>0.20</td>
<td>0.28</td>
</tr>
<tr>
<td>$</td>
<td>\mu + 2\sigma</td>
<td>$</td>
<td>0.17</td>
<td>0.12</td>
<td>0.32</td>
<td>0.16</td>
</tr>
</tbody>
</table>

The CPDS algorithm does smooth the signal significantly more than the LAAS baseline architecture according to every statistical measure. The 150-s approach segments used in statistical analysis varied in length between 8 and 9.5 km, depending on aircraft speed. Only the final 8 km segments are shown below. Standard deviations are formed from unbiased variance by dividing the sum of squares (minus the mean) by $n - 1$ before taking the square root. Rms values are formed by dividing the sum of squares simply by $n$ as in the conventional rms definition. In each case, there are $n = 150$ GPS epochs in each approach.

Summary statistics for vertical and horizontal errors for both architectures, and the percent change between the architectures, are provided in Table 6. Here the overall combined mean of the approaches has been obtained simply by averaging the means of all the approaches. This is not meant to “average out” errors across approaches, but rather to indicate any bias that may persist across all the approaches. Six
approaches are not sufficient for a statistically sound inference on this hypothesis, but they do serve as an initial indication of the performance of the CPDS algorithm. The combined maximum value is the maximum of all GPS epochs over all approaches. Assuming for the moment a common variance between all six approaches, the combined standard deviation is derived as the square root of the pooled estimator for this common variance, as in [17], dividing by \( m - 1 \) with \( m = 6 \) approaches. The combined 95% and \( |\mu + 2\sigma| \) statistics are formed in the same way. The combined rms statistic is formed by applying the conventional rms formula to the rms values for each approach and dividing by \( m \).

**Table 6: Summary NSE statistics for six approaches, LAAS vs. CPDS (m)**

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Statistic</th>
<th>LAAS</th>
<th>CPDS</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>Mean</td>
<td>0.24</td>
<td>0.15</td>
<td>38%</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>1.07</td>
<td>0.57</td>
<td>47%</td>
</tr>
<tr>
<td></td>
<td>Std Dev</td>
<td>0.13</td>
<td>0.07</td>
<td>45%</td>
</tr>
<tr>
<td></td>
<td>Rms</td>
<td>0.38</td>
<td>0.26</td>
<td>32%</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>0.56</td>
<td>0.35</td>
<td>39%</td>
</tr>
<tr>
<td></td>
<td>(</td>
<td>\mu + 2\sigma</td>
<td>)</td>
<td>0.59</td>
</tr>
<tr>
<td>Horizontal</td>
<td>Mean</td>
<td>0.18</td>
<td>0.16</td>
<td>13%</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.45</td>
<td>0.29</td>
<td>36%</td>
</tr>
<tr>
<td></td>
<td>Std Dev</td>
<td>0.05</td>
<td>0.03</td>
<td>31%</td>
</tr>
<tr>
<td></td>
<td>Rms</td>
<td>0.20</td>
<td>0.18</td>
<td>11%</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>0.28</td>
<td>0.22</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>(</td>
<td>\mu + 2\sigma</td>
<td>)</td>
<td>0.30</td>
</tr>
</tbody>
</table>

**SUMMARY AND CONCLUSIONS**

The single-frequency CPDS algorithm significantly enhances accuracy performance for differential GPS aircraft precision approach and landing operations as compared to a comparable LAAS architecture. Results over the entire data set show an improvement in vertical accuracy from 0.80 m to 0.55 m (95%), and in horizontal accuracy from 0.41 m to 0.40 m (95%), reductions by 31% and 3%, respectively. During the six 150-s approaches, the mean vertical error is reduced from 0.24 m to 0.15 m, the maximum absolute vertical error from 1.07 m to 0.57 m, the standard deviation from 0.13 m to 0.07 m, the rms error from 0.38 m to 0.26 m, and the 95% error from 0.56 m to 0.35 m. Similarly, the mean horizontal error magnitude is reduced from 0.18 m to 0.16 m, the maximum horizontal error magnitude from 0.45 m to 0.29 m, the standard deviation from 0.05 m to 0.03 m, the rms error from 0.20 m to 0.18 m, and the 95% error from 0.28 m to 0.22 m.

The benefits of the CPDS algorithm include establishment of a virtual “rail in the sky” to define the aircraft approach path with no large jumps in position (assuming continuous carrier phase tracking on 4 or more satellites), instant use of satellite’s carrier phase data when available, and the ability to patch pseudoranges following a temporary loss of lock. The considerable reductions in error described above may well be worth the implementation costs of changing air processing algorithms in future version of the LAAS specification. If implemented, however, a more sophisticated cycle slip detection and exclusion algorithm will be needed.

**ACKNOWLEDGMENTS**

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**REFERENCES**


