The following review sheet will help you prepare for the first midterm. This should not be the only material you study, however, but this sheet will represent between 80% and 90% of the exam. The remaining portion of the midterm will be word problems.

1. Find the center and radius of the circle given by the equation \( x^2 + y^2 - 6x + 10y - 15 = 0 \).
   \[ \text{ANSWER} \] \( C = (3, -5) \) and \( r = 7 \)

2. Find the lines \( l_1, l_2, \) and \( l_3 \) where \( l_1 \) is the line joining \( P(2, 3) \) and \( Q(-1, -3) \), \( l_2 \) is perpendicular to \( l_1 \) and passes through \( P \), and \( l_3 \) is parallel to \( l_1 \) and passes through the point \( (2, 0) \).
   \[ \text{ANSWER} \] \( l_1 = 2x - 1, \ l_2 = -\frac{1}{2}x + 4, \ \text{and} \ l_3 = 2x - 4 \)

3. Find the point(s) of intersection for the line \( y = -3x + 2 \) and the parabola \( y = -2x^2 - 4x + 5 \).
   \[ \text{ANSWER} \] \( p_1 = (-1.5, 6.5) \) and \( p_2 = (1, -1) \)

4. Show algebraically that \( \lfloor x - 1 \rfloor - 2 = \lfloor x + 3 \rfloor - 6 \).
   \[ \text{ANSWER} \] \( \lfloor x - 1 \rfloor - 2 = \lfloor x \rfloor - 1 - 2 = \lfloor x \rfloor - 3 = \lfloor x \rfloor + 3 - 6 = \lfloor x + 3 \rfloor - 6 \)

5. Show that \( f(x) = \frac{5}{3}x^3 + \sqrt{x} + 2 \) is one-to-one.
   \[ \text{Ask for demonstration if needed} \]

6. Show (algebraically or via example) that \( f(x) = 6x^2 + 48x - 4 \) is not one-to-one.
   \[ \text{ANSWER} \] \( f(-5) = -94 = f(-3) \)

7. Show that \( f(x) = 3x^3 + 2x \) is one-to-one.
   \[ \text{Ask for demonstration if needed} \]

8. Find the inverse function, \( f^{-1}(x) \), of \( f(x) = \sqrt{2x} + 3 \).
   \[ \text{ANSWER} \] \( f^{-1}(x) = \frac{x - 3}{\sqrt{2}} \)

9. Determine a domain such that \( f(x) = 2x^2 - 12x + 4 \) is one-to-one and find \( f^{-1} \) on that domain.
   \[ \text{ANSWER} \] Restrict \( f(x) \) to \( D = [3, \infty) \). Then, on \( D \), \( f^{-1}(x) = 3 + \sqrt{\frac{x}{2} + 7} \)

10. Let \( f(x) = \frac{3}{x} \) and \( g(x) = \sqrt{x^2 - 1} \). What is the domain of:
    (a) \( (f + g)(x) \)
    (b) \( \left(\frac{f}{g}\right)(x) \)
    (c) \( f(g(x)) \)
    (d) \( g(f(x)) - f(x) \)
   \[ \text{ANSWER} \] (a) \( (-\infty, -1) \cup [1, \infty) \) (b) \( (-\infty, -1) \cup (1, \infty) \) (c) \( (-\infty, -1) \cup (1, \infty) \) (d) \( [-3, 0) \cup (0, 3] \)
(11) Write the function \( f(x) = \sqrt{\frac{x^3 - 7x^2 + 3x + 1}{x^3 + 2x - 2}} \) as a composition of three (3) functions.

**[ANSWER]** One possible solution: \( g(x) = \sqrt{x}, \ h(x) = \frac{1}{x}, \) and \( i(x) = \frac{x^2 + 2x - 2}{x^3 - 7x^2 + 3x + 1} \)

(12) Write each interval as an inequality, or each inequality as an interval:

(a) \((-7, \infty)\)
(b) \(x > 4\)
(c) \(-3 < x \leq 4\)
(d) \([3, \pi)\)

**[ANSWER]**
(a) \(-7 < x\) (b) \((4, \infty)\) (c) \((-3, 4]\) (d) \(3 \leq x < \pi\)

(13) Find all values of \(x\) such that \(f(x) = x^2 - 3x - 4\) is less than 0.

**[ANSWER]** \((-1, 4)\)

(14) If the only roots of \(P(x)\) are at \(x = 3\) (which has a multiplicity of 2), \(x = 0\) (with a multiplicity of 1), and \(x = -2\) (with a multiplicity of 3), and we know that \(P(2) = 160\), find \(P(x)\).

**[ANSWER]** \(P(x) = \frac{5}{4}x(x - 3)^2(x + 2)^3\)

(15) As \(x \to \infty\), what value does \(f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0\) approach if \(a_n < 0\)?

**[ANSWER]** \(-\infty\)

(16) Is the remainder 0 when \(D(x) = x - 2\) is divided into \(P(x) = 3x^4 - 2x^2 + 4\)?

**[ANSWER]** No.

(17) Find all roots of \(P(x) = 3x^3 - 2x^2 + 4x - 5\).

**[ANSWER]** \(x = 1\) is the only root

(18) Find all possible rational roots of \(P(x) = \frac{3}{4}x^4 - \sqrt{3}x^3 + 7283x - \frac{4}{3}\).

**[ANSWER]**
\[\%\text{divisors}(16)\%\text{divisors}(9) = \{ \pm 16, \pm 8, \pm 4, \pm 2, \pm 1, \pm \frac{16}{3}, \pm \frac{8}{3}, \pm \frac{4}{3}, \pm \frac{2}{3}, \pm \frac{1}{3}, \pm \frac{16}{9}, \pm \frac{8}{9}, \pm \frac{4}{9}, \pm \frac{2}{9}, \pm \frac{1}{9} \}\]

(19) Sketch the following functions on graphs with \(-10 \leq x \leq 10\) and \(-10 \leq y \leq 10\) ((make sure to label important details such as X-intercepts, Y-intercepts, vertical/horizontal/slant asymptotes, etc.). If important information (such as an X-intercept or asymptote) exists outside of the \([-10,10] \times [-10,10]\) graph, indicate it below.
the graph.

\[
\begin{align*}
(a) & \quad \frac{1}{x^2 + 2x - 3} \\
(b) & \quad \frac{x^2 - 2x - 3}{x^2 - 1} \\
(c) & \quad 3[x - 2] + 2|x^2 - 3| \\
(d) & \quad \frac{x^2 - 4}{x^2 + 5x} \\
(e) & \quad \sqrt{\frac{x - 1}{x + 2}}
\end{align*}
\]

See Graphs on Last Pages

(20) Provide a rational function, \( f(x) \) that satisfies all of the following conditions:

\[
\begin{align*}
(a) & \quad f(x) \to -\infty \text{ as } x \to 3^- \\
(b) & \quad f(x) \to \infty \text{ as } x \to 3^+ \\
(c) & \quad f(x) \text{ has a horizontal asymptote at } y = 2 \\
(d) & \quad f(1) = 0
\end{align*}
\]

[ANSWER] \( f(x) = \frac{2x-2}{x-3} \)

(21) What is the domain of \( f(x) = \frac{x^2-2}{\sqrt{(x-3)x^3}} \)?

[ANSWER] \( D(f(x)) = (-\infty, 0) \cup (2, \infty) \)
\[ f(x) = \frac{1}{x^2 + 2x - 3} \]
$$g(x) = \frac{x^2 - 2x - 3}{x^2 - 1}$$
\[ h(x) = 3 \left| x - 2 \right| + 2 \left| x^2 - 3 \right| \]
\[ p(x) = \frac{x^2 - 4}{x^2 + 5x} \]
\[ q(x) = \sqrt{\frac{x - 1}{x + 2}} \]