Simple Interest

There are two types of interest: *Simple (paid on principal only)* and *Compound (paid on principal and past interest)*. We will discuss simple interest this chapter, and compound interest in chapter 13.

**Objectives**

1. Find simple interest
2. Find interest for less than a year
3. Find principal if given rate and time
4. Find rate if given principal and time
5. Find time if given principal and time

**Definitions**

Interest: price paid for borrowing money

Principal: amount of money borrowed

Rate: the percent of interest charged

**WHEN REFERENCING TIME, WE WILL WORK IN YEARS**

**Objective 1: Find Simple Interest**

The amount of interest to be paid is found using a simple formula:

$$I = P \cdot R \cdot T,$$

where I is the interest, P is the principal, R is the rate, and T is the time (in years). Since we are to use years when working with T, we must always remember to convert days/months/etc. to a fraction of years.
Example 1

In addition to using some of their own money, Gilbert Construction must also borrow some. They need to borrow $60,000 to build an 1800-square-foot home, but must decide between two lending options. Option A would be a short-term, 1 year loan with a rate of 8%, while Option B is a short-term, one-and-a-half year loan at a rate of 8.5%. Which loan should she take?

Solution

To determine which choice is better, we must use the formula we have for simple interest. Thus, for Option A we have

\[ I = PRT \]
\[ = 60,000 \times 0.08 \times 1 \]
\[ = 4,800. \]

That is, with Option A, the Gilbert Construction Company would pay $4,800 in addition to having to pay back the $60,000.

Now, for Option B, we see that the interest charged would be

\[ I = PRT \]
\[ = 60,000 \times 0.085 \times 1.5 \]
\[ = 7,650. \]

Gilbert Construction chooses to go with Option A.

In Example 1 we saw how to use our formula for interest to determine how much it would cost to borrow a given amount of money. Now, let us do the same thing with irregular time denominations.

Objective 2: Finding Interest for Less Than a Year

Since there are 12 months in a year, one surefire way to express time in years when you are given time in months is

\[ T = \frac{x}{12}. \]
where \( x \) is the number of months given. In our next example, we will use this very method.

**Example 2**

Jodi needs $2800 to buy a new car. Her uncle offers to loan her the money with a simple rate of 7%. Find the interest paid if the period is (a) 9 months; and, (b) 13 months.

**Solution**

(a) With a period of 9 months, Jodi will pay

\[
I = PRT
\]

\[
= 2800 \times 0.07 \times \frac{9}{12}
\]

\[
= $147
\]

in interest.

(b) When the period is 13 months, she pays

\[
I = PRT
\]

\[
= 2800 \times 0.07 \times \frac{13}{12}
\]

\[
= $212.33
\]

in interest.

**Objective 3: Find Principal if Rate and Time are Given**

As with any equation, we can solve for whichever variable we choose to in order to have an expression that yields to us our desired result. With our interest formula, if we want to obtain an expression for \( P \) (the principal), we simply divide both sides by \( RT \) to obtain

\[
P = \frac{I}{RT}.
\]

Likewise, we also have that
\[ R = \frac{I}{PT} \]

and

\[ T = \frac{I}{PR} \]

You do not need to remember all of these variations, just \( I = PRT \) and use algebra to find the expression you need.

**Example 3**

Gilbert Construction borrows \( P \) dollars 10.5\% for 10 months to build a home. Find \( P \) if the interest is $8000.

**Solution**

Using our expression that we found above, we have

\[
P = \frac{I}{RT} = \frac{8000}{.105 \times \frac{10}{12}} = $91,428.57.
\]

**Objective 4: Find Rate if Given Principal and Time**

**Example 4**

After a hefty down payment, Tony borrows $9000 from his local bank to finance a used Honda Accord. Find the interest rate if the loan was for 10 months and the interest was $750.

**Solution**
Using the variation of our interest formula we have

\[
R = \frac{I}{PT} = \frac{750}{9000 \times \frac{10}{12}} = .10 = 10\%.
\]

NOTE: To avoid errors, do not round any result until the very end.

Objective 5: Find Time if Given Principal and Rate

Example 5

All-American Savings and Loan wants to make $500 off a $4000, 9.25% loan. How long should they make the loan for?

Solution

The loan should be for

\[
T = \frac{I}{PR} = \frac{500}{4000 \times 0.0925} = \frac{500}{370} = 17 \text{ months}.
\]

If you punch \(\frac{500}{370}\) into your calculator, you will get an answer of 1.351351.... This means that it is nearly one and a half years, but to find how many months, we need to multiply that answer by 12. Doing this, you will get an answer of 16.216.... Why did we say that the loan should be for 17 months then? Shouldn’t 16.216... round down to 16 months? If they make the loan for 16 months, they will fall short of their desired $500 in interest revenue. Thus, rounding up to 17 months guarantees them their goal.
Simple Interest for a given number of days

Objectives
1. Find the number of days from one date to another using a table
2. Find the number of days from one date to another using the number of days in the months
3. Find exact and ordinary interest

Objective 1: Find the number of days from one date to another using a table
*The table we use can be found on page 411 (in the 8th edition).*
*Do Example 1 in class, and do a second example using birthdays*

Find the number of days from one date to another using the number of days in the months
As you all know, there are 3 possibilities for the number of days in a month depending on whether or not it is a leap year or not.

Example 2
Find the number of days from February 23 to July 2 if it is not a leap year.

Solution
There are 5 days left in February, 31 in March, 30 in April, 30 in June, and then 2 into July. Thus, there are 98 days between them.

*Walk through Example 3, detailing modulus addition*

Objective 3: Find Exact and Ordinary Interest
Recall that when we were dealing loans in months, we simply divided the number of months by 12 to get T. With days, it is not so easy. Intuition would lead one to think the only difference would be we use days on top and divide by 365, but this would only give
us exact interest. You see, 365 has as its proper divisors: 1, 5, and 73. If we consider the (close) number 360, we now have 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, and 180 as proper divisors. For this reason, lenders have long used ordinary interest (using 360 days) to simplify their calculations. Most lending agencies use exact interest, but some still do ordinary out of tradition and the fact that it produces a greater dollar amount. Let’s see what we mean with an example.

**Example**

on April 6, 1949, the optimistically thinking National Loans and Mortgages lent out $275,000 to an emerging defense company. Not knowing much about loan types, the CEO of the company agreed to a 5.75% ordinary loan, terminating on October 1, 2000. How much money did the CEO give away by not demanding an Exact loan?

**Solution**

First, we need to separate the whole years into two categories: leap and not-leap. All that we really need to know is that 1948 was a leap year. Thus, we have 12 leap years, and 37 not leap years. Thus, for the whole years we have $12 \times 366 + 37 \times 365 = 17,897$ days. Now, all we have to do is account for the partial years and we will have our total number of days. There are 269 days left in 1949 after April 6, and October 1 is the 274 day of 2000. Thus, the total number of days in the loan is 18,440. Using ordinary interest, the defense company pays out

$$I = PRT$$

$$= 275,000 \times 0.0575 \times \frac{18,440}{360}$$

$$= 809,951.39$$

Had he have chosen an exact loan, he would have only paid $798,856.16—a savings of over $11,000! Still, notice that the company is paying back over one million dollars because of interest.
Objectives

1. Find maturity value
2. Find principal if given maturity value, time, and rate
3. Find rate if given principal, maturity value, and time
4. Find time if given maturity value, principal, and rate

Objective 1: Find Maturity Value

Definitions

Maturity Value: The future value of the loan, with interest included. Its formula is \( M = P + I \).
Maturity Date: The date the loan is paid off.
Present Value: The principal (borrowed amount).

Example 1

Jim Wilcox would like to remodel his small bookstore so that he can serve customers coffee and allow them to sit and browse. To remodel the store he borrows $7200 for 21 months at 9.25% interest. Find the interest due on the loan and the maturity value.

Solution

First, we use our formula for interest to find how much interest will accrue after 21 months.

\[
I = PRT \\
= 7200 \times 0.0925 \times \frac{21}{12} \\
= $1165.50.
\]

Finally, we add the Present Value of $7200 and interest of $1165.50 to conclude that the Maturity Value is $8365.50.

*An alternate formula for maturity value is \( M = P(1 + RT) \). Use this formula to do

Example 2.*

Objective 2: Find Principal if Given Maturity Value, Time, and Rate

Using the alternate formula for the Maturity Value, we have a way to find the principal
value if we know maturity, rate, and time:

\[ P = \frac{M}{1 + RT}. \]

**Example 3**

Find the principal that would produce a maturity value of $15,300 in 4 months at 6% interest.

**Solution**

Using the formula above, we have that

\[ P = \frac{M}{1 + RT} = \frac{15,300}{1 + (0.06 \times 4/12)} = \frac{15,300}{1 + 0.02} = \frac{15,300}{1.02} = \$15,000. \]

**Objective 3: Find Rate if Given Principal, Maturity Value, and Time**

Again, we can rework the formula for Maturity Value to have that

\[ R = \frac{M - P}{PT}. \]

**Example 4**

Lin Pao invests a principal of $7200 and receives a maturity value of $7540 in 200 days. Find the interest rate.

**Solution**
Using the formula above,

\[ R = \frac{M - P}{PT} = \frac{7540 - 7200}{7200 \times \frac{200}{360}} = 0.085. \]

Notice that \( M - P = I \), so we can substitute the numerator of the expression for \( R \) with the interest accrued.

**Objective 4: Find Time if Given Maturity Value, Principal, and Rate**

Notice in our expression of \( R \) that we can multiply through by \( T \), then divide through by \( R \) to get

\[ T = \frac{M - P}{PR}. \]

If we need Time in days, all we need to do is to use

\[ T = \frac{360 \times I}{PR}. \]

**Example 5**

City Lights, INC. borrowed $18,250 at 10.125% interest for the construction of new signs and agreed to repay $19,687.19. Find the time in days and round to the nearest day if necessary.

**Solution**

Using the formula above, we have

\[ T = \frac{360 \times 1437.19}{18,250 \times 0.10125} = \frac{360 \times 1437.19}{1847.81} = 280 \text{ days}. \]
Inflation and the Time Value of Money

Objectives

1. Define inflation and the Consumer Price Index
2. Understand the time value of money
3. Define present value and future value
4. Calculate present and future values using simple interest
5. Find present value for a given maturity value
6. Find present value after a loan is made

Objective 1: Inflation and the CPI

Definitions

Inflation: A rise in the general price level of goods and services.
Consumer Price Index (CPI): Measure the average change in prices from one year to the next for a common bundle of goods and services bought by the average consumer on a regular basis. It can be used to track inflation.

Example 1

Inflation from one year to the next was 4.8% as measured by the CPI. First, find the effect of the increase on a family with an income and budget of $19,800 and also whether they gain or lose buying power if they realize a 2% pay increase.

Solution

First, we determine how inflation effects their budget:

\[
Budget\ Needed = CurrentBudget \times (1 + Inflation)
\]
\[
= 19,800 \times 1.048
\]
\[
= 20,750.40.
\]

Finally, we determine what their actual budget will be:

\[
Budget\ Actualized = CurrentBudget \times (1 + PayIncrease)
\]
\[
= 19,800 \times 1.02
\]
\[
= 20,196.
\]
Thus, the family has lost over $500 in buying power, even though they had a pay increase.

**Objective 2: Understanding the Time Value of Money**

*Discuss the definition and context in class*

**Objective 3: Define Present Value and Future Value**

\[
Future\ Value = P(1 + RT)
\]

\[
Present\ Value = \frac{M}{(1 + RT)}
\]

**Objective 4: Calculate Present and Future Values Using Simple Interest**

**Example 2**

Joan Waters loans her inheritance of $28,500 to her aunt at 9.25% simple interest on May 30. Find the future value of this amount on February 4 of the following year. Round to the nearest cent.

**Solution**

First we find the days between the loan origination and termination to be 250. How?

Next, we use our formula for maturity value to conclude that

\[
M = P(1 + RT)
\]

\[
= 28,500[1 + (0.0925 \times \frac{250}{360})]
\]

\[
= $30,330.73.
\]

*Go over Example 3 in class*

*For Objectives 5 and 6, discuss the process and why we do it. Work through the examples, being ready to answer questions.*